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QUAD DIFFERENCE LABELLING ON SOME GRAPHS

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ABSTRACT – A new labelling and a new graph called quad difference labelling and quad difference graph is defined. Let G be a graph. G is said to be quad difference labelling if there exist $f: V(G) \to \{0, 1, 2, \dots p-1\}$ such that the edge set of G has assigned a weight defined by the absolute quad difference of its end vertices, the resulting weights are distinct. We investigate quad difference labelling for some graphs. The approach will be to summarize them in different flavour and possible different labelling.

Keywords – Square difference, Cube difference, Quad difference, Labelling, difference labelling

I. INTRODUCTION

Graphs are one of the prime objects of study in discrete mathematics. In general, a graph is represented by set of nodes connected by arcs. Graphs are therefore mathematical structures used to model pairwise relations between objects. They are found on road maps, consellations, when constructing schemes and drawings.

Graph labelling is one of the fascinating area of graph theory with wide ranging application. Graph labelling was first introduced in 1960's. Labelled graph serve as useful methods for a circuit design, coding theory, communication network addressing, data base management, data mining etc., An enormous body of literature has grown around graph labelling in the last four decades.

The concept of cube difference labeling was introduced by J.Shiama[6]. J.Shiama proved that the following graphs paths, cycle, stars and trees admits cube difference labeling. Sharon Philomena.V[5] proved that the Square and cube difference labeling of graphs.

All the graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) be the vertex set and edge set of the graph respectively. We introduced the concept of quad difference labelling and proved that some graph admits this kind of labeling. Also we will discuss about some important theorems and examples based on those theorems.

II. BASIC TERMINOLOGY

Definition : 2.1 A *Labelled graph* is a graph whose vertices are each assigned an element from a set of symbols(letters, usually, but this is unimportant). The important thing to note is that the vertices can be distinguished one from another.

Definition :2.2 *A Star graph* S_k is defined to be the tree of order k with maximum diameter 2; in which case a star $k > 2$ has k-1 leaves.

Definition: 2.3 *A fan graph* $F_{m,n}$ is defined as the graph join $\overline{K}_m + P_n$, *where* $\overline{K}_m$ is the empty graph on m nodes and P_n is the path on nodes.

Definition: 2.4 *A Coconut tree* $CT(m, n)$ is the graph obtained from the path P_m by appending n new pendent edges at an end vertex of P_m.

Definition:2.5 A *shell graph* is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. Shell graphs are denoted as $C_{(n,n-3)}$. A shell graph S_n is also called fan F_{n-1}.

Definition: 2.6 Let G = (V (G), E (G)) be a graph. G is said to be *square difference labeling* if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G)→N given by $f^*(uv) = |f(u)^2 - f(v)^2|$ is injective.

Definition: 2.7 Let G = (V (G), E (G)) be a graph. G is said to be *cube difference labeling* if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G)→N given by $f^*(uv) = |f(u)^3 - f(v)^3|$ is injective.

Definition: 2.8 Let G = (V (G), E (G)) be a graph. G is said to be *quad difference labeling* if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G)→N given by $f^*(uv) = |f(u)^4 - f(v)^4|$ is injective.

III. CONCEPTS OF QUAD DIFFERENCE LABELLING

Theorem-3.1: *The Path P_n is a quad difference labelling.*

Proof: Let the graph G be a path P_n

Let $|V(G)| = n \; and \; |E(G)| = n - 1$

The mapping $f: V(G) \to \{0,1, \dots, n-1\}$ is defined by $f(u_i) = i, 0 \le i \le n-1$ and the induced function $f^*: E(G) \to N$ is defined by $f^*(uv) = \left| f(u)^4 - f(v)^4 \right|$ for every $uv \in E(G)$ are all distinct such that $f(e_i) \ne f(e_j)$ for every $e_i \ne e_j$

The edge sets are
$E_1 = \{u_i u_{i+1} : 0 \le i \le n-1\}$

and the edge labelling are

In E_1

$$f^*(u_i u_{i+1}) = 4i^3 + 6i^2 + 4i + 1, 0 \le i \le n-1$$

Here we get all the edges with distinct weights. Hence the path P_n is a quad difference labeling.

Example-3.1 – The path P_5 is aquad difference labeling.

Theorem-3.2: *The Cycle C_n is a quad difference labelling.*

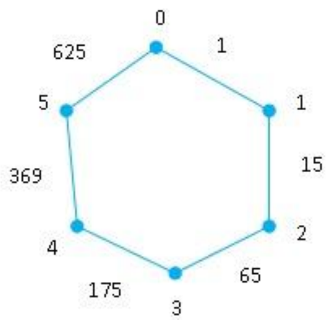
Proof: Let the graph G be a path P_n

Let $|V(G)| = n \; and \; |E(G)| = n$

The mapping $f: V(G) \to \{0,1, \dots, n-1\}$ is defined by $f(u_i) = i, 0 \le i \le n-1$ and the induced function $f^*: E(G) \to N$ is defined by $f^*(uv) = \left| f(u)^4 - f(v)^4 \right|$ for every $uv \in E(G)$

The edge sets are
$E_1 = \{u_i u_{i+1} : 0 \le i \le n-1\}$
$E_2 = \{u_{n-1} u_0\}$.
The Edge labeling are
In E_1
$f^*(u_i u_{i+1}) = 4i^3 + 6i^2 + 4i + 1$
In E_2
$f^*(u_{n-1} u_0) = (n-1)^4$
Here we get all the edges with distinct weights. Hence the Cycle C_n is a quad difference labeling.

Example-3.2 – The cycle C_6 is a quad difference labeling.



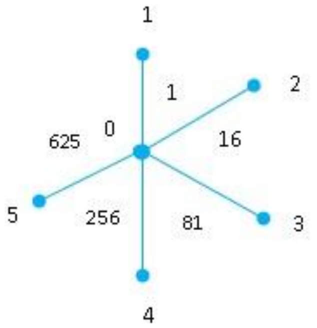
Theorem-3.3: *The Star graph $K_{1,n}$ admits a quad difference labelling.*

Proof: Let the graph G be a Star graph $K_{1,n}$

Let $|V(G)| = n + 1 \; and \; |E(G)| = n$

The mapping $f: V(G) \to \{0,1, \dots, n-1\}$ is defined by $f(u) = 0, \quad f(u_i) = i, \quad 1 \le i \le n$
The edge sets are

$E_1 = \{uu_i : 1 \leq i \leq n\}$.
The Edge labeling are
In E_1
$f^*(uu_i) = i^4, 1 \leq i \leq n$
Here we get all the edges labellings are distinct.
Hence the Star graph $K_{1,n}$ admits a quad difference labeling.
Example-3.3 – The star graph $K_{1,5}$ is a quad difference labeling.



Theorem-3.4: *The Fan graph F_n admits a quad difference labelling.*

Proof: Let the graph G be a Fan graph F_n

Let $|V(G)| = 2n + 1$ *and* $|E(G)| = 3n$

The mapping $f: V(G) \to \{0,1,\dots,n-1\}$ is defined by $f(u) = 0$, $f(u_i) = i$, $1 \leq i \leq n$ and the induced function $f^*: E(G) \to N$ is defined by $f^*(uv) = |f(u)^4 - f(v)^4|$ for every $uv \in E(G)$

The edge sets are
$E_1 = \{uu_i : 1 \leq i \leq 2n\}$
$E_2 = \{u_{2i+1}u_{2i+2} : 0 \leq i \leq n\}$

The Edge labeling are
In E_1
$f^*(uu_i) = i^4, 0 \leq i \leq 2n$
In E_2
$f^*(u_{2i+1}u_{21+2}) = 32i^3 + 72i^2 + 56i + 15$

Here all the edges are distinct.
Hence the Fan graph F_n admits a quad difference labeling.

Theorem-3.5: *The Shell graph $S_{n,n-3}$ admits a quad difference labelling.*

Proof: Let the graph G be a Shell graph $S_{n,n-3}$

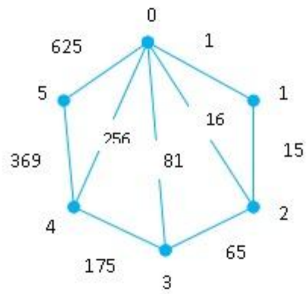
Let $|V(G)| = n$ *and* $|E(G)| = 2n - 3$

The mapping $f: V(G) \to \{0,1,\dots,n-1\}$ is defined by $f(u_i) = i$, $1 \leq i \leq n-1$
The edge sets are
$E_1 = \{u_iu_{i+1} : 0 \leq i \leq n-2\}$
$E_2 = \{u_{n-1}u_0\}$
$E_3 = \{u_0u_{i+1} : 1 \leq i \leq n-3\}$
The Edge labeling are
In E_1
$f^*(u_iu_{i+1}) = 4i^3 + 6i^2 + 4i + 1$
In E_2
$f^*(u_{n-1}u_0) = (n-1)^4$
In E_3
$f^*(u_0u_{i+1}) = (i+1)^4$

Here all the edges are distinct.

Hence the Shell graph $S_{n,n-3}$ admits a quad difference labeling.

Example-3.4 – The Shell graph $S_{n,n-3}$ is a quad difference labeling.



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