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COMPARISION AND STUDY OF NUMERICAL METHODS FOR DYNAMIC RESPONSE EVALUATION OF SDOF (LINEAR)

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Abstract*— This paper presents study and comparison of numerical methods which are used for evaluation of dynamic response. A Single Degree of Freedom (SDF)-linear problem is solved by means of Newmark's Average acceleration method, Linear acceleration method, Central Difference method, Wilson-theta method and RungeKutta method (Fourth Order) with the help of MATLAB. The advantages, disadvantages, relative precision and applicability of these numerical methods are discussed throughout the analysis.

Keywords*—Single Degree of Freedom (SDOF)Average Acceleration Method (AAM), Linear Acceleration Method (LAM), Central Difference Method (CDM), RungeKutta Method (RKM), Wilson-Theta Method(WTM)

I. INTRODUCTION

The dynamic response analysis of SDOF system is a crucial problem in dynamics. With the development of computer integrated analytical softwares, the accuracy in dynamic analysis is mainly restricted by the modelling of structure and the applicability of numerical algorithms. Also, in real life there are many dynamic problems whose closed form solution is not known. Then these types of problems can be solved using various numerical methods such as, Time-Stepping Methods, Methods based on interpolation of excitation, Central Difference Methods, Newmark's Method, Wilson-Theta method etc. Dynamic analysis of structure subjected to dynamic loads like earthquake, blast is difficult due to complexity of modelling and unavailability of closed form solution. Numerical methods prove very useful in such cases. To solve practical problems, numerical evaluation techniques must be employed to arrive at the dynamic response.The Differential equation governing the response of SDOF systems to harmonic force is;

$$m\ddot{x} + c\dot{x} + ku = p_\circ sin\omega t$$

In this paper, the problem is solved by using following methods,

1. Newmark's Method
2. Central Difference Method
3. Wilson Theta Method
4. Runge Kutta Method

Newmark's Method

In 1959, N.M.Newmark developed a family of time-stepping methods based on the following equations:

$$\dot{u}_{i+1} = \dot{u}_i + [(1-\gamma)\Delta t]\ddot{u}_i + (\gamma\Delta t)\ddot{u}_{i+1}$$
$$u_{i+1} = u_i + (\Delta t)\dot{u}_i + [(0.5-\beta)(\Delta t)^2]\ddot{u}_i + [\beta(\Delta t)^2]\,\ddot{u}_{i+1}$$

The parameters β and γ define the variation of acceleration over a time step and determine the stability and accuracy characteristics of the method. Typical selection for γ is $\frac{1}{2}$ and $\frac{1}{6} \leq \beta \leq \frac{1}{4}$ is satisfactory from all points of view, including that of accuracy. These two equations, combined with the equilibrium equation at the end of time step, provide the basis for computing$u_{i+1}, \dot{u}_{i+1}$ and $\ddot{u}_{i+1}$ at time $i+1$ from the knownu_i,$\dot{u}_i$ and $\ddot{u}_i$and time i.For linear systems it is possible to modify Newmark's original formulation, the two special cases of Newmark's method are the well known- average acceleration and linear acceleration methods.

Central Difference Method

This method is based on a finite difference approximation of the time derivatives of displacement. The Central difference method is accurate only if time step chosen is short the specific requirement for stability is, $\frac{\Delta t}{T^n} \leq \frac{1}{\pi}$. Typically, $\Delta t/_{T^n} \leq$ 0.1 to define the response analysis even shorter time step, typically $\Delta t = 0.01$ to 0.02 sec is chosen to define the ground acceleration $\ddot{u}_g$(t) accurately.

Wilson Theta Method

Wilson-Theta method is a modification of linear acceleration method and improved it to an unconditionally stable method. This modification is based on the assumption that the acceleration varies linearly over an extended time step $\delta t = \theta \Delta t$. The accuracy and stability properties of the method depend on the value of the parameterθ, which is always greater than 1.The numerical procedure can be derived merely by rewriting the basic relationships of the linear acceleration method. For SDF systems values are specialized for $\gamma = \frac{1}{2}$and$\beta = \frac{1}{6}$. The Wilson-θ method is unconditionally stable for θ=1.37. This method is subject to both phase and amplitude errors depending on the time step used. Classical methods such as the Newmark's method or the Wilson-θ method assume a constant or linear expression for the variation of acceleration at each time step.

RungeKutta Method

Runge-Kutta formula is the oldest and best understood method in numerical analysis. Despite of being an oldest technique, Runge-Kutta still continues to be a source of active research. The most suitable way of solving most initial value problems for system of ordinary differential equations are mostly provided by Runge-Kutta methods sometimes referred to as "RK" methods. They are accurate due to the closeness between the approximate solution and the exact solution. Finally, they are conditionally stable in that it is stable for some values of the parameter.

II. PROBLEM STATEMENT

Linear SDOF system is considered as lumped mass model with 5% damping. The system is subjected to half cycle sine pulse loading condition. Dynamic response is calculated by using various numerical methods like Central difference method, Newmark's method, Wilson-theta method and Runge-Kutta method. Comparison of dynamic response evaluated by closed form solution and numerical methods.

- An SDOF system has the following properties: m=0.2533 kip-sec^2/in., k=10kips/in., T_n = 1sec (ω_n =6.283 rad/sec), and ξ=0.05. Determine the response u(t) of this system to p(t) defined by the half-cycle sine pulse force using different numerical evaluation techniques with Δt=0.1 sec.

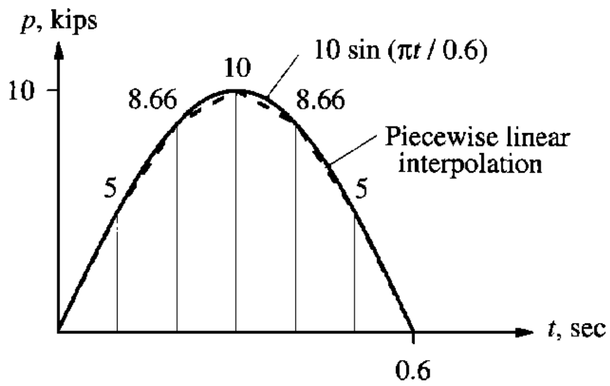

Figure 1 Half Cycle sine force

Comparision of the same problem with various Forcing Function (Linear)

The various forcing function which are to be considered are as follows:

1. Half cycle sine forces
2. Step force
3. Linear force
4. Constant force

III. RESULTS AND DISCUSSION

1. Comparisons of Dynamic Response:

Table 1 Displacement comparision of all the methods

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM	Theoretical
1	0	0	0	0	0	0	0
2	0.1	0.1913	0.0437	0.03	0.0648	0.0279	0.0328
3	0.2	0.6292	0.2326	0.2194	0.312	0.2032	0.2331
4	0.3	1.1824	0.612	0.6165	0.7296	0.5661	0.6487
5	0.4	1.5808	1.0825	1.1128	1.1629	1.0145	1.1604
6	0.5	1.5414	1.431	1.478	1.3769	1.3448	1.5242
7	0.6	0.9147	1.4233	1.4626	1.1677	1.3435	1.4816
8	0.7	-0.024	0.9627	0.952	0.5377	0.92	0.9246
9	0.8	-0.8963	0.1914	0.1282	-0.2576	0.2404	0.0593
10	0.9	-1.3724	-0.6038	-0.6945	-0.9143	-0.4357	-0.7752
11	1	-1.2942	-1.1439	-1.2203	-1.1968	-0.882	-1.2719

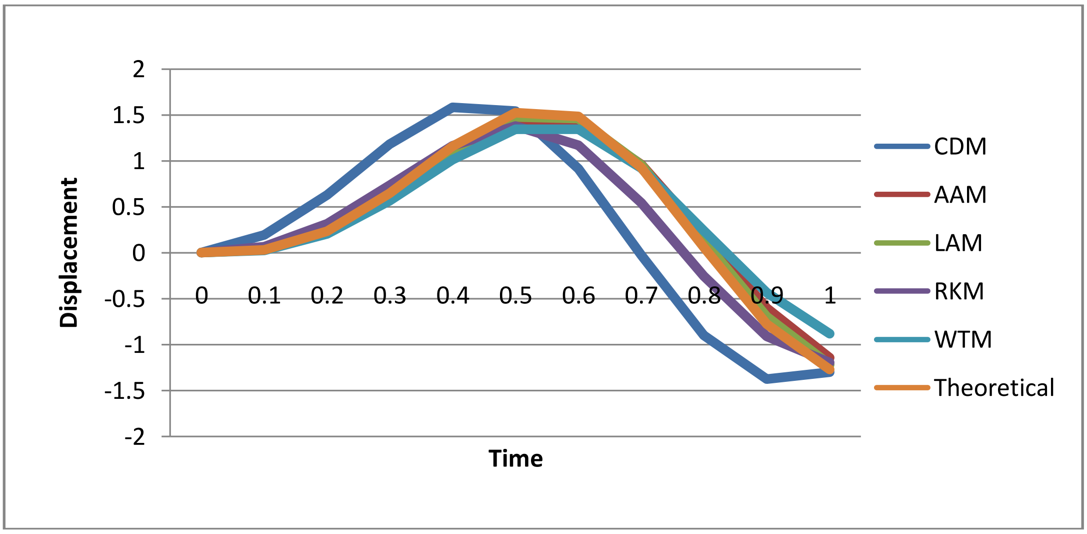

Figure 2 Graphical comparision of Displacement of all the methods

Table 2 Velocity comparisons of all the methods

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM	Theoretical
1	0	0	0	0	0	0	0
2	0.1	0.9567	0.8732	0.8993	1.3086	0.8382	0.9565
3	0.2	3.1462	2.9053	2.9812	3.2773	2.7435	3.1378
4	0.3	4.9551	4.683	4.7709	4.4185	4.3314	4.967
5	0.4	4.7578	4.7264	4.7423	3.4996	4.2707	4.8411
6	0.5	1.7952	2.2437	2.1105	0.2147	1.9478	1.98
7	0.6	-3.3306	-2.3967	-2.6869	-4.5489	-2.1921	-3.0818
8	0.7	-7.8271	-6.816	-7.1433	-7.6728	-5.9235	-7.6349
9	0.8	-9.0549	-8.6095	-8.7756	-7.7845	-7.2076	-9.0812
10	0.9	-6.742	-7.2957	-7.1572	-5.0054	-5.9158	-7.0773
11	1	-1.9894	-3.5063	-3.0567	-0.5218	-2.8001	-2.5754

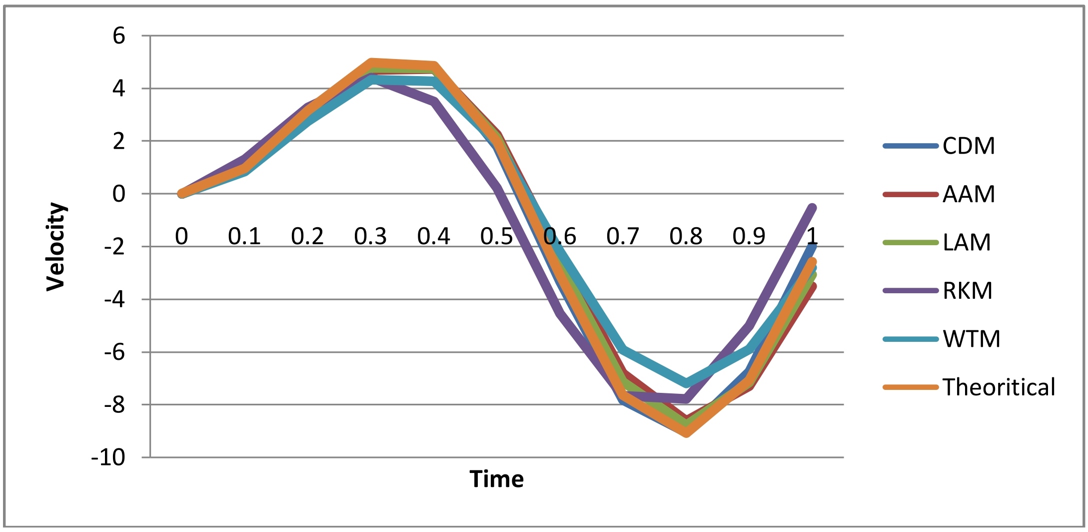

Figure 3 Graphical comparision of Velocity of all the methods

Table 1 Acceleration comparison of all the methods

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM	Theoretical
1	0	0	0	0	0	0	0
2	0.1	19.1349	17.4638	17.9853	16.3604	16.764	17.8405
3	0.2	24.6547	23.1782	23.6529	19.8135	21.3409	23.0105
4	0.3	11.5235	12.3755	12.142	7.8986	10.4171	10.75
5	0.4	-15.4708	-11.5068	-12.7144	-13.9176	-11.6298	-14.6554
6	0.5	-43.7799	-38.1467	-39.9206	-34.7532	-34.828	-41.6598
7	0.6	-58.7376	-54.662	-56.0292	-43.2424	-47.9702	-56.5324
8	0.7	-31.1923	-33.7234	-33.0969	-16.405	26.6577	-31.7033
9	0.8	6.637	-2.1475	0.4508	15.0598	0.9756	3.3634
10	0.9	39.6213	28.4231	31.9154	39.242	24.8611	35.0493
11	1	55.4308	47.3648	50.0964	47.5743	37.4528	51.8297

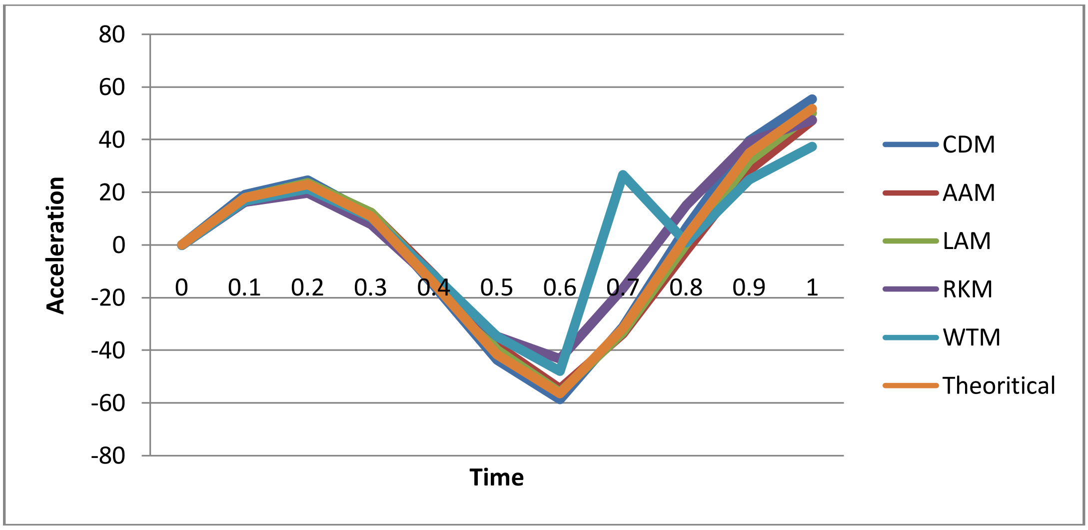

Figure 4 Graphical comparision of Acceleration of all the methods

Comparision of the same problem with various Forcing Function (Linear)

The various forcing function which are to be considered are as follows:

1. Half cycle cosine forces
2. Step force
3. Linear force
4. Constant force

i. Half cycle cosine forces for all the methods:

Table 2 Displacement comparision of all the methods for half cycle cosine

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0.1974	0	0	0	0
2	0.1	0.6387	0.163	0.1737	0.1695	0.1707
3	0.2	1.0001	0.5422	0.5726	0.5021	0.5535
4	0.3	0.9568	0.874	0.9146	0.7294	0.8764
5	0.4	0.3587	0.8821	0.908	0.61	0.8786
6	0.5	-0.6717	0.4188	0.4034	0.061	0.4387
7	0.6	-1.765	-0.4473	-0.5149	-0.7801	-0.3583
8	0.7	-2.1161	-1.3593	-1.4871	-1.4831	-1.1954
9	0.8	-1.6358	-1.833	-1.9287	-1.5995	-1.571
10	0.9	-0.5587	-1.6401	-1.6511	-1.1168	-1.3603
11	1	0.6667	-0.8849	-0.7954	-0.2464	-0.7066

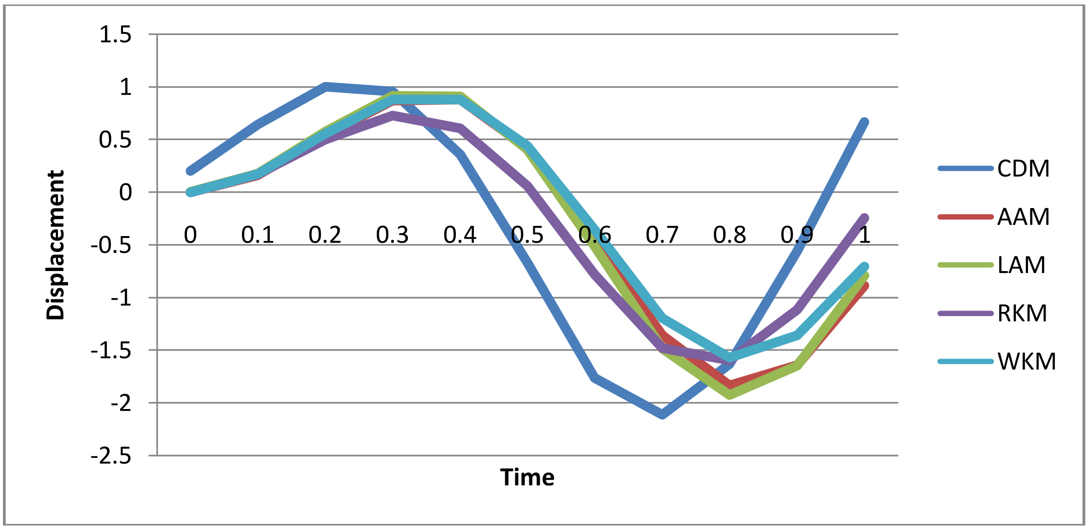

Figure 5 Displacement comparision of all the methods for half cycle cosine

Table 3 Velocity comparision of all the methods for Half cycle cosine

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0	0	0	0	0
2	0.1	3.1936	3.2594	3.2389	2.7599	3.148
3	0.2	4.0136	4.326	4.2279	3.2292	4.0126
4	0.3	1.5905	2.3098	2.0819	0.9001	1.973
5	0.4	-3.2073	-2.1476	-2.4888	-3.2487	-2.1522
6	0.5	-8.1427	-7.1197	-7.4603	-7.1853	-6.4971
7	0.6	-10.6182	-10.2021	-10.3632	-8.7405	-8.9644
8	0.7	-7.2218	-8.0369	-7.8142	-4.3451	-6.6685
9	0.8	0.6458	-1.4379	-0.801	2.0301	-0.7439
10	0.9	7.787	5.2966	6.0988	7.3081	4.6967
11	1	11.5125	9.8063	10.4094	9.5925	7.8889

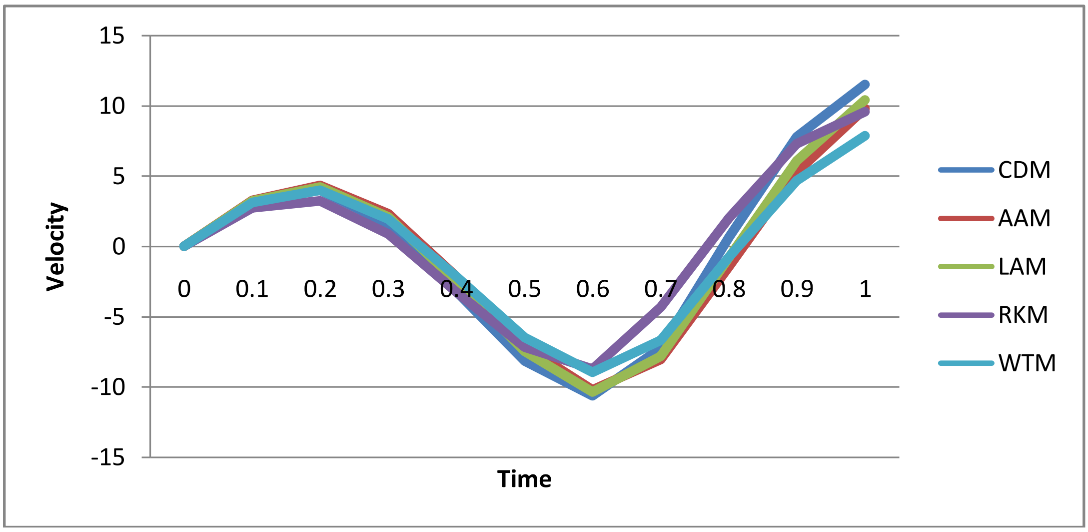

Figure 6 Velocity comparision of all the methods for half cycle cosine

Table 4 Acceleration comparision of all the methods for Half cycle cosine

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	39.4789	39.4789	39.4789	39.4789	39.4789
2	0.1	24.3922	25.7098	24.3922	25.7665	23.4813
3	0.2	-7.9912	-4.379	-7.9912	-2.1067	-6.1892
4	0.3	-40.471	-35.9452	-40.471	-29.351	-34.6038
5	0.4	-55.4844	-53.2022	-55.4844	-41.7649	-47.8996
6	0.5	-43.2231	-46.2391	-43.2231	-32.0754	-38.9991
7	0.6	-6.2883	-15.4091	-6.2883	-3.1885	-10.346
8	0.7	74.2174	58.7122	74.2174	61.2824	56.2643
9	0.8	83.1347	73.2686	83.1347	61.8704	62.2274
10	0.9	59.6878	61.4204	59.6878	39.4994	46.5851
11	1	14.8229	28.7746	14.8229	3.7013	17.2575

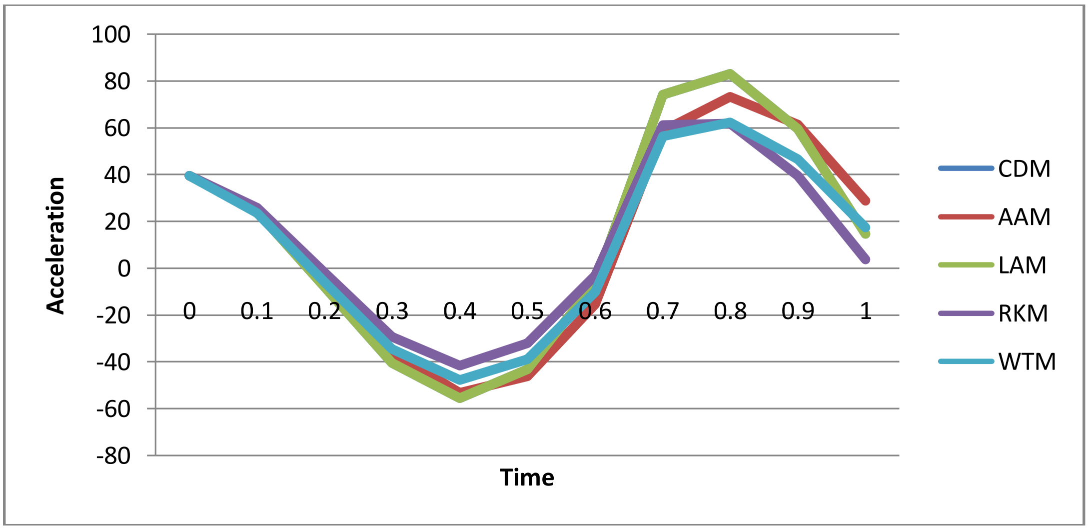

Figure 7 Acceleration comparision of all the methods for half cycle cosine

ii. Step force for all the methods

Table 5 Displacement comparision of all the methods for step force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0.0493	0	0	0	0
2	0.1	0.1725	0.0437	0.0454	0.0446	0.0467
3	0.2	0.3178	0.157	0.1619	0.1557	0.1551
4	0.3	0.524	0.3183	0.3184	0.302	0.3141
5	0.4	0.7084	0.5123	0.5162	0.4858	0.5108
6	0.5	0.8976	0.7131	0.7119	0.6688	0.7031
7	0.6	1.0187	0.8937	0.8951	0.8457	0.8687
8	0.7	0.7425	0.9486	0.9705	0.9299	0.8509
9	0.8	0.1989	0.7346	0.7375	0.737	0.5194
10	0.9	-0.3876	0.2758	0.2525	0.3241	0.0134
11	1	-0.7901	-0.2538	-0.2955	-0.1431	-0.4702

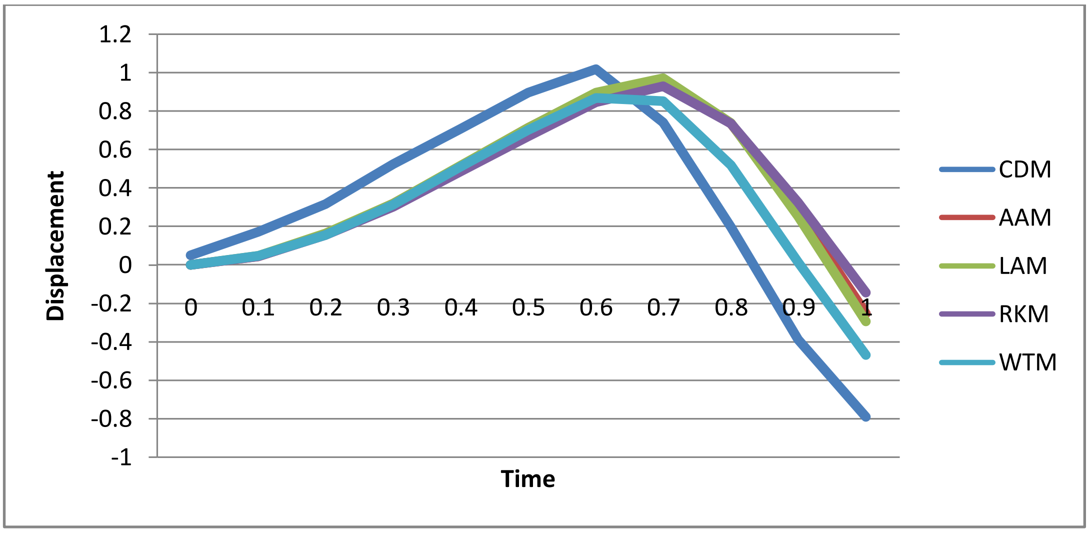

Figure 8 Displacement comparision of all the methods for step force

Table 6 Velocity comparision of all the methods for step force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0	0	0	0	0
2	0.1	0.8625	0.8733	0.8699	0.8431	0.7776
3	0.2	1.3423	1.393	1.3771	1.2992	1.202
4	0.3	1.7575	1.8339	1.8095	1.6823	1.7894
5	0.4	1.9532	2.0462	2.0158	1.873	1.7879
6	0.5	1.8678	1.9686	1.9348	1.8312	1.7323
7	0.6	1.5512	1.6443	1.6121	1.5997	1.1208
8	0.7	-0.7753	-0.5464	-0.6213	-0.3971	-1.9047
9	0.8	-4.0987	-3.7345	-3.8524	-3.2711	-4.5163
10	0.9	-5.6507	-5.4407	-5.5125	-4.6924	-5.3131
11	1	-4.9454	-5.1514	-5.0944	-4.3641	-4.0978

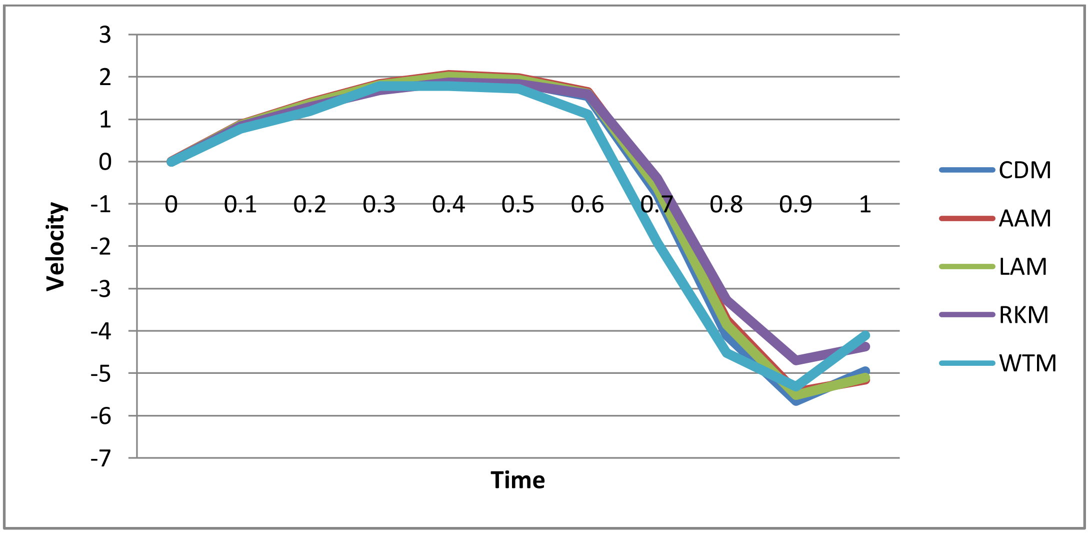

Figure 9 Velocity comparision of all the methods for step force

Table 7 Acceleration comparision of all the methods for step force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	39.4789	9.8697	9.8697	9.8697	9.8697
2	0.1	7.3796	7.5971	7.5292	6.9931	7.5372
3	0.2	2.2165	2.7968	2.6139	2.1275	2.9913
4	0.3	6.0887	6.0198	6.0339	5.5343	6.2165
5	0.4	-2.1747	-1.7727	-1.907	-1.72	-1.5509
6	0.5	0.4668	0.2207	0.2871	0.884	0.7637
7	0.6	-6.8003	-6.7072	-6.7419	-5.5143	-5.3909
8	0.7	-39.7292	-37.1071	-37.9255	-34.4217	-32.3949
9	0.8	-26.7379	-26.6538	-26.6966	-23.0574	-17.669
10	0.9	-4.3037	-7.4705	-6.5062	-5.3678	2.8079
11	1	18.4112	13.2559	14.8689	11.9319	21.1366

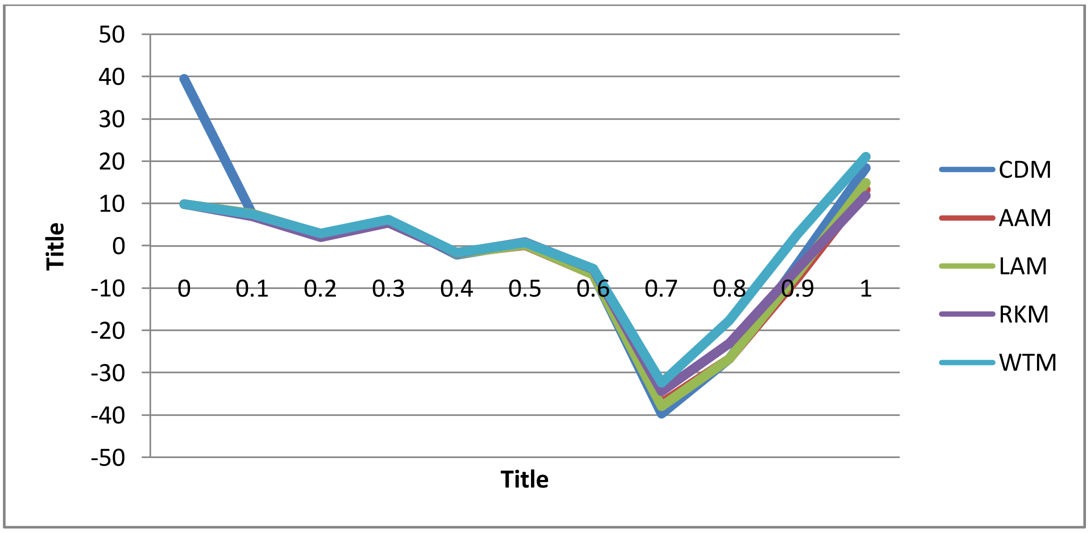

Figure 10 Acceleration comparision of all the methods for step force

iii. **Constant force for all the methods**

Table 8 Displacement comparision of all the methods for constant force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0.1974	0	0	0	0
2	0.1	0.69	0.1747	0.1818	0.1868	3.3726
3	0.2	1.2712	0.6279	0.6475	0.6204	5.1967
4	0.3	1.7132	1.186	1.2134	1.1267	5.0523
5	0.4	1.8553	1.648	1.6701	1.5142	3.2316
6	0.5	1.6614	1.858	1.8596	1.6457	0.5526
7	0.6	1.2261	1.7566	1.729	1.4863	-1.9801
8	0.7	0.348	1.3092	1.2835	0.9813	-5.2119
9	0.8	-0.6098	0.5166	0.4619	0.141	-7.9726
10	0.9	-1.2758	-0.4124	-0.4789	-0.7061	-7.7181
11	1	-1.4129	-1.1456	-1.1935	-1.2457	-4.9069

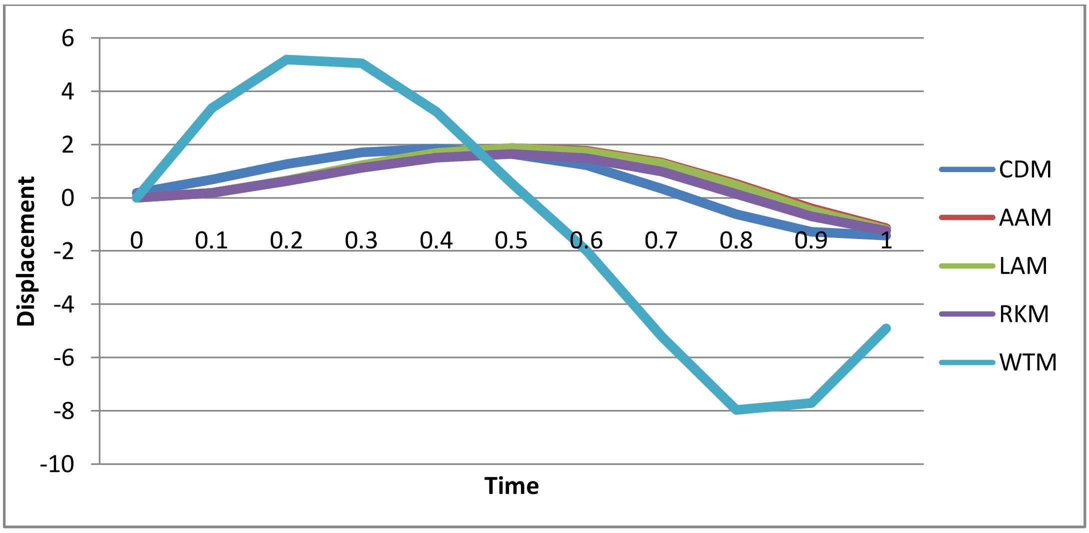

Figure 11 Displacement comparision of all the methods for constant force

Table 9 Velocity comparision of all the methods for constant force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0	0	0	0	0
2	0.1	3.4499	3.4934	3.4798	3.1104	3.3726
3	0.2	5.3691	5.5721	5.5084	4.8081	5.1967
4	0.3	5.1163	5.5888	5.4391	4.5405	5.0523
5	0.4	2.9206	3.6522	3.417	2.513	3.2316
6	0.5	-0.2593	0.5474	0.2813	-0.4279	0.5526
7	0.6	-3.1462	-2.576	-2.776	-3.1429	-1.9801
8	0.7	-6.5668	-6.3725	-6.4584	-7.2554	-5.2119
9	0.8	-9.1793	-9.4785	-9.4053	-9.0643	-7.9726
10	0.9	-8.1192	-9.1006	-8.7997	-7.4163	-7.7181
11	1	-4.0159	-5.5645	-5.063	-3.1093	-4.9069

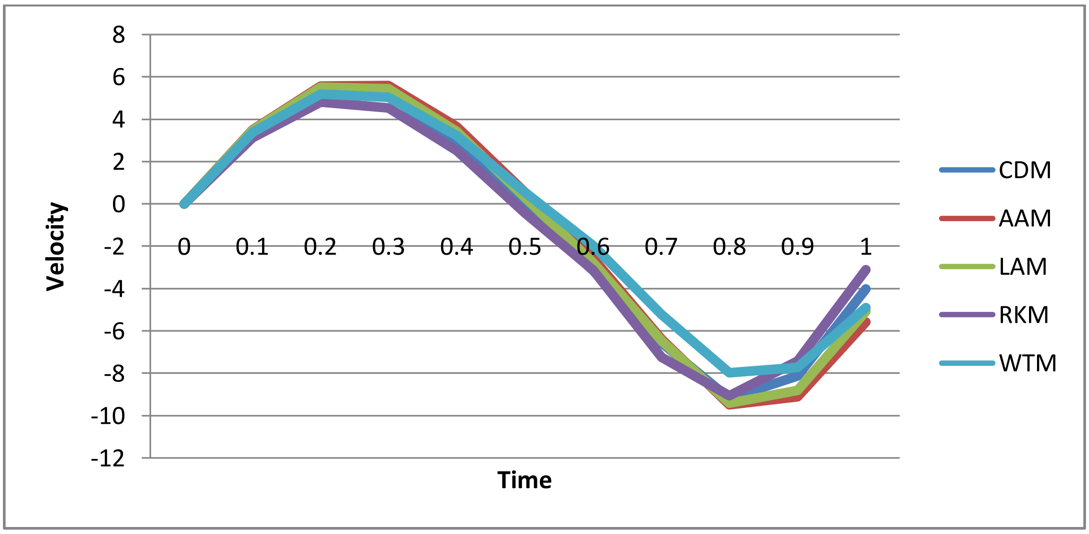

Figure 12 Velocity comparision of all the methods for constant force

Table 10 Acceleration comparision of all the methods for constant force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	9.8697	39.4789	39.4789	39.4789	39.4789
2	0.1	29.5183	30.3882	30.1168	30.1487	27.9724
3	0.2	8.866	11.1873	10.4557	11.9651	8.51
4	0.3	-13.9218	-10.8542	-11.8412	-7.8548	-11.3967
5	0.4	-29.9926	-27.8785	-28.6014	-21.8778	-25.0175
6	0.5	-33.6044	-34.2173	-34.112	-25.221	-28.5628
7	0.6	-24.1336	-28.2504	-27.0348	-17.2232	-22.091
8	0.7	-44.2786	-47.6799	-46.6133	-34.1825	-42.5462
9	0.8	-7.9719	-14.4393	-12.3241	0.1289	-12.6681
10	0.9	29.1745	21.9973	24.4356	32.5365	17.7595
11	1	52.891	48.7237	50.2993	51.1341	38.4649

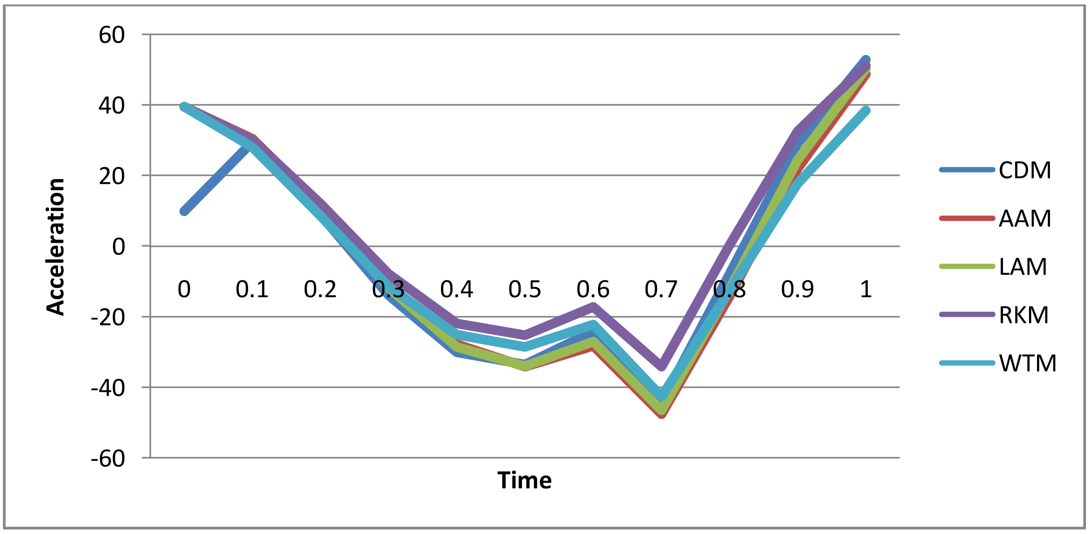

Figure 13 Acceleration comparision of all the methods for constant force

iv. Linear force for all the methods

Table 11 Displacement comparision of all the methods for linear force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0	0	0	0	0
2	0.1	0.0383	0.0087	0.006	0.013	0.0056
3	0.2	0.1361	0.0489	0.0455	0.0659	0.0421
4	0.3	0.2907	0.1396	0.1383	0.1696	0.1271
5	0.4	0.4778	0.2813	0.284	0.3155	0.2595
6	0.5	0.6619	0.4566	0.463	0.4799	0.4227
7	0.6	0.8111	0.6373	0.6452	0.6335	0.5915
8	0.7	0.6408	0.7338	0.7585	0.6618	0.7032
9	0.8	0.2356	0.621	0.6286	0.444	0.5988
10	0.9	-0.2351	0.2976	0.2799	0.0734	0.3065
11	1	-0.5872	-0.1119	-0.1495	-0.3044	-0.0524

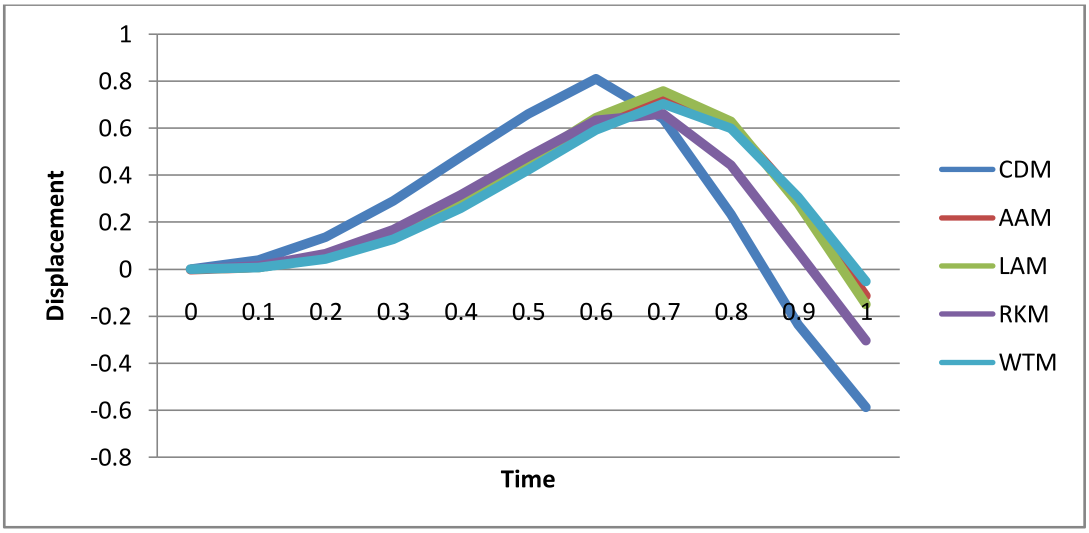

Figure 14 Displacement comparision of all the methods for linear force

Table 12 Velocity comparision of all the methods for linear force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	0	0	0	0	0
2	0.1	0.1914	0.1747	0.1799	0.2617	0.1677
3	0.2	0.6806	0.6279	0.6445	0.7256	0.5937
4	0.3	1.2623	1.186	1.2104	1.1996	1.1032
5	0.4	1.7081	1.648	1.6682	1.4983	1.5151
6	0.5	1.8558	1.858	1.8594	1.5114	1.7032
7	0.6	1.6669	1.7566	1.7305	1.2403	1.6322
8	0.7	-0.1055	0.1738	0.0869	-1.0404	0.1844
9	0.8	-2.8778	-2.4297	-2.5731	-3.1767	-2.1521
10	0.9	-4.3797	-4.0397	-4.1551	-4.0202	-3.4777
11	1	-4.1137	-4.1489	-4.1516	-3.3284	-3.4729

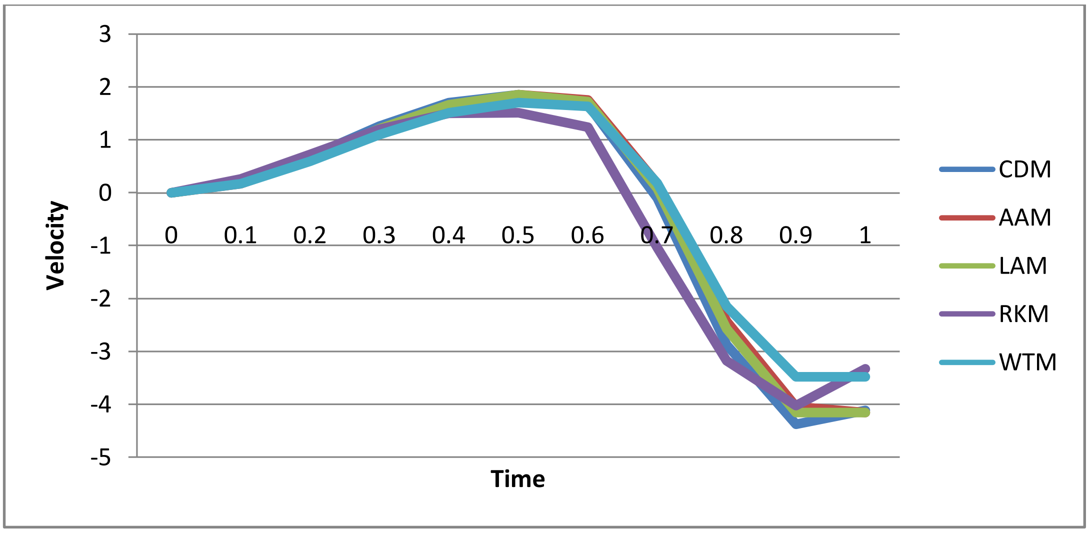

Figure 15 Velocity comparision of all the methods for linear force

Table 13 Acceleration comparision of all the methods for linear force

Sr. No.	Time	CDM	AAM	LAM	RKM	WTM
1	0	39.4789	0	0	0	0
2	0.1	3.8276	3.4934	3.5977	3.2721	3.3534
3	0.2	5.957	5.5721	5.695	4.8395	5.1671
4	0.3	5.6766	5.5888	5.6234	4.395	5.0236
5	0.4	3.2404	3.6522	3.5328	2.3936	3.2133
6	0.5	-0.2877	0.5474	0.2909	-0.1575	0.5495
7	0.6	-3.4907	-2.576	-2.8701	-2.1018	-1.9688
8	0.7	-31.956	-29.0793	-30.0011	-25.4747	-26.9888
9	0.8	-23.4897	-22.9905	-23.1993	-15.5314	-19.7409
10	0.9	-6.5483	-9.2089	-8.4406	-0.3706	-6.7703
11	1	11.8676	7.0236	8.5097	14.1075	6.8649

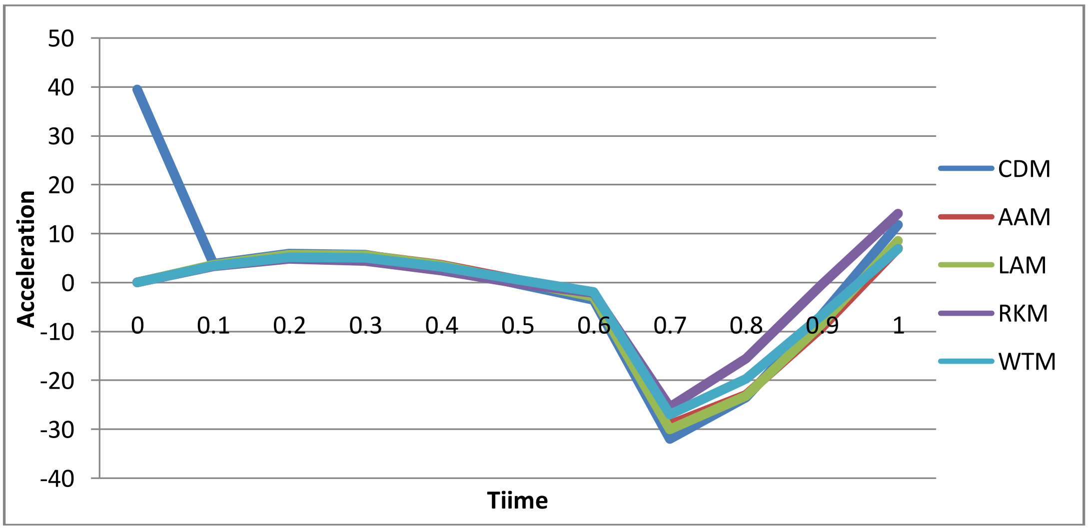

Figure 16 Acceleration comparision of all the methods for linear force

IV. CONCLUSION

The study shows that by comparing all the methods for the SDOF linear problem with the time step of 0.1 sec the Newmark's average acceleration method is the most accurate method from the other four methods which are being compared as it gives almost similar results to the theoretical one. Therefore the study concludes that to get optimum accuracy we can use average acceleration method when the time step is 0.1 sec.

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