International Journal of Technical Innovation in Modern Engineering & Science (IJTIMES)
Impact Factor: 5.22 (SJIF-2017), e-ISSN: 2455-2585
Volume 5, Issue 07, July-2019

Effect of Different Order on Characteristics and Implementation Cost of an IIR Butterworth High Pass Filter Using MATLAB

[1]Amit Kumar, [2]Rajesh Mehra, [3]Srishtee Chaudhary

[1]M.E Scholar, Department of Electronics and Communications Engineering, NITTTR Chandigarh (India)
[2]Professor, CDC, Department of Electronics and Communications Engineering, NITTTR Chandigarh (India)
[3]Ph.D. Scholar, Department of Electronics and Communications Engineering, NITTTR Chandigarh (India)

Abstract- ***In this paper, different orders of the IIR Butterworth filter have been taken and realized. The performance of different order of the Butterworth filter has been analyzed in terms of stability, implementation cost, magnitude and phase response, pole-zero plot, step and impulse response and round off noise power spectrum. It has been studied that by changing the filter order there are variations in characteristics of the filter. By keeping order high, it was observed that there is an increase in the sharpness of the pass band and stop band interface as well as there is a reduction in the pass band and stop band attenuation but eventually, it increases the complexity of the system.***

Keywords- IIR filter, filter order, Butterworth filter, Butterworth high pass filter, direct form-II IIR Butterworth filter, round off noise power spectrum of Butterworth filter, IIR using MATLAB.

1. Introduction

The development of digital filter is the determination of a realizable transfer function G(s) approximating the given frequency response specifications. IIR stands for "Infinite Impulse Response" while FIR is "Finite Impulse Response". If an IIR filter is desired, it is also necessary to ensure that G(s) is stable The process of deriving the transfer function G(s) is called Digital Filter Design. Digital filters are highly flexible and can be used at very low frequencies e.g. in biomedical applications. In digital filters, ADC (Analog to Digital converters) and DAC (Digital to Analog converters) are used therefore the speed of these filters depends upon the conversion time of ADC and settling time of DAC.
The next step is to realize it in the form of suitable filter structure and it can be implemented by IIR and FIR realization. The output of the FIR filter is delayed and it also needs a large number of components to realize it, so it can be achieved by the use of IIR filter. The IIR filter reduces the number of components used for its realization and also reduces the implementation cost. The IIR systems have an infinite number of non-zero terms, it means that the impulse response sequence of IIR filter is of infinite duration. IIR filters are implemented using structures having feedback i.e. recursive structure. There are various methods of IIR filter designing:
i. Approximation of Derivatives
ii. Impulse Invariance
iii. Bilinear Transformation
Among all above-mentioned methods, the Bilinear Transformation method is better because there is no aliasing in it. But Bilinear Transformation encounters nonlinear frequency relationship or frequency wrapping.

IIR filters are known to be stable if all its poles lie inside the unit circle of the z-plane [1].
In IIR filter design the most common practice is to convert the digital filter specifications into analog high pass prototype filter domain, to observe the analog low pass filter transfer function meeting these specifications and then to transform it into the desired digital filter domain. This approach is widely used for reasons as: i. analog approximations techniques are highly advanced. ii. extensive tables are available for analog filter design. iii. many applications need digital simulation of analog filters.
Let us suppose an analog filter $Ga(s)$, where $Pa(s)$ is the pole of the filter and $Da(s)$ is the zero of the filter in the analog domain. Therefore, the analog filter equation can be attained as under:

$$Ga(s) = \frac{Pa(s)}{Da(s)} \qquad \text{……….} (1)$$

In equation (1), 'a' is the analog domain.

$$G(z) = \frac{P(z)}{D(z)} \qquad \text{………} (2)$$

In equation (2), $Ga(s)$ is converted into $G(z)$, also $P(z)$ and $D(z)$ are the pole and zero in z-domain respectively, i.e. mapping of s-domain to the z-domain, so that the essential properties of the analog frequency response are preserved.
Mapping function should satisfy the following-

i. Imaginary axis in the s-plane to be mapped on to the unit circle of z-plane

ii. Stable analog transfer function to be transformed into a stable digital transfer function [2].

Digital filter is one of the important parts of DSP. Digital filters are highly stable, precise and flexible as compare to analog filter. Nowadays, digital technology is on its peak of development so filter using digital technology is a function of more attention by the world and used widely [3]. IIR filter has an advantage for high-speed design because the numbers of multipliers needed as compared to FIR are very small [4]. IIR filters have low stop band side lobes as compared to FIR, but IIR having nonlinear phase and can be unstable if not designed properly [5].

2. Analysis of Butterworth Filter Design

The Butterworth filter is designed to have a flat frequency response in the pass-band so that it does not offer any kind of attenuation to the wanted signal. It is also known as “maximally flat” magnitude filter. It was first described in 1930 by the British engineer Stephen Butterworth in his paper entitled "On the Theory of Filter Amplifiers” [6]. A most popular technique for designing of IIR digital filter is bilinear transformation. The simplest of all filters is the Butterworth filter. By cascading together simple first order the Butterworth filter of desired passband response, desired order and cut off frequency can be realized. The response of the Butterworth filter is maximally flat. *i.e.* having no ripples in passband and roll-off factor-20dB per pole [7]. High pass filter allows passing all high frequencies above the cut-off frequency. There is 20dB/decade increase in the gain of high pass filter with a constant rate, with an increase in the frequency starting from zero. When the frequency exceeds a certain limit, the gain attains a constant maximum value [8].

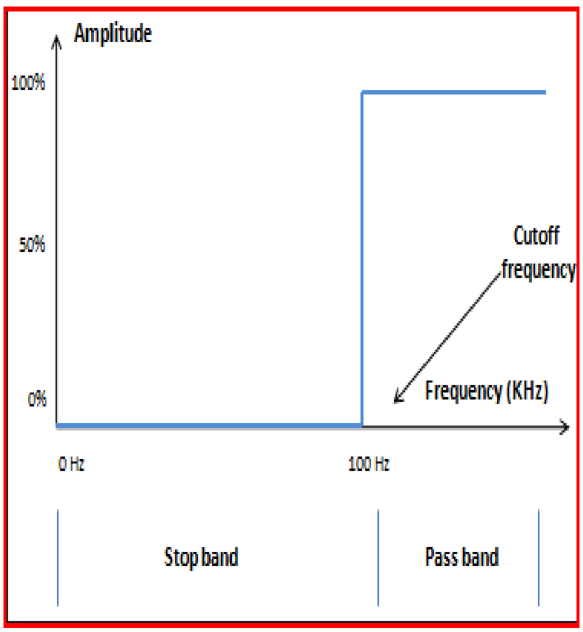

Fig.1 High pass filter response

Fig.1 shows the response of simple high pass filter with suitable stop band i.e. low frequency is stopped and after the cut-off frequency of filter it allows to pass the frequency signals i.e. is can pass high-frequency components.

3. Digital Design Specifications

The main specifications of the filters are Order of Filter, Pass Band Frequency (Fp), Stop Band Frequency (Fst), Normalized Sampling Frequency (Fc), Pass Band Attenuation (Wp), Stop Band Attenuation (Wst), and Density Factor. Order of the filter is the number of delay elements used in the filter. In the IIR filter designing there is the need of Pass Band Frequency (Fp) above which signal is desired and Sampling Frequency (Fc) at which the signal is sampled. By considering all the mentioned factors, we implement IIR Butterworth filter [9].
Order of filter means a maximum number of delay elements used in the filter, more is the number of delay elements used, better will be the result but it will increase overall cost and complexity of the filter. Pass Band Frequency (Fp) is the frequency of the

signal above which filter allows to pass signals through it. Stop Band Frequency (Fst) is the frequency of the signal up to which filter doesn't allow to pass through it. Pass Band Attenuation (Wp) and Stop Band Attenuation (Wst) is the attenuation of the signal, which exist in the pass band and stop band of the filter respectively. Normalized frequency (Fc) is the frequency at which the sampling of the signal is done.

4. Design parameters and Simulation

In order to implement the Butterworth high pass IIR filter, the design parameters are set as:
Order of filter= 16, 24, 32. One of the filter order among 16, 24 and 32 is chosen, then setting the values of different parameters given in Table 1, and observing each parameter for different filter order.

Table 1- Design Parameters:

Parameters	Values
Normalized Sampling Frequency (Fc)	48000 Hz
Pass Band Frequency (Fp)	12000 Hz

Table 1 shows the specifications for the IIR Butterworth filter type in MATLAB. By implementing the structure using defined design parameters in MATLAB, varying results for magnitude, phase, step response, impulse response, and round-off noise power spectrum are observed. All these outputs are observed and discussed one by one below.

i. Magnitude and Phase Response-

Magnitude and Phase Response shows the variations in magnitude and phase of the filter with respect to change in the frequency.

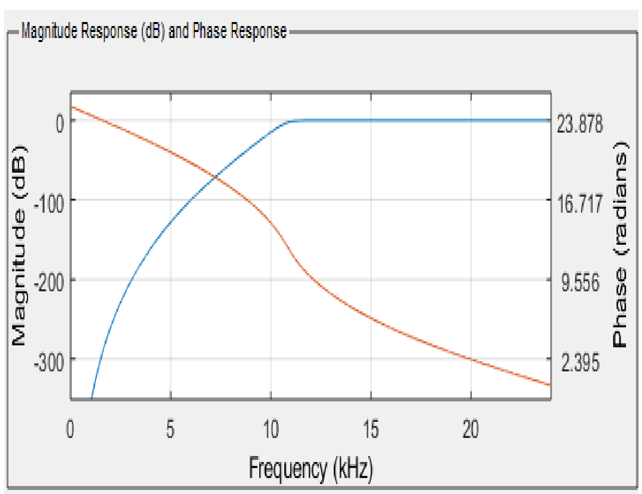

Fig.1: Magnitude and Phase response of IIR Butterworth filter of order 16.

Fig. 1 shows that magnitude is flat above frequency of 12 kHz approximately, and below 12 kHz there is an attenuation of -300 dB approximately. It means that the filter is acting as high pass filter with cut-off frequency of 12 kHz. It shows a phase of 2.395 radians at high frequency (at 20 kHz) but having an phase of 23.878 radians at lower frequency (near 1 kHz)

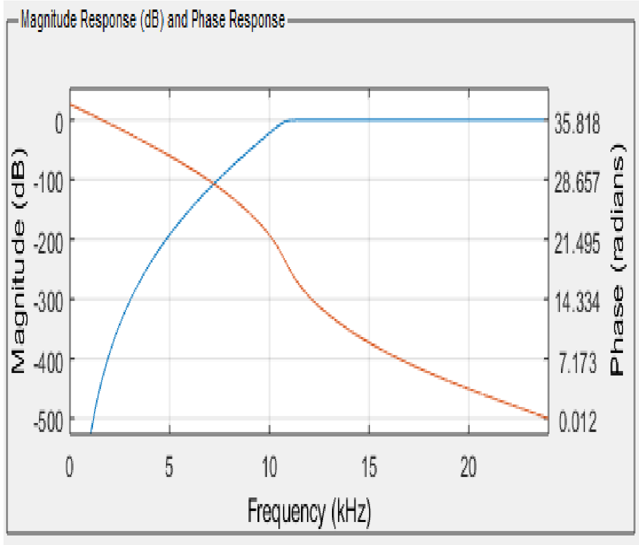

Fig. 2: Magnitude and Phase response of IIR Butterworth filter of order 24.

Fig. 2 shows an attenuation of more than -500 dB in the stop band and a flat magnitude response above 12 kHz frequency. It shows a phase of 0.012 radians at high frequency (20 kHz) but having a phase of 35.818 radians at lower frequency (near 1 kHz).

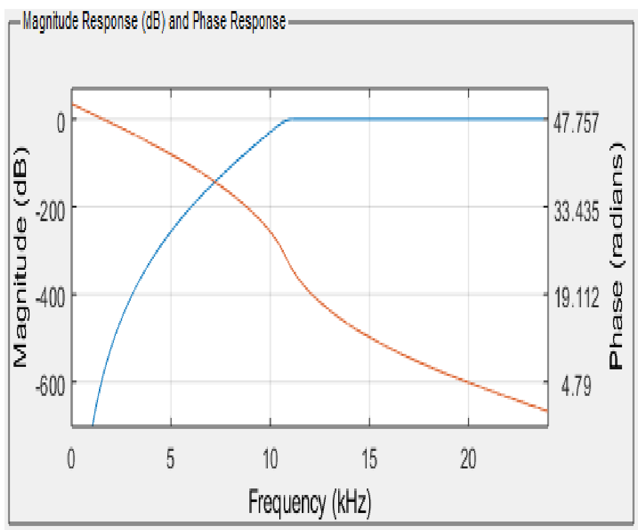

Fig. 3: Magnitude and Phase response of IIR Butterworth filter of order 32.

Fig. 3 shows the stopband attenuation of more than -600 dB and having flat magnitude response above 12 kHz. As we increase the order of filter the magnitude response of the filter becomes better. It shows a phase of 4.97 radians at high frequency (20 kHz) but having a phase of 47.8757 radians at a lower frequency (near 1 kHz).

ii. Round-off Noise Spectrum-

The power spectral density (PSD) at the output of the filter, occurs because of round off noise. This noise occurs due to the quantization error. More is the number of trials for calculating the average PSD, better will be the estimate but eventually increase the calculations and complexity [10].

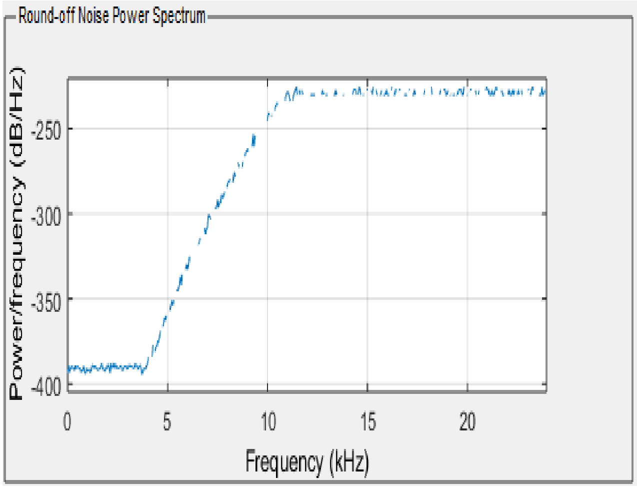

Fig.4: Round off Noise Power Spectrum of IIR Butterworth filter of order 16.

Fig. 4 shows that the slope of the graph is slightly nonlinear it means that with filter order 16 there is some noise power present with the signal. This noise is due to quantization error.

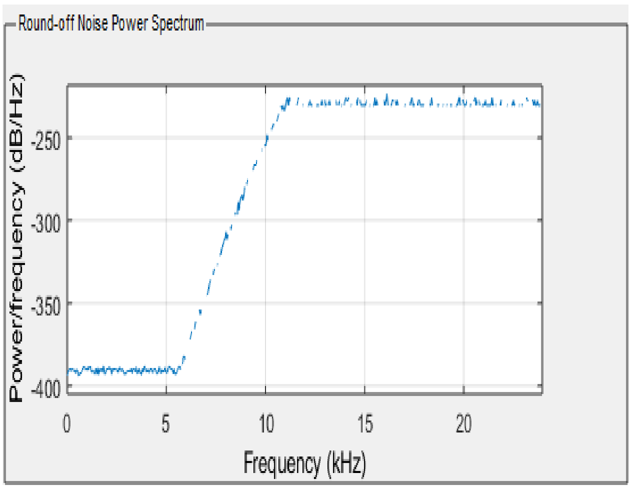

Fig.5: Round off Noise Power Spectrum of IIR Butterworth filter of order 24.

Fig. 5 shows that the slope of the graph is slightly better than filter order 16. it means that with filter order 24 there is less noise power present with the signal. There is a rise in the power of the signal just after frequency 5.7 kHz approximately and the maximum is at approximately 11 kHz.

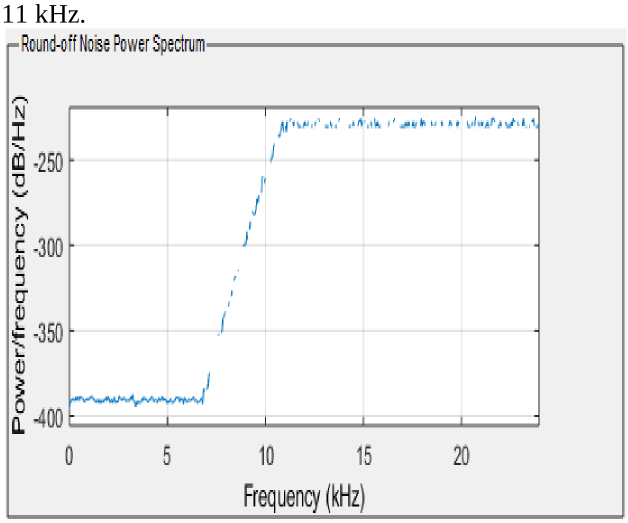

Fig.6: Round off Noise Power Spectrum of IIR Butterworth filter of order 32.

Fig. 6 shows that the slope of the graph is better than filter order 16 and 24 both. It means that with filter order 32 there is very less noise power present with the signal. There is a rise in the power of the signal just after frequency 7 kHz approximately and the maximum is at approximately 10.8 kHz. From the above plots, it is clear that as we increase the filter order the Round – off noise power spectrum is becoming better and better.

iii. Impulse Response-

The Impulse Response for a system is the output when the input is the unit impulse signal. If we know the output of a system for impulse signal, the output of any signal can be calculated by convoluting the input signal with the impulse response. It is very useful for characterizing linear time-invariant systems.

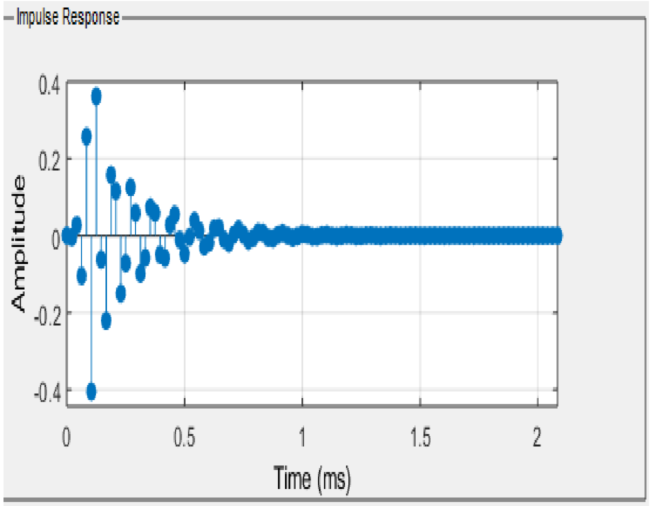

Fig.7: Impulse Response of IIR Butterworth filter of order 16.

Fig.7 shows the sampling response of the IIR Butterworth filter with filter order 16. It is showing the amplitude of the samples at different time intervals.

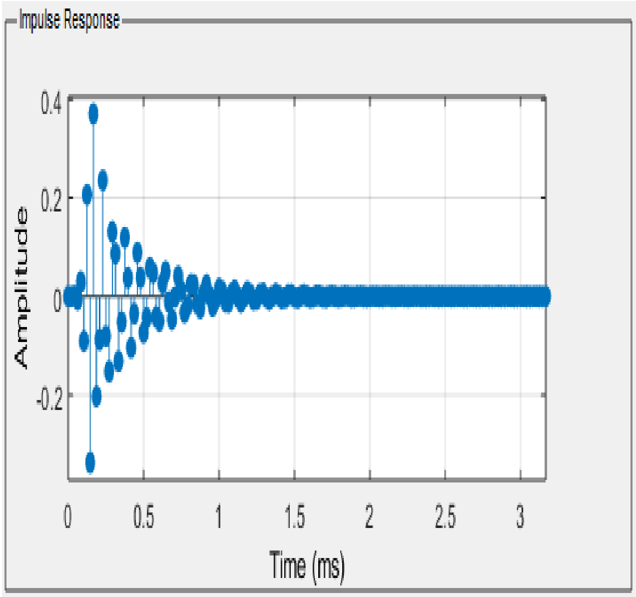

Fig.8: Impulse Response of IIR Butterworth filter of order 24.

Fig. 8 shows that the numbers of samples with filter order 24 are more than that of the filter order of 16. IIR Butterworth filter with filter order 24 is better than filter order of 16 because more is the number of samples better is the response of the filter.

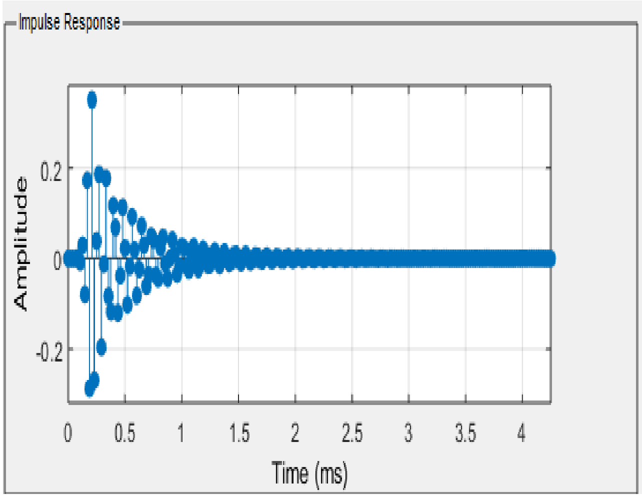

Fig.9: Impulse Response of Butterworth filter of order 32.

Fig. 9 shows that the numbers of samples with filter order 32 are more than that of the filter order of 16 as well as 24. IIR Butterworth filter with filter order 32 is better than filter order of 16 and 24 both because more is the number of samples better is the response of the filter.

It is observed that an increase in the order of the filter resulted in an increase in the number of samples of the signal. Greater is the number of samples more accurate will be our digital signal but also increase the circuit complexity and cost.

iv. Step Response-

The step response is the output of a device in response to an abrupt change in voltage. It is time behavior of the outputs of a general system when its input changes from zero to one in a very short time.

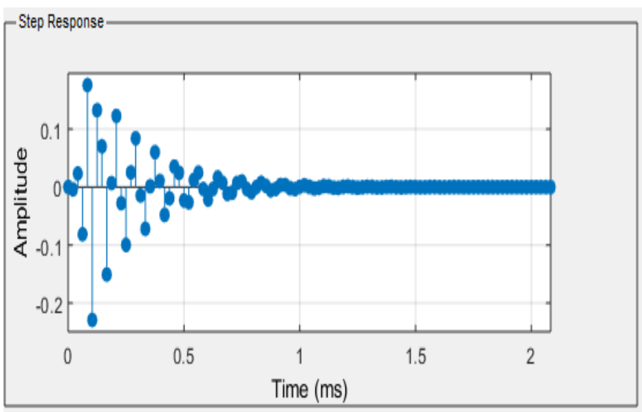

Fig.10: Step Response of IIR Butterworth filter of order 16.

Fig. 10 shows the step response of IIR Butterworth filter with filter order of 16. It shows the amplitude values of different samples at different interval of time.

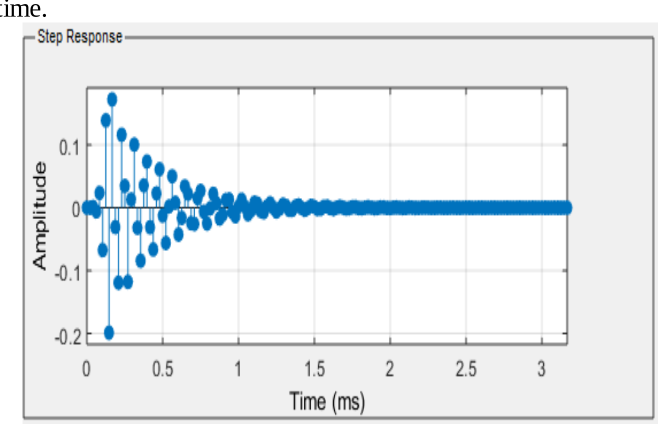

Fig.11: Step Response of Butterworth filter of order 24.

Fig. 11 shows the step response of IIR Butterworth filter with filter order of 24. It is observed that the numbers of samples in the step response of the filter in filter order 24 are more than that of the filter order 16.

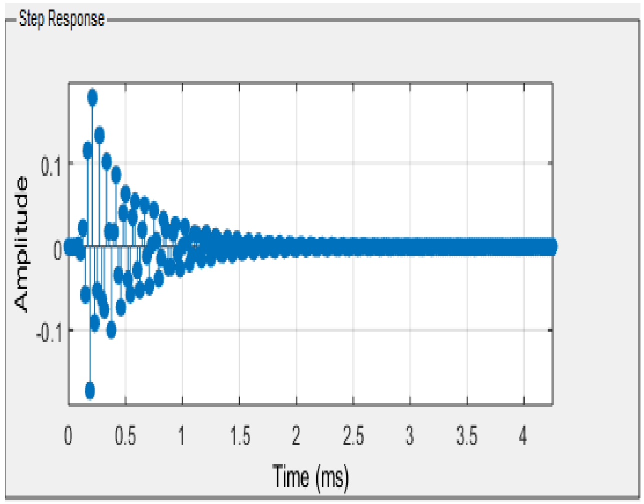

Fig.12: Step Response of IIR Butterworth filter of order 32.

Fig. 12 shows the step response of IIR Butterworth filter with filter order of 32. It is observed that the numbers of samples in the step response of the filter in filter order 32 are more than that of the filter order 16 as well as 24.
From the above plots, it is clear that as we increase the filter order then there is an increase in the number of samples of the signals. Greater is the number of samples more accurate will be our digital signal.

v. Pole-Zero Plot-

It is a graphical representation of poles and zeros of the system in the complex plane. With the help of the pole-zero plot, it can be easily determined and analyzed whether the system is stable or not. It also determines the causal or anti-causal, region of convergence (ROC) and minimum / non-minimum phase of the system.

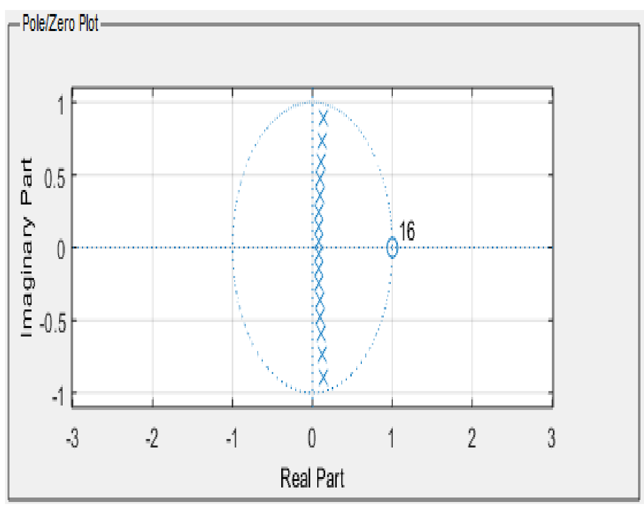

Fig.13: Pole-Zero Plot of IIR Butterworth filter of order 16.

Fig. 13 shows the pole-zero plot of IIR Butterworth filter with filter order of 16, Pole-zero plot shows the location in the complex plane of the poles and zeros of the transfer function of a dynamic system like a filter, sensor, equalizer etc. The pole-zero plot determine whether the system is stable or not and how well the system performs. The cross shows the poles and circle shows zeros.

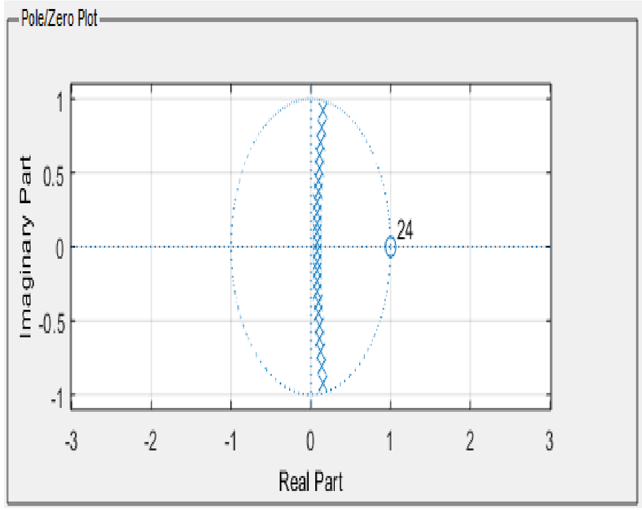

Fig.14: Pole-Zero Plot of IIR Butterworth filter of order 24.

Fig. 14 shows the pole-zero plot of IIR Butterworth filter with filter order of 24. It is observed that filter with order 24, having a pole-zero plot denser than filter order 16. It is because due to an increase in the value of filter order, there is also an increase in the number of poles and zeros, which makes the plot denser.

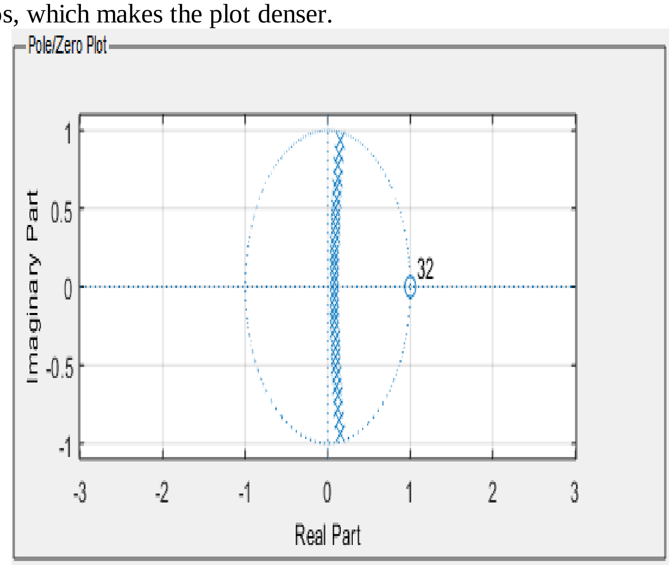

Fig.15: Pole-Zero Plot of IIR Butterworth filter of order 32.

Fig. 15 shows the pole-zero plot of IIR Butterworth filter with filter order of 32. Poles are frequencies near which magnitude of transfer function actually shoots up to hypothetically to infinity. Whereas, zeros are frequencies at which the response magnitude become zero. A pole determines the transient response of the system while zero determines the speed of response. It is observed that filter with order 32, having a pole-zero plot is denser than filter order 16 as well as 24.

From the above plots, it is clear that as we increase the filter order, the Pole-Zero Plot is getting denser. The system is stable in all cases shown above. It means that for the finite set of input variables the output for the IIR Butterworth filter is also finite. But the only difference is that on increasing filter order there is also an increase in the number of poles and zeros, which increase the calculations and cost as well as the complexity of the filter.

5. Result

Considering the filter specifications as Filter order 16, 24, and 32, Normalized Sampling Frequency (Fc) of 48000Hz, and Pass Band Frequency (Fp) of 12000Hz, Density Factor as 16 for IIR Butterworth High Pass Filter, following performance analysis has been observed:

Table 2 shows the number of sections (adders and multipliers), stability, and phase linearity, 6dB and 3dB points of the IIR Butterworth filter. These observations are taken by changing the order of the filter to 16, 24 and 32 respectively and keeping Normalized Sampling Frequency (Fc) as well as Pass Band Frequency (Fp) constant.

Table 2. Performance analysis

Type of IIR filter	No. of section needed	Stability	Phase linearity	6dB point	3dB Point
Butterworth filter order 16	8	stable	Non-linear	0.43924	0.45
Butterworth filter order 24	12	stable	Non-linear	0.44282	0.45
Butterworth filter order 32	16	stable	Non-linear	0.44461	0.45

It is observed from the table that the number of sections goes on increasing as we increase the order of the filter continuously, which practically increase the cost and complexity of the filter. IIR filters are nonlinear in nature. On increasing order of filter, 3dB and 6dB Point of the IIR filter varies accordingly.

Table 3. Implementation cost

IIR filter designing technique	No. of Multipliers or multiplications per input sample	No. of adders and addition per input sample	No. of states
Butterworth filter order 16	32	32	16
Butterworth filter order 24	48	48	24
Butterworth filter order 32	64	64	32

From table 3 it is observed that on increasing the filter order number of additions, multiplications and number of states also increases, which eventually increase the accuracy of the system but also increase the cost and complexity.

6. Conclusion

From the above, it is observed that an increase in the order of Butterworth filter results in an increase in the implementation cost of designing of the filter, i.e. a large number of components are needed to realize it.

As the order of the filter is varied, there is an increase in the magnitude of the Butterworth filter observed in the magnitude response. There is an increase in the number of samples of Butterworth filter in both impulse and step response, it will lead to a better result. But for manipulating all these samples, the number of multipliers and adders needed, which increase the complexity and cost of the system. Both the pole-zero plot and round-off noise power spectrum become more clear and denser as we increase the order of the filter. The phase of the IIR filter is nonlinear and having no phase symmetry. But, it is not desirable to increase the order of filter because it causes an increase in the implementation cost. If the concern is the accuracy and high performance of the filter, then higher filter order is considered but if the need is to design a filter with low cost then go with low filter order. If both accuracy and economical are key concerns then there is a trade-off between the order of the filter and accuracy.

7. Reference

[1] Dr. Sanjay Sharma, "Digital Signal Processing -with MATLAB programs", "S.K Kataria and Sons", Sixth Edition, February 2013, Page no.-629.

[2] Sanjit K. Mitra," Digital Signal Processing- a computer based approach" McGraw Hill Education (india) private limited, fourth edition, 2013 , page no.-492-493.

[3] Zhang Chengliang, Wang Aihong, "IIR filter design research and simulation on MATLAB", 4th international conference on signal processing system (ICSPS-2012), DOI: 10.7763/IPCSIT.2012, V58.23.

[4] Ushanandhini and Dr. S.Shivkumar Research Scholar, Prof. Kalaignor Karunanidhi Institute of Technology Coimbatore, Tamilnadu India, "Designing of IIR Filter using Radix-4 Multiplier by Precharging Technique" IJTSRD, ISSN NO: 2456-6470, Vol. -2, Issue-4.

[5] Ela Mehra and Dr. Sulochna Wadhwani, Electrical engineering Department, RGPV University, MITS, Gwalior, M.P. India, "Design of Finite Impulse Response Filter Using Genetic Algorithm"207 IJSRCSEIT /VOL.2 /Issue 4 /ISSN : 2456-3307.

[6] Elham Jasim Mohammad, Department of Physics Science from Al-Mustansiriyah University, Baghdad, Iraq, "IIR Butterworth Low-High Pass Optical Filters Design", International Journal of Scientific & Engineering Research, Volume 4, Issue 6, June-2013, ISSN 2229-5518

[7] Vijay Kr. Srivastava, Lalit Maurya,Er. Rajesh Mehra, M.E. student ECE, NITTTR Chandigarh, "International Journal of Engineering Research and Technology (IJERT)", issn:2278-0181, vol. 3, issue 2 feb-2014.

[8] OKUBANJO A..A and OYETOLA O. K, "Design construction and simulation of 3rd order Butterworth high pass filter using MATLAB", "Department of computer and electrical engineering OLABISI ONABANJO University P.M.B. 2002, Ago-I Woye, Nigeria, july 2002".

[9] Arushi Garg, Rajesh Mehra "Design Analysis of High Pass Filter using Different Technique ", International Journal of Advanced Research in Science and Engineering (IJARSE), ISSN-2319-8354(E),vol. no. 3, special issue, September 2014.

[10] Er. Rupinder Kaur, Er. Manjit Thapa, Er. Bhubneshwar Sharma, "Prediction of Round-off Noise Power Spectrum and Magnitude Response of Direct Form Second Order filter by Least Pth-norm IIR Filter Design Method", International Journal of Advanced Research in Computer Engineering & Technology (IJARCET) Volume2, Issue 10, October 2013.

8. Acknowledgment

The authors would like to thank Director, National Institute of Technical Teachers Training and Research Chandigarh, India for their inspirations and support throughout this research work.

Author

Amit Kumar received Bachelors of Technology degree in Electronics and Communications Engineering from Jawaharlal Nehru Government Engineering College Sundernagar, Distt. Mandi (H.P.), under HPTU Hamirpur (Himachal Pradesh Technical University, Hamirpur) in 2014. He is pursuing Master of Engineering in Electronics and Communications Engineering from National Institute of Technical Teachers Training and Research, Punjab University Chandigarh, Chandigarh (India).

Rajesh Mehra received Bachelors of Technology degree in Electronics and Communications Engineering from National Institute of Technology, Jalandhar, India in 1994, Maters of engineering in Electronics and Communications Engineering from National Institute of Technical Teachers Training and Research, Punjab University, Chandigarh, India, Ph.D. (engineering) from Punjab University Chandigarh India. He is Professor in the Department of CDC, NITTTR Chandigarh (India) at National Institute of Technical Teachers Training and Research Chandigarh, India. His current research and teaching interests are in signal and communication processing, very large scale integration design. He has authored more than 200 research publications including more than 100 in journals. He is also the member of IEEE and ISTE.

Srishtee Chaudhary received a Masters of Engineering degree in Electronics and Communications Engineering from National Institute of Technical Teachers Training and Research, Punjab University, Chandigarh. She is pursuing Ph.D. from National Institute of Technical Teachers Training and Research, Punjab University, Chandigarh.