

Symmetric, Asymmetric vibration mode shapes and determination of bending frequencies on windmill blades using free vibration analysis.

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Abstract— Wind mill is one of the most leading alternatives in the present energy production, where the energy of wind is converted into the mechanical energy by means of vanes called sails or blades. Force that acts on the surface of the blade is due to flow of the wind, which in turn causes vibration. This vibration, when matches with the natural frequency will produce high stresses. The materials of the blades play a prominent role in this scenario. The Light weight galvanized steel or aluminium is currently used in the outer shell of the blade. World Energy Council (WEC) projected that; this may be replaced by thermoplastic foam. In this paper the effects of vibration on these materials were studied and natural frequencies were found out, there by finding the frequencies at which the plates in the blades should not be exposed and the frequencies at which bending should be considered. Using this conditions the minimum amount of damper or stiffness addition needed for resonance avoidance can be calculated for a plate there by calculating for a whole blade. This leads to the most optimised safest design for wind mill blade. 3D Modelling is done and is been analysed using FEA.

Keywords— Composite fibre, Thermo plastic foam, Stiffness addition, Optimised Safety design.

I. INTRODUCTION

The use of fibre had increased drastically in today's world, especially the composite fibre. Fibre is a material with high size to Weight ratio and high durability which is the mandatory for several applications. The deflection produced in the composite material is lesser than the other ordinary metals^[2].

Windmill, one of the major alternate energy production had seen its future through fibre materials. Present windmills are still made of aluminium or galvanised steel. On talking about today's wind mill structure, blades are the most important part that convert the energy of air flow to rotary mechanical energy. During this process of energy conversion, the air will hit the blade causing it to vibrate. The amplitude and frequency of the vibration depends on the aerofoil shape and the angle of attack. Every material with stiffness has its own resonant frequency(Natural frequency).

Due to high value of blade length, it will be subjected to bending(Operation deflection Shape) at the blade due to external vibration. The vibration induced by the air flow when equalize the resonant frequency, the operational deflection shape will be dominated by the mode shape of resonance making the blade to vibrate vigorously causing serious damage to the blade and energy production. This can be overcome by adding stiffness to the material which can be achieved by introducing the additional structure inside the blade^[4]. This material addition is done along the midsection of the blade.

By using modal analysis, the resonant frequency and the corresponding mode shapes of the blade can be found out. The resonant frequency gives the frequency at which the blade will be prone to high relative motion and the mode shapes will give the point of maximum bending. By adding more material at the coordinate of bending and reducing the material at the non affected area will lead to the optimised design of blade^[5]. This will reduce the overall weight of the blade to good extent and will be more strong enough to overcome the resonance condition.

Cantilever plates were taken instead of blades. The frequency at which the plate subjects to bending is known as bending frequency and the frequency at which the plates prone to torsion is known as torsion frequency^[3]. Here the bending frequency alone is taken into account.

By using the basic governing equation, sum of the transverse force acting on a differential plate element, including the lateral loading, is equal to zero^[1].

II. MATHEMATICAL CALCULATION

By Lagrange's governing differential equation [1] =
$$\frac{\partial^4 W(x,y)}{\partial x^4} + 2\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} = \frac{q(x,y)}{D}$$
 (1.1)

Where $q(x, y)$ is the applied static loading.
 q is the surface loading.

The lateral force $q(x, y)$ of the of equation (1.1) must be replaced by this inertial force in order to develop the governing differential equation for the free vibration of rectangular plates. We thus obtain the governing differential equation

$$\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2\frac{\partial^4 W(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y,t)}{\partial y^4} + \frac{\rho}{D} \frac{\partial^2 W(x,y,t)}{\partial t^2} = 0$$
 (1.2)

Displacement function can be express in product of two functions as

$$W(x, y, t) = W(x, y) T(t)$$
 (1.3)

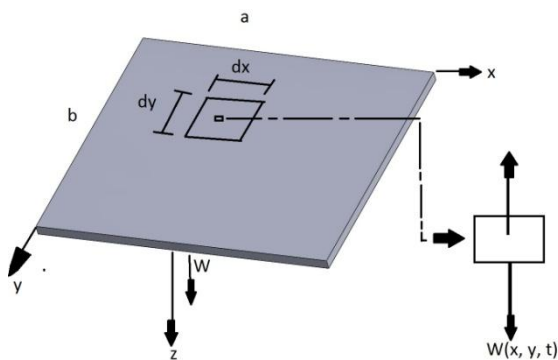


Fig 1: Rectangular plate vibration showing a typical coordinate system and a differential plate element subjected to an inertial force. Displacement W is positive downwards.

Free vibration governing differential equation

$$\frac{\partial^4 W(\xi,\eta)}{\partial \eta^4} + 2\Phi^2 \frac{\partial^4 W(\xi,\eta)}{\partial \eta^2 \partial \xi^2} + \Phi^4 \frac{\partial^4 W(\eta,\xi)}{\partial \xi^4} - \Phi^4 \lambda^4 W(\xi, \eta) = 0$$
 (1.4)

Natural frequency of the cantilever plate, $\omega_n = \frac{\lambda^2}{a^2} \sqrt{\frac{\rho}{D}}$ (1.5)

where, λ = Eigen values

a = Dimension of the edge perpendicular to the clamped edge

ρ = Density of the plate material

D = Flexural rigidity

Flexural rigidity of the cantilever plate, $D = \frac{Eh^3}{12(1-\nu^2)}$ (1.6)

where, E = Elasticity modulus

h = thickness

ν = Poison ratio

Frequency, $f = \frac{\omega}{2\pi}$ (1.7)

For Epoxy Carbon ud

Natural frequency,
$$\omega_n = \Pi^2 \sqrt{\frac{D_{11} \frac{m^4}{a^4} + 2H_1 \frac{m^2 n^2}{a^2 b^2} + D_{22} \frac{n^4}{b^4}}{m^{\eta}}}$$

III. Modal analysis

Tabulation :

Sl. no	Eigen values (λ)	a	ρ	E	thickness (h)	Poison ratio (ν)	Flexural rigidity (D)	Natural frequency (ω_n)	Frequency (f)
Galvanized steel									
Symmetric modes									
1	3.459	1	7850	2.00E+11	1.00E-02	0.3	183.1502	52.83472	8.413172
2	21.09	1	7850	2.00E+11	1.00E-02	0.3	183.1502	322.1406	51.29627
3	27.06	1	7850	2.00E+11	1.00E-02	0.3	183.1502	413.3297	65.81684
4	53.53	1	7850	2.00E+11	1.00E-02	0.3	183.1502	817.6475	130.1986
5	61.12	1	7850	2.00E+11	1.00E-02	0.3	183.1502	933.5814	148.6595
Anti Symmetric									
1	8.356	1	7850	2.00E+11	1.00E-02	0.3	183.1502	127.6343	20.32393
2	30.55	1	7850	2.00E+11	1.00E-02	0.3	183.1502	466.638	74.30541
3	63.62	1	7850	2.00E+11	1.00E-02	0.3	183.1502	971.7678	154.7401
4	70.64	1	7850	2.00E+11	1.00E-02	0.3	183.1502	1078.995	171.8145
5	92.21	1	7850	2.00E+11	1.00E-02	0.3	183.1502	1408.468	224.2783
Aluminium									
Symmetric modes									
1	3.459	1	2770	7.10E+10	1.00E-02	0.33	66.39734	53.55327	8.527591
2	21.09	1	2770	7.10E+10	1.00E-02	0.33	66.39734	326.5217	51.9939
3	27.06	1	2770	7.10E+10	1.00E-02	0.33	66.39734	418.951	66.71194
4	53.53	1	2770	7.10E+10	1.00E-02	0.33	66.39734	828.7675	131.9693
5	61.12	1	2770	7.10E+10	1.00E-02	0.33	66.39734	946.2781	150.6812
Anti Symmetric									
1	8.356	1	2770	7.10E+10	1.00E-02	0.33	66.39734	129.3701	20.60033
2	30.55	1	2770	7.10E+10	1.00E-02	0.33	66.39734	472.9842	75.31596
3	63.62	1	2770	7.10E+10	1.00E-02	0.33	66.39734	984.9838	156.8446
4	70.64	1	2770	7.10E+10	1.00E-02	0.33	66.39734	1093.67	174.1512
5	92.21	1	2770	7.10E+10	1.00E-02	0.33	66.39734	1427.623	227.3285

Table : 1 - Natural Frequency by mathematical calculation

FEA Calculation

Sl. no	Mode	Frequency value (Hz)	Deflection (m)	Name of deflection
Galvanized steel				
1.	1.	8.4543	0.22798 m	bending frequency
2.	2.	20.649	0.32955 m	torsion frequency
3.	3.	51.799	0.27635 m	bending frequency
4.	4.	66.031	0.39908 m	torsion frequency
5.	5.	75.134	0.34173 m	torsion frequency
6.	6.	131.3	0.32357 m	torsion frequency
7.	7.	149.0	0.29105 m	bending frequency
8.	8.	155.63	0.36341 m	irregular shape
9.	9.	172.3	0.42225 m	torsion frequency
10.	10.	225.03	0.37279 m	torsion frequency
Aluminium				
1.	1.	8.5494	0.38441 m	bending frequency
2.	2.	20.602	0.55695 m	torsion frequency
3.	3.	52.095	0.47374 m	bending frequency
4.	4.	66.643	0.68217 m	torsion frequency
5.	5.	75.287	0.57561 m	torsion frequency

6.	6.	131.64	0.54651 m	torsion frequency
7.	7.	150.81	0.49899 m	bending frequency
8.	8.	156.58	0.62027 m	irregular shape
9.	9.	173.97	0.72603 m	torsion frequency
10.	10.	226.51	0.63058 m	torsion frequency
Epoxy Carbon ud				
1.	1.	14.583	0.52067 m	bending frequency
2.	2.	18.603	0.83273 m	torsion frequency
3.	3.	36.499	0.91212 m	bending frequency
4.	4.	76.683	0.82351 m	torsion frequency
5.	5.	91.095	0.53333 m	torsion frequency
6.	6.	96.161	0.86661 m	torsion frequency
7.	7.	112.99	0.83666 m	bending frequency
8.	8.	140.59	0.92154 m	irregular shape
9.	9.	147.21	0.93669 m	torsion frequency
10.	10.	202.86	0.92671 m	torsion frequency

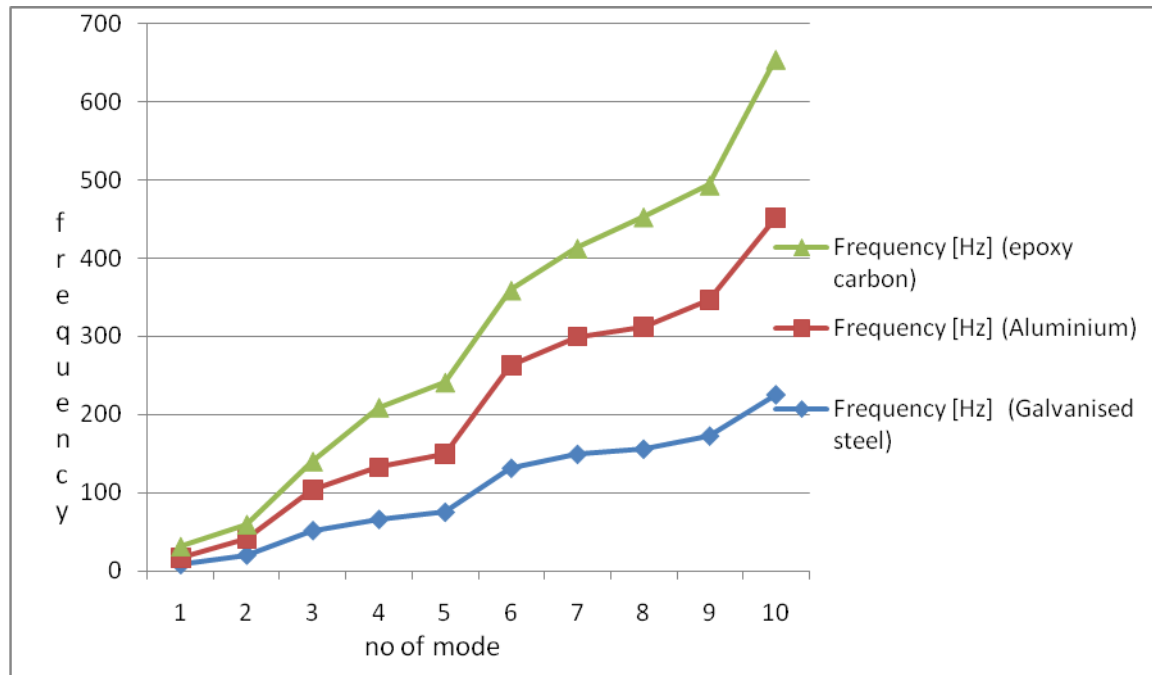
Table : 2 - frequency by FEA method

		n		
		1	2	3
m	1	359.1	875.3	1805.3
	2	1056.4	1433.6	2247.3
	3	2262.8	2561.8	3223.7

Table : 3 - Natural frequencies (rad/s) of the composite plate obtained by the finite element method

		n		
		1	2	3
m	1	359.1	876.3	1806.9
	2	1057.6	1433.2	2251.4
	3	2262.1	2566.8	3231.6

Table : 4 - Natural frequencies (rad/s) of the composite plate obtained by theoretical prediction equation



Graph : 1 - number of mode Vs frequency

Galvanized Steel

Sl. no	Frequency (mathematical)	Frequency(FEA)	Deviation	Deviation percentage
1	8.413172	8.4543	0.041128	4.1128
2	20.32393	20.649	0.32507	32.507
3	51.29627	51.799	0.50273	50.273
4	74.30541	75.134	0.82859	82.859
5	65.81684	66.031	0.21416	21.416
6	130.1986	131.3	1.1014	110.14
7	148.6595	149.0	0.3405	34.05
8	154.7401	155.63	0.8899	88.99
9	171.8145	172.3	0.4855	48.55
10	224.2783	225.03	0.7517	75.17

Table 5 : Deviation in frequency of Galvanized Steel

Aluminium

Sl. no	Frequency (mathematical)	Frequency(FEA)	Deviation	Deviation percentage
1	8.527591	8.5494	0.021809	2.1809
2	20.60033	20.602	0.00167	0.167
3	51.9939	52.095	0.1011	10.11
4	66.71194	66.643	-0.06894	-6.894
5	75.31596	75.287	-0.02896	-2.896
6	131.9693	131.64	-0.3293	-32.93
7	150.6812	150.81	0.1288	12.88
8	156.8446	156.58	-0.2646	-26.46
9	174.1512	173.97	-0.1812	-18.12
10	227.3285	226.51	-0.8185	-81.85

Table 6 : Deviation in frequency of Aluminium

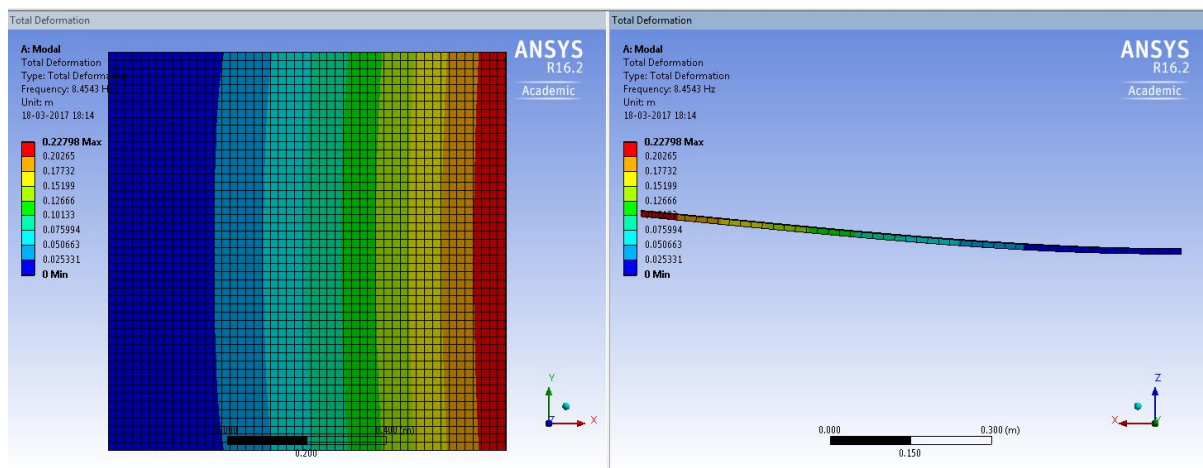


Fig 2 : Bending frequency at first mode(1st bending frequency)

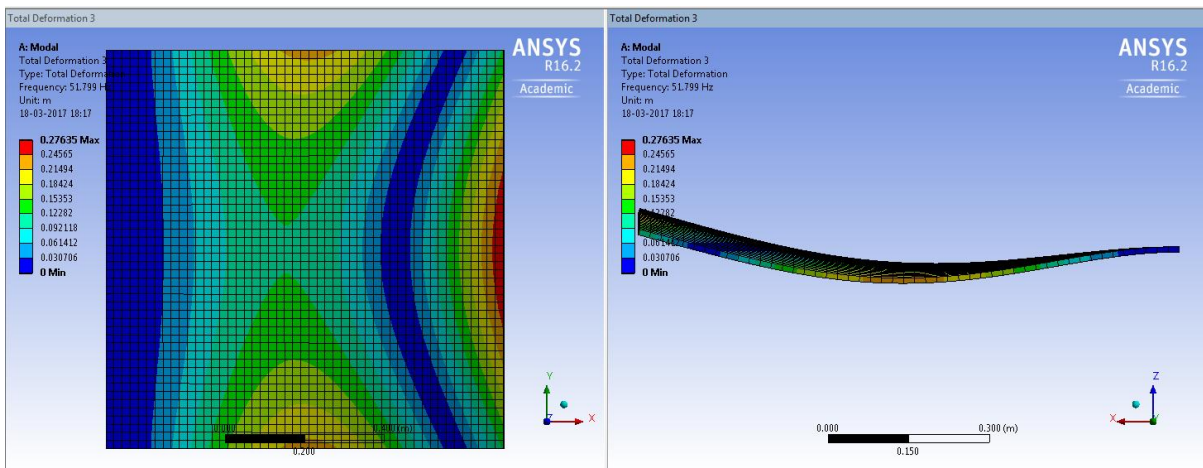


Fig 3: Bending frequency at third mode(2nd bending frequency)

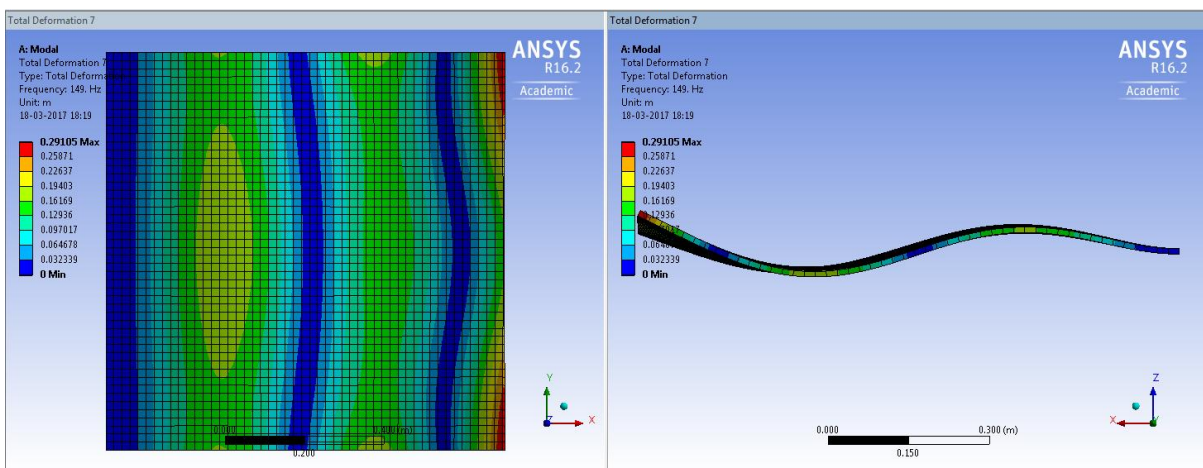


Fig 4: Bending frequency at fourth mode(3rd bending frequency)

Conclusion

The frequencies at which the material will bend more, due to over operational deflection shape is found out. The frequencies at which high stress is produced and the frequency responsible for resonance condition is found out. By finding these frequencies, the frequency at which deflection will cause along with its deflection range is found out. By providing stiffness only at the place of more bending, a optimised design of blade in which the bending is avoided.

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