

SEISMIC DAMAGE ASSESSMENT INDEX OF BUILDINGS USING FUZZY- AHP APPROACH

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ABSTRACT

Geophysical-statistics show that about sixty percent of the total land mass of the Indian subcontinent is prone to earthquakes of moderate to very high intensity. Often damages are expressed in qualitative terms like small, moderate, severe and devastating etc. However, the building damage can be expressed in quantitative terms too and compared with respect to one another. Analytical Hierarchical process (AHP) is a well-known multi-criteria decision making (MCDM) technique to express qualitative measures in quantitative terms. In order to handle ambiguity of the qualitative assessment by humans, the concept of fuzzy theory was embedded by many researchers to the AHP technique. The current study focuses on the development of seismic damage assessment index (SDAI) of buildings using fuzzy AHP technique. The index developed is applied on buildings damaged in 2011 Sikkim earthquake. These buildings are ranked on the basis of damage score. The proposed indexing model can help in comparing and ranking of the buildings for the types of damages caused to them by earthquakes and also for identifying and studying the damage levels in a seismic event. Seismic damage indexing model presented based on fuzzy-AHP is simple, and hence, embraces the possibility for various civil, structural and earthquake engineering applications.

KEYWORDS: Damages, Lateral-load resisting elements, Analytic hierarchy process (AHP), Fuzzy logic

1. INTRODUCTION

Whenever a moderate or large earthquake occurs in a large metropolitan city, a great majority of buildings are subjected to different degrees of damage. Affected structural and non-structural elements can endanger the human life in different ways. The damage of a building can put in danger the structural stability, depending on the structural configuration or redundancy and on the adverse ground conditions.

The population and government officials usually become very concerned about the security of their lives and they need to know if their buildings are safe or not. This question only can be answered by engineers and architects experts in structural and soil mechanics, damage evaluation and building rehabilitation as suggested by Carreño *et al.* (2007). Usually, National or Local technical officers deploy and manage through building inspection in order to estimate the level of damage and to assess residual building capacity.

Damage assessment of buildings during earthquake is critical to determine safety of buildings and their suitability for future occupancy. It is also necessary to conduct Post-earthquake damage assessment for the establishment of a proper reconstruction strategy. Assessing damage is of great importance to evaluate the long term use of the building concerning to suggestion and subsequent decisions regarding repair, retrofit of buildings and also to identify buildings in need of urgent demolition.

Seismic damage assessment of a reinforced concrete building is a critical and complex task. So, the decision making about assessing seismic damage of a building is decomposed into hierarchical models and subsequently, a Fuzzy-AHP based SDAI for reinforced concrete buildings has been developed. To handle the ambiguity and uncertainty among the opinions of experts, α – cut method was employed.

A survey questionnaire was developed for the collection of experts' opinion for assigning relative weightage among the different parameters at each level of the proposed model. In total 21 experts survey were collected and data was analyzed for the generation of SDAI. After the development of this model, it was applied to different sample buildings.

2. FUZZY-AHP (ANALYTICAL HIERARCHICAL PROCESS)

Damage assessment of a building is a complex process and assessment of the seismic damage of the reinforced concrete building requires knowledge of its strength, its response characteristics, and quantitative and qualitative data concerning current state of building and a methodology to integrate various types of information into a decision making process for assessing the damage of the entire building.

The Analytic Hierarchy Process (AHP), presented by Thomas Saaty (1980), is an effective tool for dealing with complex decision making process and helps in setting priorities and arrive at the final decision. AHP is a multi-criteria decision making method that uses hierarchical structures to represent a problem and then to develop priorities for alternatives based on judgment and experts' opinion. By reducing complex decisions to a series of pairwise comparisons, and then combining the results, AHP can be utilized to capture both subjective and objective aspects of a decision.

In the conventional AHP formulation procedure, human judgment is represented as exact numbers. In many situations, the human preference model is uncertain and decision makers might be reluctant or unable to assign exact numerical values to make comparison judgments (Chang et al. 2009). In order to handle ambiguity of the qualitative assessment by humans, the concept of fuzzy theory was embedded by many researchers to the AHP technique. Therefore, to resolve the imprecision and the ambiguity in assessing the relative importance, fuzzy set theory, introduced by Zadeh (1965) has been used and adopted herein.

Fuzzy logic is an innovative way of representing qualitative or subjective information in numerical form, very useful for technologic and engineering applications where expert criteria are required. Fuzzy set theory attempts to select, prioritize or rank a finite number of courses of action by evaluating a set of Pre-determined criteria.

This study adopts the concepts of Saaty (1980) and Chang et al. (2009) to analyze data and reach a consensus among experts. The eigenvector method is used to calculate the weights.

3. DECISION MAKING USING AHP

In order to make a decision in an organized way to generate weights or priorities, it is required to decompose the decision into the following steps.

STEP 1 Define the problem & determine the kind of knowledge to be sought.

STEP 2 Structure the decision problem into a hierarchical model, as shown in Fig.1. Decomposition of the decision problem into elements according to their common characteristics.

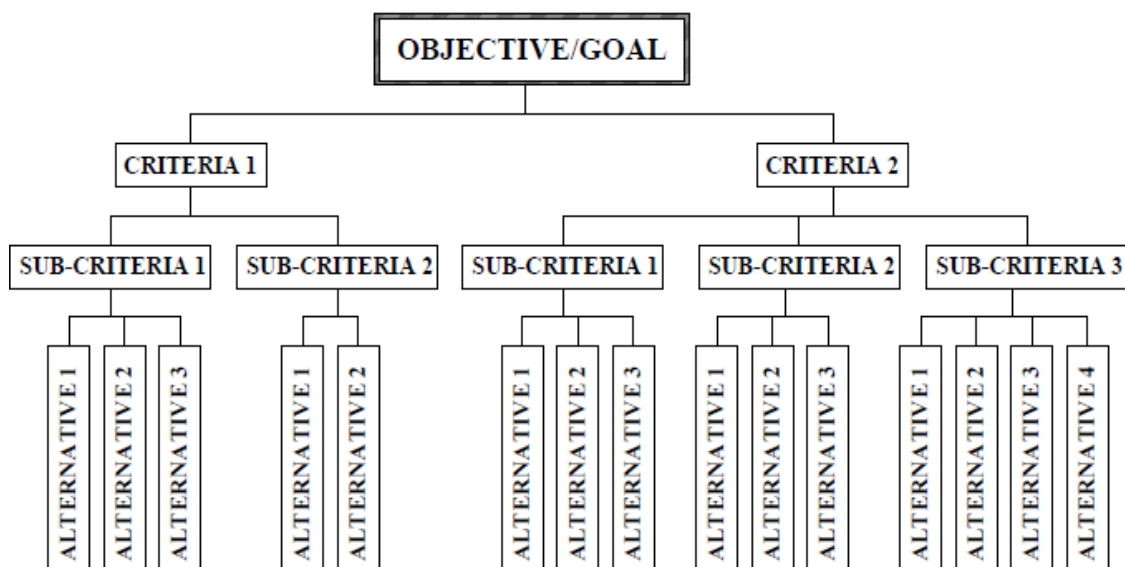


Fig. 1 Decision Making Hierarchical Model

STEP 3 The elements of a particular level are compared pairwise w.r.t. a specific element in the immediate upper level. To make comparisons, there is a need of a scale of numbers that indicates how many times more important one element is over another element w.r.t. the criterion or property to which they are compared. Saaty (1980) suggested the use of a 9-point scale to transform the verbal judgments into numerical quantities representing the values of a_{ij} as shown in Table 1. A judgmental matrix is formed and used for computing the weights (or priorities) of the corresponding elements as shown in Table 2. The matrix A is an $n \times n$ real matrix, where n is the number of evaluation criteria considered.

Table 1: Saaty’s fundamental scale of absolute numbers

Intensity of Importance	Definition	Explanation
1	Equal importance	Two criteria contribute equally to the objective
3	Slightly more important	Experience and judgment slightly favour one criterion over other
5	Strongly more important	Experience and judgment strongly favour one criterion over other
7	Very strongly important	A criterion is favoured very strongly over another, its dominance demonstrated in practice
9	Absolutely more important	The evidence favouring one criterion over another is of the highest possible validity
2, 4, 6, 8	Intermediate values	Evaluation between two intensity values
Reciprocals of above	If criterion p has one of the above non-zero numbers assigned to it when compared with criterion q , then q has the reciprocal value when compared with p	

Table 2: Pairwise criteria comparison

Criteria	C1	C2	C3
C1	a_{11}	a_{12}	a_{13}
C2	a_{21}	a_{22}	a_{23}
C3	a_{31}	a_{32}	a_{33}

$$A = [a_{ij}] = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \quad (1)$$

$$a_{ij} > 0 ; a_{ij} = \frac{1}{a_{ji}} ; a_{ii} = 1 \text{ for all } i \quad (2)$$

Because of these constraints or rules, the judgmental matrix A is a positive reciprocal pair-wise comparison matrix.

STEP 4 Computation of local priority weights of criteria and consistency of the judgments. Saaty (1980) stated that priorities of criteria can be estimated by finding the principal Eigen vector w of the matrix. Once the matrix A is built, next step is to derive from A the normalized pairwise comparison matrix A_{norm} by making equal to 1 the sum of the entries on each column, i.e., each entry

$$a_{ij} = \frac{a_{ij}}{\sum_{l=1}^n a_{lj}} \quad (3)$$

Finally, the criteria weight vector w (that is an n -dimensional column vector) is built by averaging the entries on each row of A_{norm} i.e.,

$$w_i = \frac{\sum_{l=1}^n \bar{a}_{il}}{n} \quad (4)$$

The weight vector w (or Eigen vector) thus obtained gives the priorities of the criteria w.r.t. the goal. AHP incorporates an effective technique for checking the consistency of the evaluations made by the decision maker or experts’ judgment. The technique relies on the computation of consistency index for matrix A .

Once the weight vector w (or Eigen vector) is obtained, then matrix A is multiplied with weight vector w to obtain vector $A.w$. Next step is to calculate another vector $A.w/w$ whose i^{th} element is the ratio of the i^{th} element of the vector $A.w$ to the corresponding element of the vector w . Now Eigen value, λ_{max} (say, scalar x) is computed by averaging all the elements of the vector $A.w/w$.

The consistency of the obtained judgmental matrix can be determined by a ratio known as the Consistency Ratio (C.R.) given by

$$\text{Consistency Ratio} = \frac{\text{Consistency Index}}{\text{Random Index}} \quad (5)$$

The value of Consistency Index (C.I.) is obtained from the expression

$$\text{Consistency Index} = \frac{\lambda_{\max} - n}{n - 1} \quad (6)$$

For a perfectly consistent decision the value of C.I. obtained will be zero, but small values of consistency may be tolerated.

To be specific, if

$$\frac{\text{Consistency Index}}{\text{Random Index}} < 0.1 \quad (7)$$

implies that the consistencies are tolerable, and a reliable result may be expected from the AHP.

R.I. is the *Random Index*, i.e., the obtained value of consistency index when the entries of matrix A are of completely random nature. The values of R.I. for problems with values of n being less than or equal to ten are shown in Table 3.

Table 3: Values of Random Index (R.I.) (Saaty 1980)

n	1	2	3	4	5	6	7	8	9	10
R.I.	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

STEP 5 Once the local priorities of elements of different levels are available, they are combined to obtain final priorities of the alternatives. Simple weighted summation technique is utilized for this.

α-Cut Method

The opinions of experts are demonstrated as a triangle between L and U values representing the lower and upper limits of the membership function respectively and the M is the geometric mean of experts' opinion representing the major value of the shape function. Graphically the triangular fuzzy number

$$\tilde{a}_{ij} = (L_{ij}, M_{ij}, U_{ij}) \quad (8)$$

where $L_{ij} \leq M_{ij} \leq U_{ij}$
 and $L_{ij}, M_{ij}, U_{ij} \in \left[\frac{1}{9}, 1\right] \cup [1, 9]$

i.e. \tilde{a}_{ij} is an element of fuzzy comparison matrix, where L_{ij}, M_{ij}, U_{ij} are lowest, geometric mean and highest values of the experts' opinions respectively.

$$L_{ij} = \min (B_{ijk}), \quad (9)$$

Buckley (1985) suggested that the geometric mean of experts' opinion for fuzzy comparison values for each criterion may be calculated as given in the Equation (10) below

$$M_{ij} = \sqrt[n]{\prod_{k=1}^n B_{ijk}} \quad (10)$$

(k = 1,...,n) and

$$U_{ij} = \max (B_{ijk}) \quad (11)$$

B_{ijk} represent opinions of expert k for the relative comparison of two criteria i and j. The graphical representation of a triangular fuzzy number is shown in Fig.2.

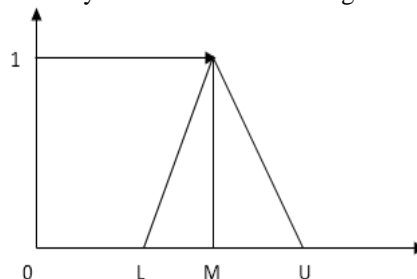


Fig. 2 Graphical representation of Triangular Fuzzy Number

The data obtained from different experts' opinion comparison matrices are obtained to form a fuzzy comparison matrix (using the values of L, M and U) which is represented as:

$$[\tilde{A}] = \tilde{a}_{ij} = \begin{pmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \frac{1}{\tilde{a}_{12}} & 1 & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\tilde{a}_{1n}} & \frac{1}{\tilde{a}_{2n}} & \dots & 1 \end{pmatrix} \quad (12)$$

The “~” (tilde) represents the triangular number demonstration, for example, \tilde{a}_{12} represents the experts’ preference of first criterion over second criterion, and equals to $\tilde{a}_{12} = (L_{12}, M_{12}, U_{12})$.

Once the single crisp values of a fuzzy numbers are obtained, the rest of the process is carried out as adopted in AHP. This α -cut method has good capacity to show clearly the preference or uncertainty (α) and risk tolerance (λ) in the fuzzy opinion of the decision makers.

$$(a_{ij}^\alpha)^\lambda = (\lambda \cdot L_{ij}^\alpha + (1 - \lambda)U_{ij}^\alpha) \quad (13)$$

where

$$L_{ij}^\alpha = (M_{ij} - L_{ij}) \alpha + L_{ij} \quad (14)$$

represents the left end value and

$$U_{ij}^\alpha = U_{ij} - \alpha (U_{ij} - M_{ij}) \quad (15)$$

represents the right end value.

Finally the resultant de-fuzzified single pair wise comparison matrix is expressed as

$$[(A^\alpha)^\lambda] = (a_{ij}^\alpha)^\lambda = \begin{pmatrix} 1 & (a_{12}^\alpha)^\lambda & \dots & (a_{1n}^\alpha)^\lambda \\ (a_{21}^\alpha)^\lambda & 1 & \dots & (a_{2n}^\alpha)^\lambda \\ \vdots & \vdots & \ddots & \vdots \\ (a_{n1}^\alpha)^\lambda & (a_{n2}^\alpha)^\lambda & \dots & 1 \end{pmatrix} \quad (16)$$

Eigenvalues, eigenvector and consistency test are the next steps involved after obtaining the de-fuzzified matrix of the above form.

4. STUDY AREA

A M_w 6.9 earthquake struck near the Nepal-Sikkim border on September 18, 2011 at 18:10 local time, about 68 km North-West of Gangtok and at a focal depth of 19.7 km as reported by United States Geological Survey (USGS). The earthquake triggered a large number of landslides and caused significant damage to buildings and infrastructure. Sikkim was the most severely affected state of India, followed by West Bengal and Bihar. Neighbouring countries of Nepal, Bhutan, Tibet (China) and Bangladesh sustained damage and losses to varying extent. The maximum shaking intensity was estimated to be around VI+ on the MSK scale. The earthquake was followed by a series of aftershocks, two of which were M4.5 and M5.0 and hit within 75 minutes of the main shock. Most multi-storey reinforced concrete buildings were Non-engineered and sustained considerable damage due to earthquake shaking; a small number of these collapsed or suffered irreparable structural damage as mentioned in EERI special report (2012).

The seismic damage index model is applied on the buildings which were damaged in 2011 Sikkim earthquake. The sample buildings upon which the proposed Fuzzy-AHP (Analytical Hierarchical Process) based model is applied are summarized in Table 4.

Table 4: Buildings for the proposed SDAI model application

Earthquake	Buildings	Location
Sikkim 2011	Moonlight School	Chuntang
	Residential Building	Singtam
	House of BDO	Chuntang
	Boys’ Hostel at SMIT	Gangtok
	Himalchuli hotel	Gangtok

The information regarding the types of seismic damage observed in these buildings during the 2011 Sikkim earthquake are derived from the database of EERI reports. The seismic damage index score for each of the buildings in Table 4 is calculated.

The complex problem of seismic damage assessment of a building can be grouped into simple and manageable hierarchical structures. The hierarchical structure follows a logical order where the causal relationship for each supporting argument is further subdivided into specific contributors.

The type of structural force resisting system used in a building plays a major role in terms of its seismic resiliency. Two reinforced concrete building types considered in this study are moment resisting frames with infill masonry walls and shear wall buildings.

Shear walls of sufficient rigidity when employed in buildings resist almost entire seismic forces. Though the term “shear wall” is well accepted and used within the engineering community, their predominant mode of behaviour can be flexure when used in medium to high-rise buildings. They typically act as vertical cantilevers and provide lateral bracing to the entire system while receiving lateral forces from diaphragms and transmitting them to the foundation. Shear wall structures have been reported to behave well under moderate to strong earthquake excitations (Saatcioglu et al. 2001).

5. PROPOSED SEISMIC DAMAGE ASSESSMENT INDEX MODEL

The damage index model proposed in this study was divided into three sub-components, namely, damages in horizontal structural elements, lateral load resisting elements and, in foundation. Structural components taken under horizontal structural elements are damages in beams, slab and joints. Damages in the lateral load resisting elements consists of three sub-criteria, namely, damages in columns, shear walls and infill walls. Types of damages studied under beams are, extensive cracks in tension zone, separation of beam from wall frame and bending shear failure at supports. Flexural cracks, separation from columns below and failure at Stair-slab intersection are the damages counted under damages in slabs. Damages in slabs includes cracking at beam-column joint, damage at expansion joints and cracking between infill and concrete frame. These damages are further divided on the basis of levels of severity namely, low, moderate and severe. Damages in column comprises of cracks, concrete spalling and buckling of reinforcement. Fine cracks in mortar, diagonal cracks extending all over the surface and disintegration of concrete included as different levels of cracks in column at low, medium and severe levels respectively. Fine cracks in mortar, diagonal cracks extending all over the surface, disintegration of concrete and complete collapse have been counted as different damage levels in shear walls. Similarly, damage in infill walls are divided according to different levels of severity i.e., low cracking, moderate cracking, severe cracking and complete collapse. Settlement of foundation and damage due to liquefaction are considered under damages to foundations.

6. DISCUSSION ON DAMAGE ASSESSMENT MODEL

The hierarchical model of seismic damage index is developed and calculations and results are derived based on the aforementioned methodology. Only one alternative of any measurable item, which has been observed in the building can have the value equal to 1, the rests are all set to be zeroes. The global score obtained for a given option is actually the weighted sum of the scores obtained with respect to all the criteria. Likewise, the total score at the topmost level is determined. The maximum damage value is likely to occur when all the damages occur in a building and at the severest level. The corresponding maximum damage value is found to be 0.9066.

Damage index score for all the buildings listed in Table 4 was calculated depending upon the types of damages observed in each building. The final damage score of the buildings are tabulated in Table 5.

Table 5: Total Damage index score of buildings

Buildings	Damage Index Score
Moonlight School	0.5502
Residential Building	0.4454
House of BDO	0.3938
Boys' Hostel at SMIT	0.1437
Himalchuli hotel	0.4589

The obtained weighted score can be normalized and converted to a relative index score at any desired base. The scale has been mapped at the scale of 100 as shown below.

Let WS be the weighted score of any building under consideration, WS_{max} be the maximum possible score in the analysis. The seismic-damage index for buildings at the relative scale of 100 can be given a name as “DI 100” and calculated as

$$DI = \frac{(\text{Weighted score of building}) WS}{(\text{Maximum possible score}) WS_{\max}} \times 100 \quad (17)$$

The relative damage index of different buildings are calculated as shown in Eqs. 18(a-e).

For example, relative damage index of Moonlight school is calculated as

$$DI = \frac{0.5502}{0.9066} \times 100 = 60.688 \quad (18a)$$

Relative Damage Index of Building at Singtam is

$$DI = \frac{0.4454}{0.9066} \times 100 = 49.129 \quad (18b)$$

Relative Damage Index of House of BDO is

$$DI = \frac{0.3938}{0.9066} \times 100 = 43.437 \quad (18c)$$

Relative Damage Index of Boys Hostel at SMIT is

$$DI = \frac{0.1437}{0.9066} \times 100 = 15.850 \quad (18d)$$

Relative Damage Index of Himalchuli Hotel is

$$DI = \frac{0.4589}{0.9066} \times 100 = 50.618 \quad (18e)$$

Table 6 shows ranking of buildings in terms of relative DI (or Relative Seismic Damage Score) obtained for each building.

Table 6: Ranking of buildings based on relative seismic damage score

Buildings	Relative Damage Score	Rank
Moonlight School	60.688	1
Residential Building	49.129	3
House of BDO	43.437	4
Boys' Hostel at SMIT	15.850	5
Himalchuli hotel	50.618	2

The damage index and ranking of Moonlight School is highest. Extensive cracks in tension zone, shear failure in beams, severe cracking at beam-column joint, severe cracking between infill and concrete frame, concrete spalling in columns, out-of-plane failure of infill walls and settlement of the foundation were the factors that accounted for such a high damage score.

Large boulder in the foreground which seems to serve as foundation rock for the building was uplifted by the earthquake as mentioned in one of the reports of EERI. The Moonlight School suffered a pancake collapse of the third storey, and incipient collapse of the fourth storey. The building suffered severe damage during the earthquake. Furthermore, Table 5 shows that Himalchuli Hotel has obtained second highest damage score out of the five buildings considered in the study. This building was damaged to such an extent during the 2011 Sikkim earthquake that it was ordered to be demolished. This high seismic damage score obtained from the damage index model is in compliance to the severity of destruction caused to the building.

7. PLOTTING OF SEISMIC DAMAGE VALUES OF STUDIED BUILDINGS

A graph was plotted for comparative Seismic Damage Index for these five buildings and the obtained graphs are shown in Figs. 3 & 4.

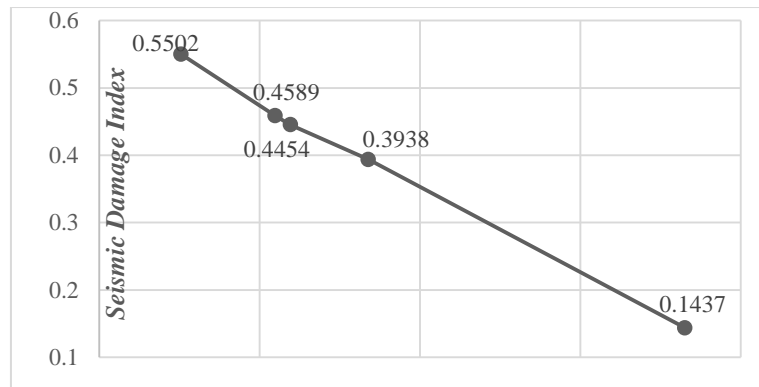


Fig. 3 Seismic Damage Index of buildings

In other words, it can be concluded that a building with lowest damage score suffered less seismic damage and vice-versa.

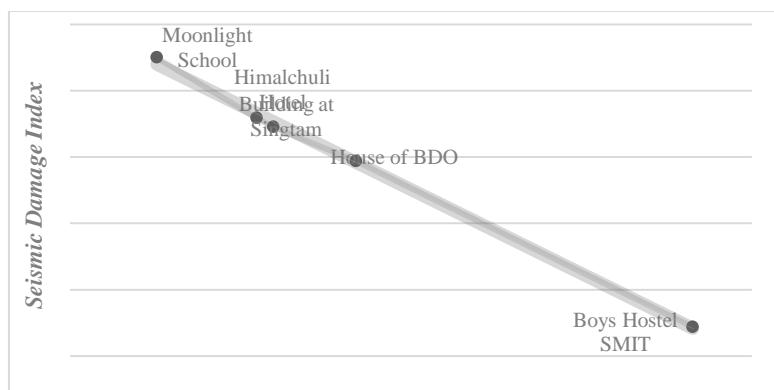


Fig. 4 Buildings in order of their Seismic Damage Index

8. CONCLUSION

- Identification, assessment and quantification of seismic damage of a reinforced concrete building is often a critical and complex task. In order to ease the decision making process, the problem has been decomposed and structured in the form of a hierarchical model and subsequently, by identifying, characterizing and assigning comparative weightage to different factors and attributes, a seismic damage assessment index (SDAI) for R.C. buildings has been developed.
- The damage assessment and indexing model developed has been demonstrated on a set of buildings situated at Sikkim that were damaged in devastating 2011 earthquake. The model is applied and utilized to compute the seismic damage score of these buildings.
- These buildings are then ranked on the basis of their seismic damage score.
- It was found that building with the highest damage score suffered maximum damage compared to other sample buildings during the earthquake.
- Also, it is found that building with lowest damage score actually survived and performed well compared to other during the earthquake.
- The presented model is very simple and easy to implement as the value of measurable items or elements for seismic damage are easily identifiable and quantifiable.
- The seismic damage index based on Fuzzy-AHP is simple, and hence holds the potential for practical application.

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