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HYBRID APPROACH FOR PEST MANAGEMENT MODEL WITH IMPULSIVE CONTROL

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Abstract*— *In this paper we study the dynamics of plant-pest virus, organic pesticides and natural enemy food chain model. Here our control input will be impulsive releasing of virus, natural enemy and organic pesticides. The main aim of this paper is to study two periodic solutions namely, Plant pest extinction and pest extinction periodic solutions using the above mentioned model. Using Floquet theory of impulsive differential equations and small amplitude perturbation technique we establish pest control through the local stability of both the periodic solutions. Also numerical examples have been given in order to demonstrate the effectiveness of the presented theoretical results.

Keywords— Impulsive Differential Equations, Pest Management, Stability, Periodic solution, Floquet theory

I. INTRODUCTION

It is knows to everyone, that impulsive differential equations serve as basic models to study the dynamics of processes that are subject to sudden changes in their states. They have been extensively studied in the past several years, see Bainov et. al. [1], Haddad et. al. [5], Ignatyev and Alexander O.[6], Lakshmikantham, Bainov, and Simeonov [8], Li and Xiaodi [9], Samoilenko and Perestyuk [10], Stamova et. al. [11]. A very basic and important qualitative problem in the study of impulsive differential equations concerns the stability and attractiveness of periodic solutions.

Many important and interesting mathematical models on this topic have been reported. On the other hand, impulsive mathematical models has become a very important direction in the theory of impulsive differential equations, stimulated by their numerous applications to problems arising in pest control, orbital transfer of satellite, ecosystems management, electrical engineering, and so on.

Pest control is a most important application of this model. Tremendous benefits have been derived from the use of pesticides in different sectors, especially in agriculture, a sector in which the Indian economy depends. Many reports have shown that considerable economic losses would be suffered without pesticide use. But pesticides are known to pollute the environment, contaminate water and soil, deplete soil fertility, and affect non target organisms. Also it is seen that pests can build up resistance to the pesticides by regular use. So farmers are forced to use strong pesticides in large quantity while its outstanding performance is poisonous to the warm blooded animals. Chemical pesticides are toxic and which only causes a number of health effects, but it is linked to a range of serious illnesses and diseases in humans, like respiratory problems to cancer etc... To solve this problem recently, the models for pest control were studied by some authors [2–4, 7, 12, 13] and some results were obtained.This paper is divided into 3 sections :-

- In section II, we describe a mathematical model which discuss the complete behaviour of plant pest virus, natural enemy and organic pesticides. This model is actually a modified form of the food chain model developed by wang et. al. [13].
- In section III, some important lemmas are discussed. Also an effort has been put forth to check the boundedness of the system.
- In section IV, the stability of plant pest eradication periodic solution is discussed.
- In section V, numerical examples have been given in order to demonstrate the effectiveness of the presented theoretical results.

II. MATHEMATICAL MODELLING

The following assumptions are made prior to the proposal of the mathematical model which discuss the complete behavior of plant pest virus, natural enemy and organic pesticides.

Hypothesis : we make following hypothesis for formulate mathematical model

- Susceptible pest attacks plant.
- Virus attacks susceptible pest and make them infected.
- Infected pest when dies release virus.
- Natural enemy attacks susceptible pest and consumes them directly.
- Virus and natural enemy are released periodically.
- Effect of natural pesticide on natural enemy is negligible.
- Pesticides are sprayed in an impulsive and periodic fashion, with the same period as the action of releasing infected pests but at different moments. As a result, fixed proportions $1 - q_1$ and $1 - q_2$ of susceptible pests and infected pests respectively are killed each time.

With these assumptions, the model proposed by Wang et. al. [13] is modified and following mathematical model is proposed:

$$\left.\begin{cases}\frac{dx(t)}{dt} = x(t)\left(1-x(t)\right)-cx(t)s(t)\\ \frac{ds(t)}{dt} = cx(t)s(t)-\alpha s(t)w(t)-\beta s(t)e(t)\\ \frac{dI(t)}{dt} = \alpha s(t)w(t)-\beta I(t)\\ \frac{dw(t)}{dt} = \mu r_2 I(t)-r_2 w(t)\\ \frac{de(t)}{dt} = \beta s(t)e(t)-r_3 e(t)\end{cases}\right\} \begin{matrix} t\neq nT \\ t\neq (n+\lambda-1)T \\ 0<\lambda<1\end{matrix}$$

$$\left.\begin{cases}x(t^+) = x(t)\\ s(t^+) = s(t)\\ I(t^+) = I(t)\\ w(t^+) = w(t)+\eta_1(t)\\ e(t^+) = e(t)+\eta_2(t)\end{cases}\right\} t = nT$$

$$\left.\begin{cases}x(t^+) = x(t)\\ s(t^+) = q_1(t)\\ I(t^+) = q_2 I(t)\\ w(t^+) = w(t)\\ e(t^+) = e(t)\end{cases}\right\} \begin{matrix} t\neq (n+\lambda-1)T \\ 0<\lambda<1\end{matrix} \tag{2.1}$$

where x(t), s(t), I(t), w(t) and e(t) are densities of plant, suspectable pest, infected pest, virus particles and natural enemy respectively, 'c' is predation rate of plant by suspectable pest, 'α' is conversion rate of suspectable pest to infected pest 'β' is rate of predation by natural enemy, 'μ' is production rate of virus from infected pest, r_1, r_2 and r_3 are natural death rates of infected pests, virus particles and natural enemies respectively, η_1 and η_2 are pulse releasing amount of virus particles and natural enemies at t = nT, n = 1,2,... and T is the period of impulsive effect.

III. PRELIMINARIES

The solution of system (2.1) is denoted by $Y(t) = \left(x(t), s(t), I(t), e(t)\right)'$ and is a piecewise continuous function $Y(t): R_+ \to R_+^5$ that is , $Y(t)$ is continuous in the interval $(nT,(n+\lambda)T]$, $((n+\lambda)T,(n+1)T]$ and $n \in Z^+$. The smoothness properties of variables guarantee the global existence and uniqueness of a solution of the system (2.1) for details, see [8]. Before proving the main results, we firstly state and establish some Lemmas which are useful in coming section.

Lemma 3.1. [8]: *The function $m \in PC'[R^+,R]$ and $m(t)$ be left continuous at t_k, k = 1,2,... satisfy the inequalities*

$$\begin{cases} m'(t) \le p(t)m(t)+q(t), & t\ge t_0, t\neq t_k,\\ m(t^+)\le d_k m(t_k)+b_k, & t = t_k, k=1,2,3,... \end{cases} \tag{3.1}$$

where $p, q \in PC[R^+,R]$ and $d_k \ge 0$, b_k are constants, then

$$m(t) \le m(t_0) \prod_{t_0<t_k<t} d_k \exp\left(\int_{t_0}^{t} p(s)ds\right) + \sum_{t_0<t_k<t}\left(\prod_{t_0<t_k<t} d_j \exp\left(\int_{t_0}^{t} p(s)ds\right)\right) b_k$$
$$+\int_{t_0}^{t} \prod_{t_0<t_k<t} d_k \exp\left(\int_{s}^{t} p(\sigma)d\sigma\right) q(s)ds, \qquad t \ge t_0 \tag{3.2}$$

If all the directions of the inequalities in (3.1) *are reversed, then* (3.2) *also holds true for the reversed inequality.*

Lemma 3.2. : *There exists a constant $L > 0$, such that $x(t) \le L$, $s(t) \le L$, $I(t) \le L$, $e(t) \le L$ and $w(t) \le L$ for all solutions $Y(t) = (x(t), s(t), I(t), w(t), e(t))$ of system (1) with t large enough.*
Proof: Define $U(t) = x(t) + s(t) + I(t) + e(t)$ and $0 < r < \min\{r_1, r_3\}$.
Then for $t \ne nT$, We get,

$$R^+U(t) + \bar{r}U(t) \le (1+\bar{r})x(t) - x^2(t)$$
$$\le M_0, \qquad \text{where } M_0 = \frac{\left(1+\bar{r}\right)^2}{4}$$

When $t = nT$, $U(t^+) \le U(t) + \eta_2$. Using Lemma 3.1 for $t \in (nT,(n + 1)T]$, then we have

$$U(t) \le U(0)\exp\left(-\bar{r}t\right) + \int_0^t M_0\eta_2 \exp\left(-\bar{r}(t-s)\right)ds + \sum_{0<nT<t} \eta_2 \exp\left(-\bar{r}(t-nT)\right)$$
$$\to \frac{M_0}{\bar{r}} + \eta_2 \exp\left(-\bar{r}T\right)\big/\left(\exp(\bar{r}t)-1\right), \qquad \text{as} \quad t \to \infty$$

Then we have $U(t)$ is uniformly bounded and hence, by the definition of $U(t)$, there exits a constant L_1
$L_1 := \dfrac{M_0\eta_2}{r} + \eta_2 \exp\left(-\bar{r}T\right)\big/\left(\exp(\bar{r}t)-1\right)$ such that $x(t) \le L_1$, $s(t) \le L_1$, $I(t) \le L_1$, $e(t) \le L_1$ for all t large enough.
Now consider the subsytem (2.1)

$$\begin{cases} v'(t) = \gamma I(t) - \gamma_2 v(t) \le \gamma L_1 - \gamma_2 v(t), & t \ne nT \\ v(t^+) = v(t) + \eta_1, & t = nT \end{cases}$$

Using Lemma 3.1, we get

$$v(t) \le v(0)\exp(-\eta t) + \int_0^t \gamma L_1 \exp\left(-r_2(t-s)\right)ds + \sum_{D<kT<t} \eta_1 \exp\left(-r_2(t-kT)\right)$$
$$\to \frac{\gamma L_1}{\gamma_2} + \eta_1 \exp\left(r_2 T\right)\big/\left(\exp(r_2 T)-1\right) \equiv L_2(\text{say}) \quad \text{as} \quad t \to \infty$$

Choosing $L = \max\{L_1, L_2\}$, we get the required results.

Lemma 3.3. [7]: *Consider the following impulsive system*

$$\begin{cases} u'(t) = c - du(t), & t \ne nT \\ u(t^+) = u(t) + \mu, & t = nT, \quad n = 1,2,3,\ldots \end{cases} \tag{3.3}$$

Then system (3.3) *has a positive periodic solution $u^*(t)$ and for every solution $u(t)$ of* (3.3), *we have $|u(t) - u^*(t)| \to 0$ as $t \to \infty$, where for $t \in (nT,(n + 1)T]$,*

$$u^*(t) = \frac{c}{d} + \frac{\mu\exp\left(-d(t-nT)\right)}{1-\exp(-dt)} \ \textit{with}\ \ u^*(0^+) = \frac{c}{d} + \frac{\mu}{1-\exp(-dt)}$$

Now we proceed to find pest extinction periodic solutions for the model (2.1). For the case of pest-extinction, we obtain the following impulsive system

$$\left\{\begin{array}{l}\frac{dx(t)}{dt}=x(t)\left(1-x(t)\right)\\ \frac{dw(t)}{dt}=-\gamma_2 w(t)\\ \frac{de(t)}{dt}=-\gamma_3 p(t)\end{array}\right\}\begin{array}{l}t\neq nT\\ t\neq (n+\lambda-1)T\\ 0<\lambda<1\end{array} \tag{3.4}$$

$$\left\{\begin{array}{l}w(t^+)=w(t)+\eta_1\\ e(t^+)=e(t)+\eta_2\end{array}\right\} t=nT$$

Considering the first equation of above subsystem, which is independent from the rest of the equations, we get two equilibrium points, namely x(t) = 0 and x(t) = 1.
For rest of the system (3.4), using Lemma 3.3 we obtain that,

$$w^*(t)=\frac{\eta_1\exp\left(-r_2(t-nT)\right)}{1-\exp(-r_2T)} \text{ and } e^*(t)=\frac{\eta_2\exp\left(-r_3(t-nT)\right)}{1-\exp(-r_3T)}$$

is a positive solution of the subsystem is globally asymptotically stable.

IV. STABILITY ANALYSIS

Theorem 4.1. *Let* ($x(t)$, $s(t)$, $I(t)$, $w(t)$, $e(t)$) *be any solution of the system* (2.1), *the plant-pest eradication periodic solution* (0, 0, 0, $w^*(t)$, $e^*(t)$) *is unstable.*
Proof: The local stability of periodic solution (0, 0, 0, $w^*(t)$, $e^*(t)$), we define

$$\begin{array}{l}x(t) = 1 + \psi_1(t),\\ s(t) = \psi_2(t),\\ I(t) = \psi_3(t),\\ w(t) = w^*(t) + \psi_4(t),\\ e(t) = e^*(t)+ \psi_5(t)\end{array}$$

The system (2.1) can be expanded in the following linearized form:

$$\left\{\begin{array}{l}\frac{d\psi_1(t)}{dt}=\psi_1(t)\\ \frac{d\psi_2(t)}{dt}=-\left(\alpha w^*(t)+\beta e^*(t)\right)\psi_2(t)\\ \frac{d\psi_3(t)}{dt}=\alpha\psi_2(t)w^*(t)-r_1\psi_3(t)\\ \frac{d\psi_4(t)}{dt}=\mu r_1\psi_3(t)-r_2\left(\psi_4(t)+w^*(t)\right)\\ \frac{d\psi_5(t)}{dt}=\beta\psi_2(t)e^*(t)-r_3\left(\psi_3(t)+e^*(t)\right)\end{array}\right\}\begin{array}{l}t\neq nT\\ t\neq (n+\lambda-1)T\\ 0<\lambda<1\end{array}$$

$$\left\{\begin{array}{l}\psi_1(t^+)=\psi_1(t)\\ \psi_2(t^+)=\psi_2(t)\\ \psi_3(t^+)=\psi_3(t)\\ \psi_4(t^+)=\psi_4(t)\\ \psi_5(t^+)=\psi_5(t)\end{array}\right\} t=nT$$

$$\left\{\begin{array}{l}\psi_1(t^+)=\psi_1(t)\\ \psi_2(t^+)=q_1\psi_2(t)\\ \psi_3(t^+)=q_2\psi_3(t)\\ \psi_4(t^+)=\psi_4(t)\\ \psi_5(t^+)=\psi_5(t)\end{array}\right\}\begin{array}{l}t=(n+\lambda-1)T\\ 0<\lambda<1\end{array}$$

From (4.1) if we get a fundamental matrix $\psi(t)$ then

$$\frac{d\psi(t)}{dt} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -(\alpha w^*(t)+\beta p^*(t)) & 0 & 0 & 0 \\ 0 & \alpha w^*(t) & -r_1 & 0 & 0 \\ 0 & 0 & \mu r_1 & -r_2 & 0 \\ 0 & \beta e^*(t) & 0 & 0 & -r_3 \end{pmatrix} \psi(t) \qquad (4.2)$$

$$= A\,\psi(t)$$

The linearization of impulsive conditions of (2.1), the equations of (2.1) precisely sixth to tenth becomes

$$\begin{pmatrix} \psi_1(t^+) \\ \psi_2(t^+) \\ \psi_3(t^+) \\ \psi_4(t^+) \\ \psi_5(t^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \\ \psi_4(t) \\ \psi_5(t) \end{pmatrix}$$

The linearization of impulsive conditions of (2.1) i.e. equations eleven to fifteenth of (2.1) is

$$\begin{pmatrix} \psi_1(t^+) \\ \psi_2(t^+) \\ \psi_3(t^+) \\ \psi_4(t^+) \\ \psi_5(t^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & q_1 & 0 & 0 & 0 \\ 0 & 0 & q_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \\ \psi_4(t) \\ \psi_5(t) \end{pmatrix}$$

Thus the monodromy matrix corresponding to (4.1) is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & q_1 & 0 & 0 & 0 \\ 0 & 0 & q_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \psi(T)$$

Thus from (4.2), we get

$$\psi(t) = \psi(0) \exp\left(\int_0^T A\,dt\right)$$

where $\psi(0)$ is identity matrix. And the eigen values of the monodromy matrix M are given by

$$\lambda_1 = \exp(-T) > 1$$

$$\lambda_2 = q_1 \exp\left(-\left(\int_0^T (\alpha w^*(t) + \beta e^*(t)dt)\right)\right) < 1$$

$$\lambda_3 = q_2 \exp(-r_1 T) < 1$$

$$\lambda_4 = \exp(-r_2 T) < 1$$

$$\lambda_5 = \exp(-r_3 T) < 1$$

Since $|\lambda_1| > 1$, From Floquet theory of impulsive differential equations we get the plant-pest extinction periodic solution of the system (2.1) is unstable .

Theorem 4.2. *Let* $(x(t), s(t), I(t), w(t), e(t))$ *be any solution of the system (2.1) the pest eradication periodic solution* $(1, 0, 0, w^*(t), e^*(t))$ *is locally asymptotically stable iff* $T \le T'_{max}$.

Proof: In order to discuss the stability of $(1, 0, 0, w^*(t), e^*(t))$ we define $x(t) = 1 + \psi_1(t)$, $s(t) = \psi_2(t)$, $I(t) = \psi_3(t)$, $w(t) = w^*(t) + \psi_4(t)$, $e(t) = e^*(t) + \psi_5(t)$, where the system (2.1) can be expanded in the following linearized form:

$$\left\{\begin{aligned}&\frac{d\psi_1(t)}{dt} = -\psi_1(t) - c\psi_2(t)\\&\frac{d\psi_2(t)}{dt} = -\left(-c + \alpha w^*(t) + \beta e^*(t)\right)\psi_2(t)\\&\frac{d\psi_3(t)}{dt} = \alpha\psi_2(t)w^*(t) - d_1\psi_3(t)\\&\frac{d\psi_4(t)}{dt} = \mu r_1\psi_3(t) - r_2\left(\psi_4(t) + w^*(t)\right)\\&\frac{d\psi_5(t)}{dt} = \beta\psi_2(t)e^*(t) - r_3\left(\psi_3(t) + e^*(t)\right)\end{aligned}\right\}\begin{aligned}&t \neq nT\\&t \neq (n+\lambda-1)T\\&0<\lambda<1\end{aligned}$$

$$\left\{\begin{aligned}&\psi_1(t^+) = \psi_1(t)\\&\psi_2(t^+) = \psi_2(t)\\&\psi_3(t^+) = \psi_3(t)\\&\psi_4(t^+) = \psi_4(t)\\&\psi_5(t^+) = \psi_5(t)\end{aligned}\right\} t = nT$$

$$\left\{\begin{aligned}&\psi_1(t^+) = \psi_1(t)\\&\psi_2(t^+) = q_1\psi_2(t)\\&\psi_3(t^+) = q_2\psi_3(t)\\&\psi_4(t^+) = \psi_4(t)\\&\psi_5(t^+) = \psi_5(t)\end{aligned}\right\}\begin{aligned}&t = (n+\lambda-1)T\\&0<\lambda<1\end{aligned} \tag{4.3}$$

Choose $\psi(t)$ be the fundamental matrix of (4.3), it must satisfy

$$\frac{d\psi(t)}{dt} = \begin{pmatrix} -1 & -c & 0 & 0 & 0 \\ 0 & -(-c+\alpha w^*(t)+\beta e^*(t)) & 0 & 0 & 0 \\ 0 & \alpha w^*(t) & -r_1 & 0 & 0 \\ 0 & 0 & \mu r_1 & -r_2 & 0 \\ 0 & \beta e^*(t) & 0 & 0 & -r_3 \end{pmatrix}\psi(t) \tag{4.4}$$

The linearization of impulsive conditions of (2.1) precisely sixth to tenth of (2.1) is

$$\begin{pmatrix}\psi_1(t^+)\\\psi_2(t^+)\\\psi_3(t^+)\\\psi_4(t^+)\\\psi_5(t^+)\end{pmatrix} = \begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}\psi_1(t)\\\psi_2(t)\\\psi_3(t)\\\psi_4(t)\\\psi_5(t)\end{pmatrix}$$

The linearization of impulsive conditions of (2.1) i.e. equations eleven to fifteenth of (2.1) is

$$\begin{pmatrix}\psi_1(t^+)\\\psi_2(t^+)\\\psi_3(t^+)\\\psi_4(t^+)\\\psi_5(t^+)\end{pmatrix} = \begin{pmatrix}1&0&0&0&0\\0&q_1&0&0&0\\0&0&q_2&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}\psi_1(t)\\\psi_2(t)\\\psi_3(t)\\\psi_4(t)\\\psi_5(t)\end{pmatrix}$$

Thus the monodromy matrix of (4.3) is

$$M = \begin{pmatrix}1&0&0&0&0\\0&q_1&0&0&0\\0&0&q_2&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}\psi(T)$$

Thus from (4.4), we get

$$\psi(t) = \psi(0)\exp\left(\int_0^T A\,dt\right)$$

where $\psi(0)$ is identity matrix. And the eigen values of the monodromy matrix M are given by

$$\lambda_1 = \exp(-T) < 1$$
$$\lambda_2 = q_1 \exp\left(\int_0^T (c - \alpha w^*(t) + \beta e^*(t)dt)\right)$$
$$\lambda_3 = q_2 \exp(-r_1 T) < 1$$
$$\lambda_4 = \exp(-r_2 T) < 1$$
$$\lambda_5 = \exp(-r_3 T) < 1$$

From Floquet theory of impulsive differential equations we get the plant-pest extinction periodic solution of the system (2.1) is locally asymptotically stable iff $|\lambda_2| \le 1$ that is $T_{max} < T'_{max}$.

Theorem 4.3. *Let* (1, 0, 0, $w^*(t)$, $e^*(t)$) *be the pest eradication periodic solution of system* (2.1). *Then the maximum period to become the solution is locally asymptotically stable,* $T'_{max} = \frac{1}{c}\left[\left(\frac{\alpha\eta_1}{r_1} + \frac{\beta\eta_2}{r_2}\right) + ln\left(\frac{1}{q_1}\right)\right]$

Corollary 4.1. *If* $T_{max} = \frac{1}{c}\left[\frac{\alpha\eta_1}{r_1} + \frac{\beta\eta_2}{r_2}\right]$ *be the maximum period to become the pest extinction periodic solution of the system*

$$\left.\begin{cases} \frac{dx(t)}{dt} = x(t)(1-x(t)) - cx(t)s(t) \\ \frac{ds(t)}{dt} = cx(t)s(t) - \alpha s(t)w(t) - \beta s(t)e(t) \\ \frac{dI(t)}{dt} = \alpha s(t)w(t) - \beta I(t) \\ \frac{dw(t)}{dt} = \mu r_2 I(t) - r_2 w(t) \\ \frac{de(t)}{dt} = \beta s(t)e(t) - r_3 e(t) \end{cases}\right\} t \ne nT$$

$$\left.\begin{cases} x(t^+) = x(t) \\ s(t^+) = s(t) \\ I(t^+) = I(t) \\ w(t^+) = w(t) + \eta_1(t) \\ e(t^+) = e(t) + \eta_2(t) \end{cases}\right\} t = nT$$

that makes the solution locally asymptotically stable then $T_{max} < T'_{max}$

V. NUMERICAL ANALYSIS

At first a pest control model is introduced with virus particles and natural enemies as the control inputs. They are released impulsively. In this section we analyze the theoretical findings numerically. For this Table 1 is given.

Table 1: **Parametric values for numerical**

Parameter	Description	Value per week
c	Predation rate of plant	0.5
$d1$	Natural death rate of infected pest population	0.1
$d2$	Natural death rate of virus particles	0.2
$d3$	Natural death rate of natural enemy	0.2
μ	Production rate of virus from infected pest	0.5
α	Conversion rate of plant to pest	0.5
β	Conversion rate of pest to natural enemy	0.2
$\eta 1$	Impulsive releasing amount of virus particle	2
$\eta 2$	Impulsive releasing amount of natural enemies	2

It includes the values of various parameters of the system (2.1) which are chosen per week, with $x(0^+) = 1$, $s(0^+) = 1$, $I(0^+) = 1$, $w(0^+) = 1$, and $z(0^+) = 1$ see [2].

In this paper we propose a new mathematical model by extending Corollary 4.1, that can be practically implemented. In this model we add an additional control input, organic pesticides which is released impulsively. Hence we add a new quantity $1 - q_1$ and $1 - q_2$, Table 2 with the parameters used in the above model.

Table 2: **Parametric values for numerical**

Parameter	Description	Value at each time
$1 - q_1$	Fixed proportion of susceptible pests killed after spraying pesticide	0.4
$1 - q_2$	Fixed proportion of infected pests killed after spraying pesticide	0.4

And thus we obtain a new threshold limit T'_{max} using Theorem 4.2 and Theorem 4.3

$$T'_{max} = T_{max} + \frac{1}{c}\ln\left(\frac{1}{q_1}\right)$$
$$\approx T_{max} + 1.02$$
$$\text{Where } T_{max} = \frac{1}{c}\left[\frac{\alpha\eta_1}{r_1} + \frac{\beta\eta_2}{r_2}\right]$$
$$= 14$$
$$\text{So } T'_{max} \approx 15.02 \approx 15$$
$$\text{Then } T_{max} < T'_{max}$$

From this analysis it is clear that in this model the period of impulsive release of virus particle and natural enemy is lengthened.

CONCLUSIONS

We have analysed the dynamics of plant-pest-virus, organic pesticide and natural enemy food chain model. Spraying organic pesticide may lengthen the period of impulsive release of virus particle and natural enemy. It reduces the cost of pest control. The impulsive control is then used. Theoretically, pest control is successful while applying impulsive inputs, provided it should be applied often enough (T is small), adequate number of pests should die due to pesticide spraying ($1 - q_1$ is large) or adequately many virus particle and natural enemy should be released periodically (η_1, η_2 is large). However, it is practically not possible to provide arbitrary large η_1 and η_2, also time period can be effected by human activities. So that the active time may not enough. So we depend on organic pesticide spraying alone.

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