

CERTAIN RESULTS INVOLVING POLYBASIC HYPERGEOMETRIC FUNCTIONS AND INFINITE PRODUCTS

Neera A. Herbert, Pooja Patel

Department of Mathematics and Statistics

Sam Higginbottom University of Agriculture, Technology and Sciences, Allahabad.

Abstract : *In this paper , making use of certain Polybasic hypergeometric functions and infinite products, an attempt has been made to establish results involving polybasic hypergeometric functions and infinite products.*

Keywords : *Polybasic hypergeometric functions, infinite products, summations formulae, truncated series.*

1. Introduction

In 2011 Srivastava et. al. [4] by making use of Bailey’s transformation of truncated series, have established transformation formulae involving polybasic hypergeometric functions.

2. Notations

In this section , we list some standard summation and transformation formulae for the basic hypergeometric series which are used in this paper.

Assuming that $|q| < 1$, where q - is non-zero complex number, this condition ensures that all the infinite product will converge.

$$(a; q)_n = (a, q)_n = \begin{cases} 1 ; n = 0 \\ (1 - a)(1 - aq) \dots (1 - aq^{n-1}) ; n = 1, 2, 3, \dots \end{cases} \quad (2.1)$$

$$(a_1, a_2, a_3, \dots, a_r; q)_n = (a_1; q)_n (a_2; q)_n (a_3; q)_n \dots (a_r; q)_n \quad (2.2)$$

$$(a; q)_{-n} = \frac{\left(\frac{-q}{a}\right)_n q^{\frac{n(n-1)}{2}}}{\left(\frac{q}{a}; q\right)_n} (a; q)_n \quad (2.3)$$

$$(a; q)_{2n} = (a; q^2)_n (aq; q^2)_n \quad (2.4)$$

$$(a; q)_n = \frac{(a; q)_\infty}{(aq^n; q)_\infty} \quad (2.5)$$

$$(a; q)_\infty = \prod_{r=0}^{\infty} (1 - aq^r) \quad (2.6)$$

The polybasic hypergeometric series is defined,

$$\begin{aligned} & \Phi \left[\begin{matrix} a_1, a_2, \dots, a_r; c_{1,1}, \dots, c_{1,r_1}; \dots; c_{r,q,1}, \dots, c_{m,r_m}; q, q_1, \dots, q_m; z \\ b_1, b_2, \dots, b_{r-1}; d_{1,1}, \dots, d_{1,r_1}; \dots, d_{m,1}, \dots, d_{m,r_m} \end{matrix} \right] \\ &= \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q)_n z^n}{(q, b_1, b_2, \dots, b_{r-1}; q)_n} \prod_{j=1}^m \frac{(c_{j,1}, \dots, c_{j,r_j}; q_j)_n}{(d_{j,1}, \dots, d_{j,r_j}; q_j)_n} \end{aligned} \quad (2.7)$$

A truncated basic hypergeometric series is

$${}_r\Phi_s \left[\begin{matrix} a_1, a_2, a_3 \dots a_r; q; Z \\ b_1, b_2, b_3 \dots b_s \end{matrix} \right]_N = \sum_{n=0}^N \frac{[a_1, a_2, a_3 \dots a_r; q]_n z^n}{[b_1, b_2, b_3 \dots b_s; q]_n} \quad (2.8)$$

where , $\max (|q|, |z| < 1)$ and no zero appears in the denominator.

The other notations appearing in this paper have their usual meaning. We shall use the following summation formulae in our analysis.

Slater [3]

$${}_2\phi_1 \left(\begin{matrix} a, y, q; q \\ ayq \end{matrix} \right)_n = \frac{[aq, yq; q]_n}{[q, ayq; q]_n} \quad (2.9)$$

Agarwal [1]

$${}_4\phi_3 \left(\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, e; q; 1/e \\ q, \sqrt{\alpha}, -\sqrt{\alpha}, \alpha q/e \end{matrix} \right)_n = \frac{(\alpha q, e q; q)_n}{(q, \alpha q/e; q)_n e^n} \quad (2.10)$$

Gasper and Rahman [2]

$${}_6\phi_5 \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\beta q}{\gamma}, \frac{\gamma q}{\delta} \end{matrix} ; q, q \right]_n = \frac{(\alpha q, \beta q, \gamma q, \delta q; q)_n}{(q, \frac{\alpha q}{\beta}, \frac{\beta q}{\gamma}, \frac{\gamma q}{\delta}; q)_n} \quad \text{provided } \alpha = \beta\gamma\delta \quad (2.11)$$

Gasper and Rahman [2]

$$\sum_{r=0}^n \frac{(1-ap^r)q^r (1-bp^r q^{-r}) (c, \frac{a}{bc}; q)_r q^r}{(1-a)(1-b)(q, \frac{aq}{b}; q)_r (\frac{ap}{c}, bcp; p)_r} = \frac{(ap, bp; p)_n (cq, \frac{aq}{bc}; q)_n}{(q, \frac{aq}{b}; q)_n (\frac{ap}{c}, bcp; p)_n} \quad (2.12)$$

In 1966 Slater established the following simple but very useful Bailey transformation in the form, if

$$\beta_n = \sum_{r=0}^{\infty} \alpha_r u_{n-r} v_{n+r} \quad (2.13)$$

$$\gamma_n = \sum_{r=0}^{\infty} \delta_{r+n} u_r v_{2n+r} \quad (2.14)$$

Where $\alpha_r, \delta_r, u_r, v_r$ are functions of r alone and the series for γ_n is convergent.

3. Main results

$$\Phi \left[\begin{matrix} a, y; \alpha q; \beta q; p, q; p \\ ayp; q, \alpha\beta q \end{matrix} \right] - \Phi \left[\begin{matrix} \alpha, \beta; a, y; q, p; pq \\ \alpha\beta q; p, ayp \end{matrix} \right] + \Phi \left[\begin{matrix} \alpha, \beta; ap, yp; q, p; q \\ \alpha\beta q; p, ayp \end{matrix} \right] = \frac{[(\alpha q, \beta q; q)_{\infty}]}{[(q, \alpha\beta q; q)_{\infty}]} \frac{[(ap, yp; p)_{\infty}]}{[(p, ayp; p)_{\infty}]} \quad (3.1)$$

$$\Phi \left[\begin{matrix} \alpha q, \beta q; a, y; q, p; p\beta \\ \frac{\alpha q}{\beta}; p, ayp \end{matrix} \right] + \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; ap, yp; q, p; q\beta \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; p, ayp \end{matrix} \right] - \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; a, y; \beta p \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; p, ayp \end{matrix} \right] = \frac{[(ap, yp; p)_{\infty}]}{[(p, ayp; p)_{\infty}]} \sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_n} \beta^n \quad (3.2)$$

$$\begin{aligned} & \Phi \left[\begin{matrix} \alpha q, \beta q, \gamma q, \delta q; a, y; \\ \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; p, ayp; q, p; p \end{matrix} \right] + \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; a, y; \\ q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; p, ayp; q, p; p \end{matrix} \right] \\ & - \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; a, y; q, p; qp \\ q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; p, ayp \end{matrix} \right] = \left[\frac{(\alpha q, \beta q, \gamma q, \delta q; q)_{\infty}}{(q, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; q)_{\infty}} \right] \left[\frac{(ap, y; p)_{\infty}}{(p, ayp; p)_{\infty}} \right] \end{aligned} \quad (3.3)$$

$$\begin{aligned} & \Phi \left[\begin{matrix} x, y: ap, bp: cq, \frac{aq}{bc}; \\ xyP: a: b: \frac{ap}{c}, bcp: q, \frac{aq}{b}; \end{matrix} P, pq, \frac{p}{q}, p, q; P \right] \\ & + \Phi \left[\begin{matrix} xP, yP: ap, bp: cq, \frac{aq}{bc}; \\ xyP: a: b: \frac{ap}{c}, bcp: q, \frac{aq}{b}; \end{matrix} P, pq, \frac{p}{q}, p, q; q \right] \\ & - \Phi \left[\begin{matrix} x, y: apq: \frac{bp}{q}: a, b: c, \frac{a}{bc}; \\ xyP: a: b: \frac{ap}{c}, bcp: q, \frac{aq}{b}; \end{matrix} P, bq, \frac{p}{q}, p, q; Pq \right] \\ & = \frac{(xP, yP; P)_{\infty} (ap, bp; p)_{\infty} (cq, \frac{aq}{bc}; q)_{\infty}}{(P, xyP; P)_{\infty} (q, \frac{aq}{b}; q)_{\infty} (\frac{ap}{c}, bcp; p)_{\infty}} \end{aligned} \quad (3.4)$$

4. Proof of Main Results

Now first we take $u_r = v_r = 1$ in (2.13) and (2.14) Bailey's transformation takes the following form:

$$\text{If } \beta_n = \sum_{r=0}^n \alpha_r \quad (4.1)$$

$$Y_n = \sum_{r=n}^{\infty} \delta_r$$

$$Y_n = \sum_{r=0}^{\infty} \delta_{r+n} \quad (4.2)$$

$$\text{then } \sum_{n=0}^{\infty} \alpha_n Y_n = \sum_{n=0}^{\infty} \beta_n \delta_n \quad (4.3)$$

By putting the value of β_n and γ_n above equation can be written as

$$\begin{aligned} & \sum_{n=0}^{\infty} \alpha_n \sum_{r=0}^{\infty} \delta_r + \sum_{n=0}^{\infty} \alpha_n \delta_n \\ & = \sum_{n=0}^{\infty} \delta_n \sum_{r=0}^n \alpha_r + \sum_{n=0}^{\infty} \alpha_n \sum_{r=0}^n \delta_r \end{aligned} \quad (4.4)$$

Proof of result (3.1) Taking $\alpha_r = \frac{(\alpha, \beta; q)_r q^r}{(q, \alpha \beta q; q)_r}$ and $\delta_r = \frac{(a, y; p)_r p^r}{(p, ayp; p)_r}$ and putting the value of α_r and δ_r in (4.4) respectively, we get the following transformation:

$$\begin{aligned} & \sum_{n=0}^{\infty} \frac{(\alpha, \beta; q)_n q^n}{(q, \alpha \beta q; q)_n} \sum_{r=0}^{\infty} \frac{(a, y; p)_r p^r}{(p, ayp; p)_r} + \sum_{n=0}^{\infty} \frac{(\alpha, \beta; q)_n q^n (a, y; p)_n p^n}{(q, \alpha \beta q; q)_n (p, ayp; p)_n} \\ & = \sum_{r=0}^{\infty} \frac{(\alpha, \beta; q)_r q^r}{(q, \alpha \beta q; q)_r} + \sum_{n=0}^{\infty} \frac{(\alpha, \beta; q)_n q^n}{(q, \alpha \beta q; q)_n} \sum_{r=0}^n \frac{(a, y; p)_r p^r}{(p, ayp; p)_r} \end{aligned} \quad (4.5)$$

Now using result (2.9) we get the following transformation:

$$\Phi \left[\begin{matrix} a, y; \alpha q; \beta q; p, q; p \\ \alpha y p; q, \alpha \beta q \end{matrix} \right] - \Phi \left[\begin{matrix} \alpha, \beta; a, y; q, p; p q \\ \alpha \beta q; p, \alpha y p \end{matrix} \right] + \Phi \left[\begin{matrix} \alpha, \beta; \alpha p, y p; q, p; q \\ \alpha \beta q; p, \alpha y p \end{matrix} \right]$$

$$= \frac{[(\alpha q, \beta q; q)_{\infty}]}{[(q, \alpha \beta q; q)_{\infty}]} \frac{[(\alpha p, y p; p)_{\infty}]}{[(p, \alpha y p; p)_{\infty}]}$$

Which on simplification gives the result (3.1)

Proof of result (3.2) Taking $\alpha_r = \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_r} \beta^r$ and $\delta_r = \frac{(a, y; p)_r p^r}{(p, \alpha y p; p)_r}$ and putting the value of α_r and δ_r in (4.4) respectively, we get the following transformation:

$$\sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_n} \beta^n \sum_{r=0}^{\infty} \frac{(a, y; p)_r p^r}{(p, \alpha y p; p)_r} + \sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_n} \beta^n \frac{(a, y; p)_n p^n}{(p, \alpha y p; p)_n} =$$

$$\sum_{n=0}^{\infty} \frac{(a, y; p)_n}{(p, \alpha y p; p)_n} p^n \sum_{r=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_r} \beta^r \sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_n} \beta^n \sum_{r=0}^{\infty} \frac{(a, y; p)_r p^r}{(p, \alpha y p; p)_r}$$

Now using result (2.10) we get the following transformation:

$$\Phi \left[\begin{matrix} \alpha q, \beta q; a, y; q, p; p \beta \\ \frac{\alpha q}{\beta}; p, \alpha y p \end{matrix} \right] + \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; \alpha p, y p; q, p; q \beta \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; p, \alpha y p \end{matrix} \right] - \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; a, y; \beta p \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; p, \alpha y p \end{matrix} \right]$$

$$= \frac{[(\alpha p, y p; p)_{\infty}]}{[(p, \alpha y p; p)_{\infty}]} \sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_n} \beta^n$$

Which on simplification gives the result (3.2)

Proof of result (3.3) Taking $\alpha_r = \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; q)_r} q^r$ and $\delta_r = \frac{(a, y; p)_r p^r}{(p, \alpha y p; p)_r}$ and putting the value of α_r and δ_r in (4.4) respectively, we get the following transformation:

$$\sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; q)_n} \sum_{r=0}^{\infty} \frac{(a, y; p)_r p^r}{(p, \alpha y p; p)_r} + \sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; q)_n} \frac{(a, y; p)_n p^n}{(p, \alpha y p; p)_n}$$

$$= \sum_{n=0}^{\infty} \frac{(a, y; p)_n p^n}{(p, \alpha y p; p)_n} \sum_{r=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_r}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; q)_r} q^r + \sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; q)_n} \sum_{r=0}^{\infty} \frac{(a, y; p)_r p^r}{(p, \alpha y p; p)_r}$$

Now using result (2.11) we get the following transformation:

$$\Phi \left[\begin{matrix} x, y; \alpha p, \beta p; c q, \frac{\alpha q}{bc}; \\ xy p; a: b: \frac{\alpha p}{c}, b c p; q, \frac{\alpha q}{b}; P, p q, \frac{p}{q}, p, q; P \end{matrix} \right] + \Phi \left[\begin{matrix} x p, y p; \alpha p, \beta p; c q, \frac{\alpha q}{bc}; \\ xy p; a: b: \frac{\alpha p}{c}, b c p; q, \frac{\alpha q}{b}; P, p q, \frac{p}{q}, p, q; q \end{matrix} \right]$$

$$- \Phi \left[\begin{matrix} x, y; \alpha p q; \frac{b p}{q}; a, b: c, \frac{a}{bc}; \\ xy p; a: b: \frac{\alpha p}{c}, b c p; q, \frac{\alpha q}{b}; P, b q, \frac{p}{q}, p, q; P q \end{matrix} \right]$$

$$= \frac{(x p, y p; P)_{\infty}}{(P, x y p; P)_{\infty}} \frac{(a p, \beta p; p)_{\infty}}{(q, \frac{\alpha q}{b}; q)_{\infty}} \frac{(c q, \frac{\alpha q}{bc}; q)_{\infty}}{(\frac{\alpha p}{c}, b c p; p)_{\infty}}$$

Which on simplification gives the result (3.3)

Proof of result (3.4) Taking $\alpha_r = (a p q; p q)_r (\frac{b p}{q}; \frac{p}{q})_r (a, b; p)_r (c, \frac{a}{bc}; q)_r q^r (a; p q)_r (b; \frac{p}{q})_r (q, \frac{\alpha q}{b}; q)_r (\frac{\alpha p}{c}, b c p; p)_r$ and $\delta_r = \frac{(x, y; P)_r P^r}{(P, x y p; P)_r}$ and putting the value of α_r and δ_r in (3.4) respectively, we get the following transformation:

Now using result (2.12) we get the following transformation:

$$\begin{aligned} & \Phi \left[\begin{matrix} x, y: ap, bp: cq, \frac{aq}{bc}; \\ xyP: a: b: \frac{ap}{c}, bcp: q, \frac{aq}{b}; \end{matrix} P, pq, \frac{p}{q}, p, q; P \right] \\ & + \Phi \left[\begin{matrix} xP, yP: ap, bp: cq, \frac{aq}{bc}; \\ xyP: a: b: \frac{ap}{c}, bcp: q, \frac{aq}{b}; \end{matrix} P, pq, \frac{p}{q}, p, q; q \right] \\ & - \Phi \left[\begin{matrix} x, y: apq: \frac{bp}{q}: a, b: c, \frac{a}{bc}; \\ xyp: a: b: \frac{ap}{c, bcp: q, \frac{aq}{b}}; \end{matrix} P, bq, \frac{p}{q}, p, q; Pq \right] \\ & = \frac{(xP, yP; P)_{\infty} (ap, bp; p)_{\infty} (cq, \frac{aq}{bc}; q)_{\infty}}{(P, xyP; P)_{\infty} (q, \frac{ap}{b}; q)_{\infty} (\frac{ap}{c}, bcp; p)_{\infty}} \end{aligned}$$

Which on simplification gives the result (3.4)

5. Special cases

1. Replacing $p = q$ in (3.1) we obtain

$$\begin{aligned} & {}_4\Phi_3 \left[\begin{matrix} a, y; \alpha q, \beta q; q; q \\ \alpha y q; q, \alpha \beta q \end{matrix} \right] - {}_4\Phi_3 \left[\begin{matrix} \alpha, \beta; a, y; q; q^2 \\ \alpha \beta q; q, \alpha y q \end{matrix} \right] + {}_4\Phi_3 \left[\begin{matrix} \alpha, \beta; aq, yq; q; q \\ \alpha \beta q; q, \alpha y q \end{matrix} \right] \\ & = \frac{[(\alpha q, \beta q; q)_{\infty}]}{[(q, \alpha \beta q; q)_{\infty}]} \frac{[(aq, yq; q)_{\infty}]}{[(q, \alpha y q; q)_{\infty}]} \end{aligned}$$

Replacing $p = q$ in (3.2) we obtain

$$\begin{aligned} & {}_4\Phi_3 \left[\begin{matrix} \alpha q, \beta q; a, y; q; q\beta \\ \frac{\alpha q}{\beta}; q, \alpha y q \end{matrix} \right] + {}_4\Phi_3 \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; aq, yq; q; q\beta \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q, \alpha y q \end{matrix} \right] \\ & - {}_4\Phi_3 \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; a, y; \beta q \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q, \alpha y q \end{matrix} \right] = \frac{[(aq, yq; q)_{\infty}]}{[(q, \alpha y q; q)_{\infty}]} \sum_{n=0}^{\infty} \frac{(\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta; q)_n}{(q, \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}; q)_n} \beta^n \end{aligned}$$

6. Reference

1. Agarwal, R.P. (1978), Generalized hypergeometric series and its application to the theory of combinational analysis and partition theory *Oxford University Press, Ely House, London.*
2. Gasper and Rahman (1991) Basic hypergeometric series, Encyclopedia of mathematics and its Applications, *Cambridge University press, New York, NY, USA.*
3. Slater, L.J (1966), Generalized Hypergeometric Functions, *Cambridge university Press, Cambridge*
4. Srivastava P. and Rudraravapu M, (2011) Certain Transformation Formulae for Polybasic Hypergeometric Series, *International Scholariv Research Network, (1-10) ISRN Algebra.*