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Elasto-Plastic analysis of a square plate with hole using higher order shear deformation theory

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***Abstract**—The plates are major load carrying structural elements in building, automobile, aerospace structures, etc. The situations like mat foundations, manholes in tank, certain machine parts, ships, submarines, etc, make it necessary to provide a hole in the plate. The pressure at hole causes stress concentration around holes and a complicated stress distribution throughout the plate. An accurate prediction of behaviour of plate not only improves the safety and economy of these elements, but also the safety and economy of the whole structure. This study intends to model such plates for loads and stresses ranging from elastic to plastic behaviour using a higher order shear deformation theory (HOSDT), which accounts for warping of the cross section. In the Elasto-Plastic analysis, the structure is designed beyond the yield stress, which allows a redistribution of stresses beyond elastic limit, thus increasing the load carrying capacity. An incremental iterative finite element method has been used for the analysis of such plates. The square plates with clamped and simply supported edges have been studied for a central square hole. The spread of plastic zone is demonstrated for such plates.*

***Keywords**— Elasto-Plastic; Higher Order Shear Deformation Theory; Square Plate; Hole; Spread of Plasticity*

I. INTRODUCTION

A plate is an important structural element in buildings, automobiles, naval, aerospace industries, etc. It is the major load carrying element in most of these applications. Plates are capable of taking bending moments and shear forces perpendicular to their own plane, as well as forces lying in their own plane. An accurate prediction of behaviour of plate not only improves the safety and economy of these elements, but also the safety and economy of the whole structure through a better monitoring of stress and load transfers to the other connected elements.

The proper prediction of bending behaviour, of plates resulted in the development of various bending theories. Kirchhoff's classical plate theory was based on over simplifying assumptions involving neglect of the transverse normal strain, transverse normal stress and transverse shear strain which have limited the scope of their theory to thin plates only. The first order shear deformation theory (FOSDT), which accounted for the transverse shear strain were also applicable to moderately thick plates and had a limited improvement in accuracy. Some heavy structural engineering applications such as nuclear reactors, deep tunnels, heavy machine foundation, etc., involve the use of thick plates where even these theories prove insufficient.

If the plate is considered as three dimensional body and the equations of elasticity are applied, definitely it renders a most accurate solution, but the resulting governing partial differential equations become so complicated that their solutions are either too costly or very often impossible for many boundary conditions, loads, etc. The higher order shear deformation theories were degenerated from three dimensional elasticity equations to two dimensional representation and account for warping of the cross section by higher order assumed displacement field. These refined theories are based on a linear elastic relationship between the stresses and strains and thus a design based on them restricted the material to be stressed to this limit while most of the material remains under stressed. This renders the elastic design highly uneconomical. An Elasto-Plastic design, in which almost the whole material is stressed to its yield limit and even beyond that, has to be very economical. The Elasto-Plastic design is also essential for structures which are occasionally stressed beyond the elastic limit of their materials, like aerospace structures or the structures which are subjected to blast and earthquake loads. Further, almost all the materials, particularly the ductile ones, can withstand strains much higher than those encountered within the elastic limit. The design based on elastic behaviour therefore fails to take the advantage of some of these materials to carry stresses above yield stress.

There are many situations where it is necessary to provide a hole in the plate. Such situations are mat foundations, manhole in tank, utility ducts in building, certain machine parts, ship, submarines, pressure vessels, chimney stacks, walls of building subjected to wind loads, roofs of building etc. The pressure at hole causes stress concentration around holes and a complicated stress distribution throughout the plate. It also makes the computation of load-deflection and stress-strain behaviour difficult. This study intends to model such plates for all ranges of load and stresses. Even in elastic analysis, there is a complex stress distribution around holes which to be dealt with accurate methods of analysis. Material around hole is therefore more prone to be stressed beyond yield stress which further cause redistribution of stresses after yield. The Elasto-Plastic analysis of such problem is of utmost important for modeling its actual behaviour.

II. LITERATURE SURVEY

2.1 Development in the elastic plate bending theories

Kirchhoff was developed a theory based on assumption that straight lines perpendiculars to mid-surface (i.e. transverse normal) before deformation remain straight and perpendicular to mid-surface after deformation. This assumption limited the application of this theory to the thin plates only. In thick plate applications however, the effect of shear deformation is significant. This guided the research towards the developments of more accurate theories, which account, at least approximately, for the effect of the shear deformation on the plate behaviour. Reissner whowas first attempted to incorporate the effect of shear deformation. He used a variational formulation technique to correct the transverse displacement only, while the stresses σ_x, σ_y and τ_{xy} remained linear in the thickness coordinate $'z'$. Srinivas and Rao [1] conducted a three-dimensional elasticity analysis of flexure of thick rectangular plates and compared with the corresponding classical thin plate theory results. Their results serve as standard benchmark solutions for the assessment of shear-deformation theories. Kant [2] analysed the rectangular plate with refined higher-order theory based on three-dimensional Hooke's law. The higher-order displacement model adopted gives rise to a more realistic quadratic variation of transverse shear strain and a linear variation of transverse normal strain through the plate thickness. He used the segmentation method and suggested its use for analysis of plates simply supported on two opposite edges for an efficient, economical and accurate solution.

Reddy [3] presented a higher-order shear deformation theory accounting for the cubic variation of in-plane displacements and the parabolic distribution of transverse shear strains though the thickness of the plates and von Karmann strains. Lee *et al*. [4] simplified a higher order theory with the assumption of in-plane rotation tensor not varying through the thickness. The theory accounts for a cubic variation of in-plane stress and parabolic variation of transverse shear stress with zero values at free surfaces. As most of the problems were either difficult or impossible to solve by the analytical methods, the solutions were sought using the numerical techniques. Many elements accounting for transverse shear deformation have been developed for finite element solutions, although initially most of these were either restricted to rectangular shape or required many degrees of freedom.

2.2 Recent development in elastic and Elasto-Plastic incremental non-linear analysis

The designs based on the elastic bending analysis, however accurate they may be, will not be the most economical as these analyses limit the stresses within the elastic limit. However, when the structure is loaded beyond elastic limit, the plastic strain occurs, which causes a redistribution of stresses. The computation of this redistribution is not easy, and limit analysis has been the usual option for such an analysis. The limit analysis is applicable to rigid perfectly-plastic behaviour while almost all the materials are elastic before yielding. The current state of stress in a yielded material again depends upon the history of loading. Therefore, an analysis starting from the loading in the elastic range and gradually increasing to plastic range till failure would be closer to the true behaviour of the plate.

Ukadgaonker and Rao [5] gave a general solution for two dimensional stress distribution around triangular holes in isotropic plate and orthotropic plate with oriented fibres and multilayered symmetric laminates. Generalised plane stress with an equivalent single layer approach is adopted. Uniaxial, biaxial and shear stresses at infinity were considered in general solution for arbitrary biaxial loading. Tangential shear, uniform pressure around hole were considered in other solutions. Results were obtained for single layered and multi layered plates for graphite/epoxy and other materials.

Cesim and Onur [6] studied an elastic-plastic stress analysis and expansion of plastic zone in layers of stainless steel fiber-reinforced aluminum metal laminated plates. Plates with simply supported and clamped edges were considered for Elasto-plastic stress analysis using finite element method and first order shear deformation theory (FOSDT) for small deflection. Conclusions were drawn on the basis of yielding point and intensity of residual stress component in composite plate.

Tripathi and Rode [7] successfully applied higher order shear deformation theory proposed by Kant and later modified by Kant and Pandya for nonlinear analysis of plate bending using an incremental finite element formulation. They have concluded that HOSDT predicts higher collapse load as compared to other formulations based on rather first order shear deformation theory or classical plate theory. Tripathi *et al.* [8] applied higher order shear deformation theory for elasto-plastic analysis of plate bending layered model. The incremental finite elemental formulation is used with nine noded Heterosis element by provision of selective and reduced integration. For solving non-linear equations modified Newton-Raphson method is applied. The spread of plastic zone with respect to the load at the cross section of the plate can be studied by incremental finite element formulation. Von Mises and Tresca yield criteria have been applied for yielding of the material along with associated flow rule.

Rao *et al*. [9]presented solution useful for finding stress distribution around holes in symmetric laminates as well as isotropic plates. They have studied Graphite/epoxy and Glass/epoxy laminates with square and rectangular holes. It is noted that the maximum stress and its location is mainly due to type of loading and the large stresses are obtained for shear loading.Kant *et al.*[10]utilizes a higher order shear deformation theory for Elasto-plastic analysis of thick plate bending using incremental finite element formulation. Von Mises yield criteria and associated flow rule has been modeled for yielding of the material. The results are compared with available benchmark and other solutions.

III. THEORETICAL FORMULATION

3.1 Theoretical formulation based on higher order shear deformation theory

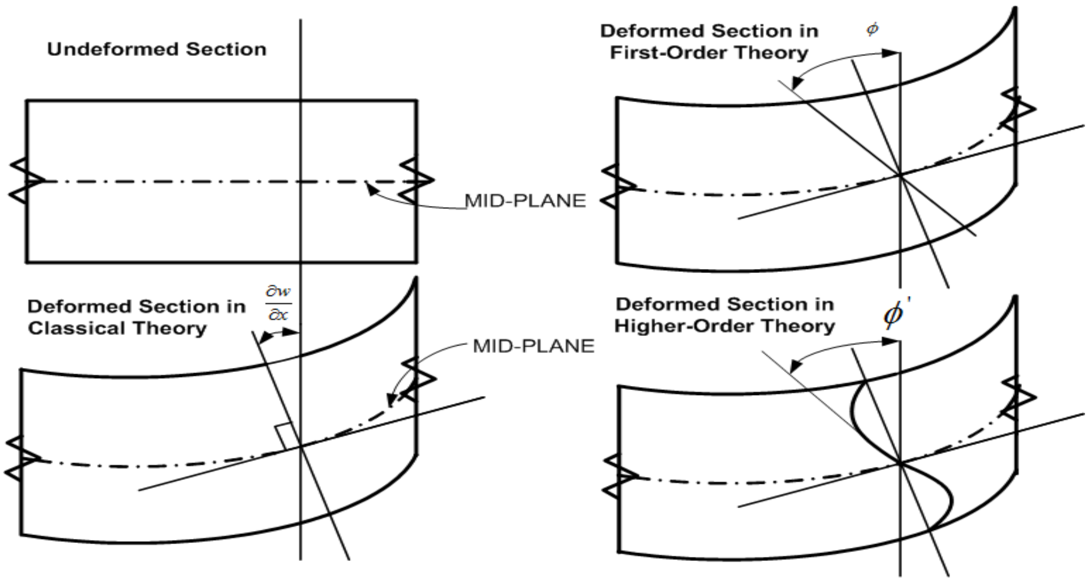

Fig. 1. Kinematics of deformation of a plate edge in various plate theories.

The displacement model for plates for these conditions is given by:

$$\left.\begin{aligned} u &= z\theta_x + z^3\theta_x^* \\ v &= z\theta_y + z^3\theta_y^* \\ w &= w_0 \end{aligned}\right\} \qquad (1)$$

θ_x and θ_y are the rotations of the normals to the mid-plane about y and x axes respectively. θ_x^* and θ_y^* are higher order term degenerated from Taylor's series for improving accuracy. The 'u' and 'v' are in-plane displacements and 'w' isthe transverse displacement.

3.2 Strain-displacement relationship and stress-strain relationship

The linear relationships between these displacements and strains can be obtained by using the definitions of strains from the theory of elasticity (Mendelson, [11]):

$$\begin{aligned}
\sigma_x &= \frac{E}{1-\nu^2}\left[\left(z\chi_x + z^3 \chi^*_x\right)+\nu\left(z\chi_y + z^3 \chi^*_y\right)\right] \\
\sigma_y &= \frac{E}{1-\nu^2}\left[\left(z\chi_y + z^3 \chi^*_y\right)+\nu\left(z\chi_x + z^3 \chi^*_x\right)\right] \\
\tau_{xy} &= G\left(z\chi_{xy} + z^3 \chi^*_{xy}\right) \\
\tau_{yz} &= G\left(\phi_y + z^2 \phi^*_y\right) \\
\tau_{zx} &= G\left(\phi_x + z^2 \phi^*_x\right)
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = z\chi_x + z^3 \chi^*_x \\
\varepsilon_y &= \frac{\partial u}{\partial y} = z\chi_y + z^3 \chi^*_y \\
\varepsilon_z &= 0 \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = z\chi_{xy} + z^3 \chi^*_{xy} \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \phi_y + z^2 \phi^*_y \\
\gamma_{zx} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi_x + z^2 \phi^*_x
\end{aligned} \tag{3}$$

The computer program, incorporating the higher order shear deformation and the corresponding finite element formulation for Elasto-Plastic analysis, developed by Rode and Kant [12], has been modified and validated for simply supported and clamped full plate and then used for analysis of plate with holes for various boundary conditions.

IV. NUMERICAL EXAMPLE

Example No. 1: Clamped square plate with 20% hole of size (a') 2.68 X 2.68, with a/h= 5, E= 30000, ν = 0.3, G = 11500 and a = 6.0 units. It is subjected to uniformly distributed load. The non-dimensional parameters used in the formulation are:

For non-dimensional displacement : $W_c = W_0 \dfrac{Eh^3}{12\left(1-\nu^2\right)\left(a^2 - a'^2\right)M_p}$ (4)

For non-dimensional uniformly distributed load: $\bar{q} = \dfrac{q_0\left(a^2 - a'^2\right)}{M_p}$ (5)

Example No. 2: Simply supported square plate with 20% hole of size (a') 2.68 X 2.68, with a/h= 5, E= 30000, ν = 0.3, G = 11500 and a = 6.0 units. It is subjected to uniformly distributed load The non-dimensional parameters used as above:

V. Result and Discussion

Table 1: Non-dimensional displacement and non-dimensional load for simply supported and clamped plate subjected to uniformly distributed load.

Plate	a/h	Actual Load App. (units)		Non-dim. Disp. (units)		Non-dim. Load (units)	
		Elastic	Collapse	Elastic	Collapse	Elastic	Collapse
Simply Supp.	05	4.1	6.0	0.07362	0.11297	10.9319	15.9979
	20	0.25	0.416	0.06284	0.13873	10.6652	17.747
	40	0.075	0.1135	0.07419	0.41231	12.8021	19.3738
	80	0.0175	0.028	0.0686	0.26441	11.9472	19.1155
	100	0.0125	0.0181	0.0764	0.35699	13.3315	19.3041
Clamped	05	12.00	20.25	0.05099	0.12724	31.997	53.9928
	20	0.7125	1.35	0.02928	0.15322	30.3959	57.5966
	40	0.15	0.345	0.02359	0.10786	25.596	58.8721
	80	0.0412	0.08625	0.02552	0.08384	28.1479	58.8547
	100	0.0285	0.054	0.02747	0.22101	30.3959	57.5923

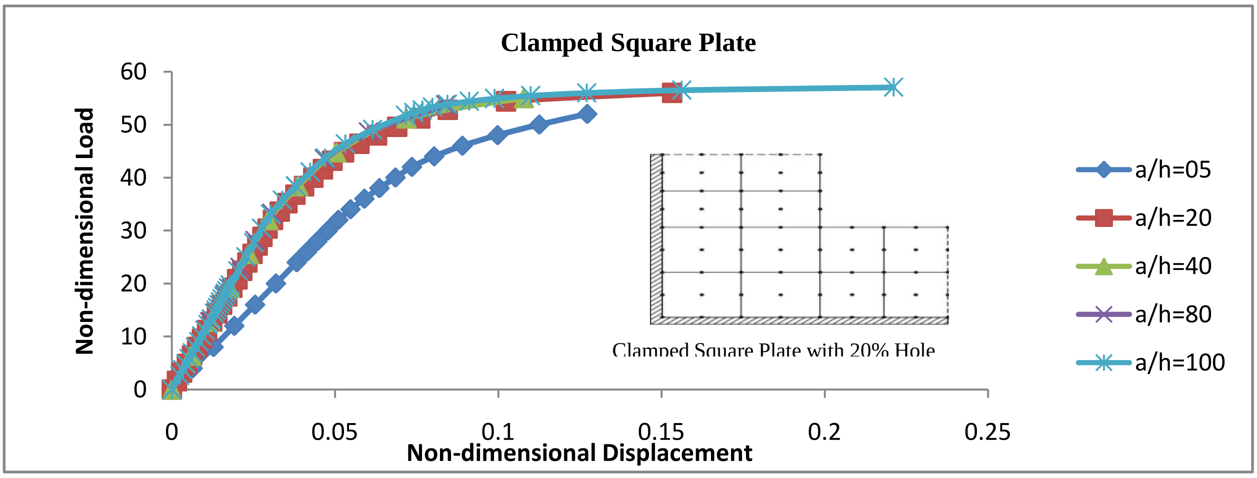

Fig. 2. Clamped square plate subjected to uniformly distributed load with 20% hole: Effect of thickness variation.

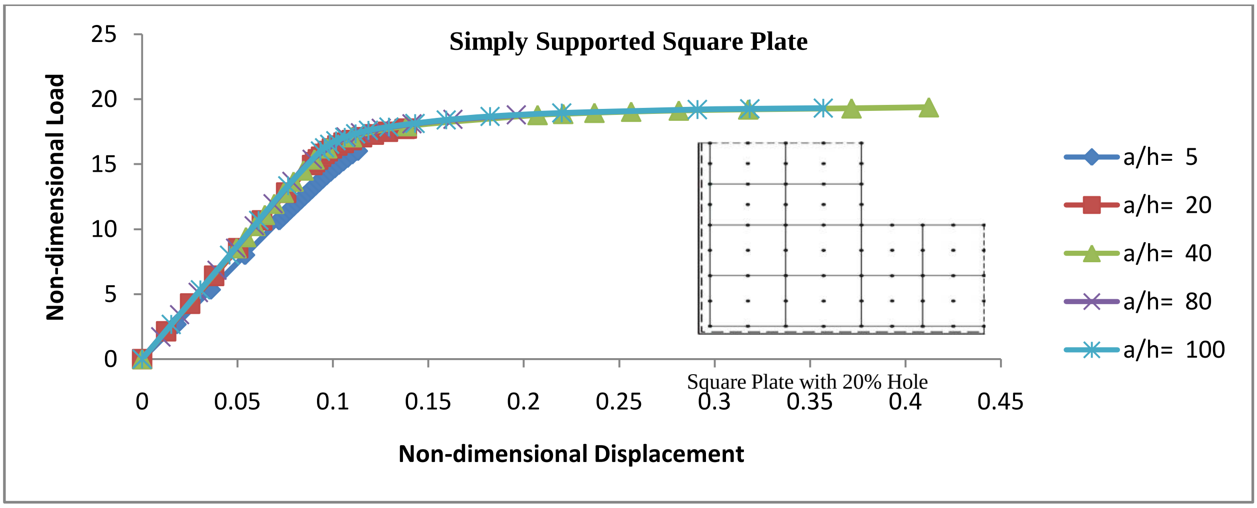

Fig. 3. Simply supported square plate subjected to uniformly distributed load with 20% hole: Effect of thickness variation.

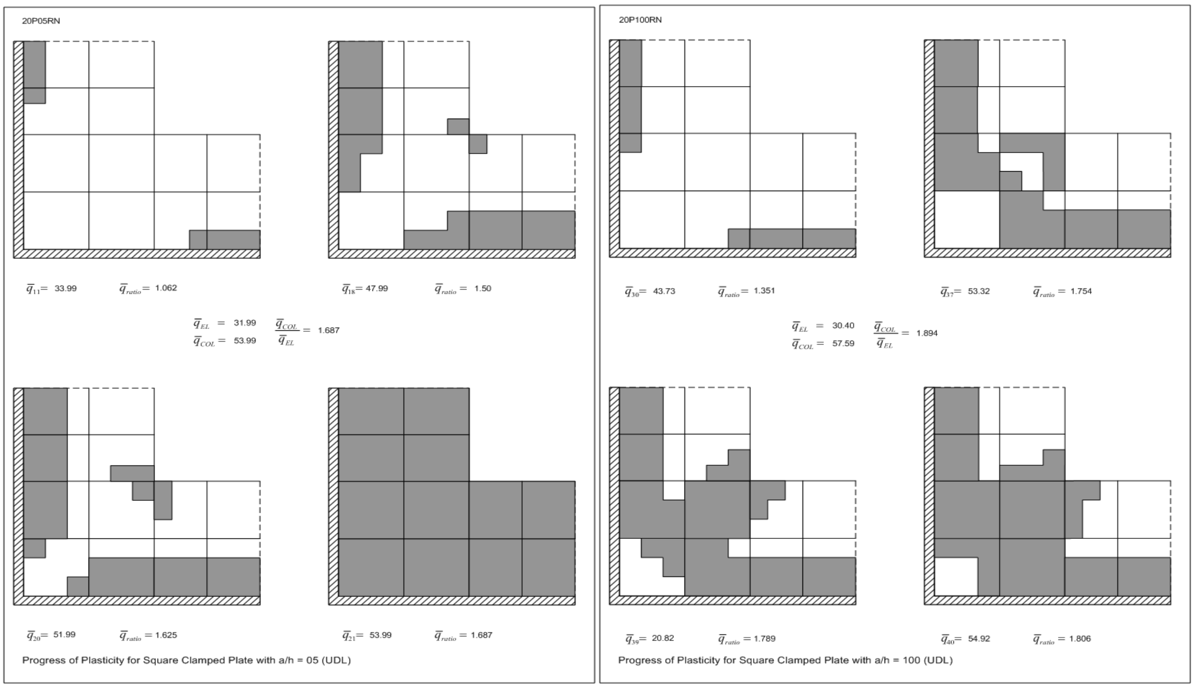

Fig. 4. Progress of plasticity for clamped square plate.

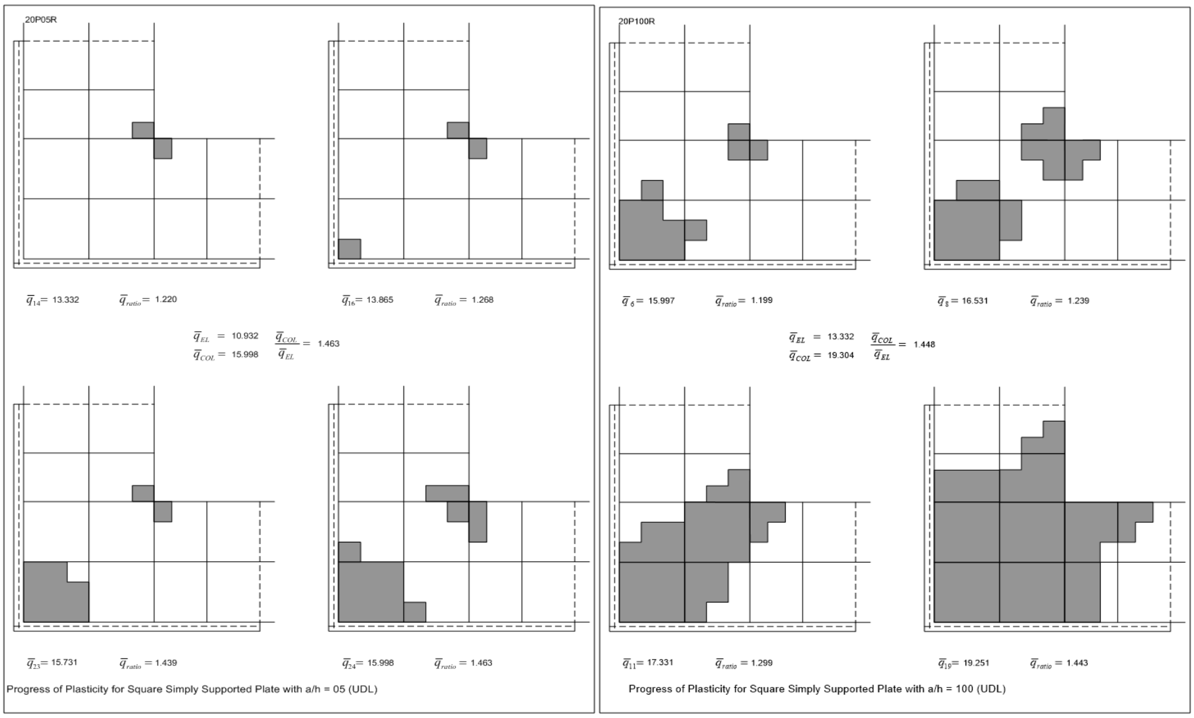

Fig. 5. Progress of plasticity for simply supported square plate.

Table 1 shows non-dimensionalized elastic and collapse loads to make them comparable. The effect of thickness variation can also be seen from the table.The Fig. 2 shows non-dimensional load versus non-dimensional displacement graph for clamped square plate having a central square hole with area 20% of the area of full plate, while Fig. 3 shows a similar graph for a simply supported square plate. The material properties considered in both the cases are same. The length of side of 20% hole area is 2.68 units, where side of full plate is 6.0 units. Five thicknesses ranging from thick to thin plate have been considered. The load has been applied incrementally from elastic to plastic range upto collapse. The gradual spread of plastic zone have been shown in Fig 4 for clamped plate and in Fig 5 for simply supported case. The a/h ratios considered in these figures are a/h=05 (Thick plate) and a/h=100 (Thin plate).

VI. CONCLUSION

A clamped and simply supported square plates with 20% hole analysed with higher order shear deformation theory and presented here for various thicknesses. The other geometric and material properties have been kept constant. The load has been applied incrementally. The start and gradual spread of plasticity has been depicted in the form of tabulated results and plan diagrams of the plate. The actual collapse load of thick to thin plate reduces drastically with decreasing thickness as their stiffness is proportional to cube of their thickness. On non-dimensionalization, the collapse loads almost converge to the same value, which endorses the selection of our non-dimensionalizing parameters.

Thus, normalization by non-dimensional parameters is essential to make them comparable on the same graph. The normalized displacements for simply supported plates are much higher as compared to those for clamped plates as seen in the graphs. The thick simply supported plate shows a brittle collapse while other plates depict ductile behaviour before collapse.

The observation of the normalized load versus normalized maximum displacement, graph indicates a flexible normalized behaviour of thick plates as compared to thin plates as opposite to actual behaviour. Although the yielding of plate starts at a lower load, the load versus displacement graph shows linearity upto some higher load inspite of spread of plasticity across the plate.

The plan view of the plate at increasing loads show that the plasticity starts at the centre of clamped edges of the plate followed by formation of another plastic zone at the centre of plate. At collapse, the two zones meet each other. On the other hand, plasticity starts at corners of the plate and corners of the hole. These two plastic zones spread towards each other as load increases, until they merge at collapse. However, simply supported thick plate collapse before they merge.

The comparison of the limit elastic load with collapse load of plate shows that the collapse load is almost double that of elastic limit load for all thicknesses. This emphasizes the importance of the present study to ascertain the reserve strength of a plate even after the start of plasticity.

REFERENCES

[1] Srinivas S and Rao A K, “Flexure of thick rectangular plates,” ASME Journal of Applied Mechanics, 1973, 40, 298-299.
[2] Kant T, “Numerical analysis of thick plates,” Computer Methods in Applied Mechanics and Engineering, 1982, **31**, 1-18.
[3] Reddy J N, “A refined nonlinear theory of plates with transverse shear deformation,” International Journal of Solids and Structures, 1984, **20**(9/10), 881-896.
[4] Lee K H, Senthilnathan N R, Lim S P and Chow S T, “Simple higher-order nonlinear shear deformation plate theory,” International Journal of Nonlinear Mechanics, 1989, **24**(2), 127-137.
[5] Ukadgaonker V G and Rao D K N, “Stress distribution around triangular holes in anisotropic plates,” Composite Structures, 1999, **45**, 171-183.
[6] Cesim A and Onur S,“Elastic plastic stress analysis and expansion of plastic zone in clamped and simply supported aluminum metal matrix laminated plates,” Composite Structures, 2000, **49**, 9-19.
[7] Tripathi R K and Rode V, “Elasto-plastic analysis of plate bending using higher order shear deformation theory,” Journal of the Institution of Engineers India Division, 2002, **83**, 191-200.
[8] Tripathi R K, Rode V, Verma M K and Thripathi N, “Finite element Elasto-plastic analysis of plate bending using layered higher order shear deformation theory,” IJSE10910, The IUP Journal of Structural Engineering, 2009, **10**, 1-18.
[9] Rao D K N, Babu M R, Reddy K R N and Sunil D, “Stress around square and rectangular cutouts in symmetric laminates,” Composite Structures, 2010, **92**, 2845-2859.
[10] Kant T, Tripathi R K and Rode V, “Elasto-Plastic behaviour of thick plates with a higher-order shear deformation theory,” Proc. Indian Natn. Sci.Acad, 2013, **79** No. 4, Spl.Issue PartA, 563-574.
[11] Mendelson A, Plasticity: “Theory and Applications”. Macmillan Publishing Co, New York, 1968.
[12] Rode V R and Kant T,“A program and user’s manual for elasto-plastic analyses of plates based on a higher-order shear deformation theory,” Rep. CE/SW/95/1, April 1995, Department of Civil Engineering, Indian Institute of Technology, Bombay.