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Takagi-Sugeno fuzzy modelling and control of two rule inverted pendulum

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Abstract— ***This paper gives a framework to model and control complex nonlinear system via Takagi-Sugeno fuzzy modelling and control. The nonlinear plant is first converted into regional fuzzy models for which the control gains are derived from linear matrix inequalities (LMIs). Then the local fuzzy controller are aggregated using a concept called parallel distributed compensation which yields the final control law. MATLAB simulation result are presented to analyse the stability and control of a two rule inverted pendulum and to show the effectiveness of the T-S fuzzy design approach.***

Keywords— ***TS fuzzy model; Linear matrix inequality; Parallel distributed compensation; Inverted pendulum; MATLAB simulink***

I. Introduction

In recent years, Takagi-Sugeno fuzzy model based control design have found variety of applications including ball and beam system [1], robot manipulator [2], control of DC series motor [3], induction motor [4], Permanent magnet synchronous motor, bidirectional inverter [5], DC-DC converter and many more. In T-S fuzzy control approach a nonlinear system is first converted into a T-S fuzzy model, then parallel distributed compensation scheme (PDC) is applied to design the T-S fuzzy controller, and the state feedback gains of which are calculated from the Linea matrix inequality (LMI) approach.

The T-S fuzzy model based control provides the environment of developing systematic tools for the analysis and design of fuzzy control system and it does not require an expert knowledge thus reducing errors due to human input.

In this paper we investigate the stability analysis of the continuous time system for the two rule inverted pendulum system and finding the solution to the LMIs formed using lyapunov stability conditions [6].

This paper is organised as follows: Section 2 describes the procedure of the construction of T-S fuzzy model. Section 3 gives the overview of parallel distributed compensation technique. In section 4, stability analysis and LMIs involved are presented. In section 5, Simulink model for two rule inverted pendulum is given. Section 6 gives the MATLAB simulation results. Section 7 finally gives the concluding remarks.

II. T-S Fuzzy Model

A T-S fuzzy system is described by fuzzy IF-THEN rules that represent locally linear input-output relationships of a system. The overall fuzzy model of the system is obtained by fuzzy aggregation of linear system models. The i^{th} rule of this continuous fuzzy system is of the following form:

$$IF\ z_1(t)\ is\ M_{i1}\ and\ \cdots\ and\ z_p(t)\ is\ M_{ip},$$

$$THEN \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), \\ y(t) = C_i x(t), \end{cases} \qquad i = 1,2,\ldots,r.$$

Here and r is the number of model rules;

Where $z(t) = [z_1(t),\ldots,z_p(t)]$ is the premise variable vector whose elements may be function of the states, external disturbances, and/or time, $x(t) \in R^n$ is the state vector, , Mij is the fuzzy input set, $u(t) \in R^m$ is the control input vector, $y(t) \in R^q$ is the output vector and $A_i \in R^{n\times n}, B_i \in R^{n\times m}, and\ C_i \in R^{q\times n}$ are state matrix, input matrix and output matrix respectively. Given a pair of $(x(t),u(t))$, the final outputs of the fuzzy systems is inferred as:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))\{A_i x(t) + B_i u(t)\}}{\sum_{i-1}^{r} w_i(z(t))} \tag{1}$$

$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)), \tag{2}$$

where M_{ij} is the membership function of the j[th] fuzzy set in the i[th] fuzzy rule. Let,

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))} \tag{3}$$

where $h_i(z(t))$ is the normalized weight for each rule. Then, (1) can be expressed as:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\} \tag{4}$$

Since $w_i(z(t)) \geq 0$ and $\sum_{i-1}^{r} w_i(z(t)) > 0$, we have $h_i(z(t)) \geq 0$ and $\sum_{i-1}^{r} h_i(z(t)) = 1$.

III. PARALLEL DISTRIBUTED COMPENSATION (PDC)

In designing a fuzzy controller for the control of a nonlinear system described by a T-S fuzzy model, PDC provides an easy and straightforward procedure [7]. In PDC design, a fuzzy control rule has to be designed for each corresponding rule of the fuzzy model. The designed fuzzy controller and the T-S fuzzy model share the same fuzzy sets in premise part. Consequent parts of the control rules consist of linear controllers. The i[th] rule of fuzzy controller is given as:

$$IF\ z_1(t)\ is\ M_{i1}\ and \cdots and\ z_p(t)\ is\ M_{ip},$$
$$THEN\ u(t) = -F_i x(t), \qquad i = 1,2,\ldots,r.$$

The fuzzy control rules have linear controllers (state feedback controllers) in consequent parts, the overall nonlinear controller is obtained by fuzzy blending of the linear controllers and the following is obtained:

$$u(t) = -\frac{\sum_{i=1}^{r} w_i(z(t))F_i x(t)}{\sum_{i=1}^{r} w_i(z(t))}$$

(6)

$$u(t) = -\sum_{i=1}^{r} h_i(z(t))F_i x(t).$$

The design problem is now to calculate local feedback gains in consequent parts.

IV. Stability Analysis and Linear Matrix Inequality

The main objective here is to select the feedback gain coefficient matrices, F_i that stabilizes the nonlinear system. Tanaka and sugeno derived the stability condition corresponding to a quadratic lyapunov function [6].

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\{A_i-B_iF_j\}x(t). \tag{7}$$

$$\dot{x}(t)=\sum_{i=1}^{r}h_i(z(t))h_j(z(t))G_{ii}x(t)+2\sum_{i=1}^{r}\sum_{i<j}h_i(z(t))h_j(z(t))\{\frac{G_{ij}+G_{ji}}{2}\}x(t) \tag{8}$$

Theorem 1: The equilibrium of the continuous fuzzy control system described by (8) is globally asymptotically stable if there exists a common positive definite matrix P such that

$$G_{ii}{}^T P+PG_{ii}<0, \qquad i=1,2,\ldots,r$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P+P\left(\frac{G_{ij}+G_{ji}}{2}\right)\le 0 \qquad i<j \;\; s.t.\;\; h_i\cap h_j\ne\phi.$$

where

$$G_{ij}=\frac{\{A_i-B_iF_j\}+\{A_j-B_jF_i\}}{2}$$

LMI formulation for stable controller design

We first present a stable fuzzy controller design problem which is to determine the feedback gains "*Fi*" for the continuous system using the stability conditions of Theorem 1. The conditions are not jointly convex in "*Fi*" and "*P*". Now multiplying the inequality on the left and right by P^{-1} and defining a new variable $X=P^{-1}$, we rewrite the conditions as

$$\begin{aligned}&-XA_i{}^T-A_iX+XF_i{}^TB_i^T+B_iF_iX>0,\\&-XA_i{}^T-A_iX+XF_i{}^TB_j^T+B_jF_iX-XA_j^T-A_jX+XF_j{}^TB_i^T+B_iF_jX\ge 0.\end{aligned} \tag{9}$$

Define $M_i=F_iX$ so that for $X>0$ we have $F_i=M_iX^{-1}$. Substituting into the above inequalities yields:

$$\begin{aligned}&-XA_i{}^T-A_iX+M_i{}^TB_i^T+B_iM_i>0,\\&-XA_i{}^T-A_iX+M_i^TB_j^T+B_jM_i-XA_j^T-A_jX+M_j^TB_i^T+B_iM_i\ge 0.\end{aligned} \tag{10}$$

Decay Rate Controller Design

The speed of response is related to decay rate, that is, the largest Lyapunov exponent. the largest lower bound on the decay rate that we can find using a quadratic Lyapunov function can be found by solving the following GEVP (Generalized eigenvalue minimization problem)in *X* and α :

Maximize α

$X,M_1,\ldots,M_r$

subject to conditions:

$X > 0$,

$$-XA_i^T - A_iX + M_i^TB_i^T + B_iM_i - 2\alpha X > 0,$$
$$-XA_i^T - A_iX + M_i^TB_j^T + B_jM_i - XA_j^T - A_jX + M_j^TB_i^T + B_iM_i - 4\alpha X \geq 0, \quad i < j \tag{11}$$

Where X = P^{-1} , $M_i = F_iX$. Decay rate fuzzy controller reduces to the stable fuzzy controller design when α = 0.

Constraint on control input

Assume that the initial condition x(0) is known. The constraint ‖ u(t) ‖$_2$ ≥ u is enforced at all times t ≥ 0 if the below LMIs hold in addition to above,

$$\begin{bmatrix} 1 & x(0)^T \\ x(0) & X \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} X & M_i^T \\ M_i & u^2I \end{bmatrix} \geq 0 \tag{12}$$

Where X = P^{-1} and $M_i = F_iX$.

Using these LMI conditions, we define a stable fuzzy controller design problem.

V. T-S FUZZY MODELLING AND CONTROL OF INVERTED PENDULUM

Consider the problem of balancing and swing-up of an inverted pendulum on a cart. Recall the equations of motion for the pendulum:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = \frac{g\sin(x_1(t)) - amlx_2^2(t)\sin(2x_1(t))/2 - a\cos(x_1(t))u(t)}{(4l/3) - aml\cos^2(x_1(t))} \tag{13}$$

Where $x_1(t)$ is angle (in radians) of inverted pendulum from the vertical, $x_2(t)$ is angular velocity, m is the mass of the pendulum, M is the mass of the cart, 2l is length of pendulum, u is force applied to the cart (in newtons), a=1/(m+M) and g is gravity constant.

Two-Rule Modeling and Control

The control objective of this subsection is to balance the inverted pendulum for the approximate range $x_1(t) = (-\pi/2, \pi/2)$. We first represent the system equations by a Takagi-Sugeno fuzzy model. To minimize the design effort and complexity, we try to use as few rules as possible. Therefore by using local approximation,

When $x_1(t)$ is near zero, the nonlinear equations can be simplified as

$$\dot{x}_1(t) = x_2(t),$$
$$\dot{x}_2(t) = \frac{gx_1(t) - au(t)}{4l/3 - aml} \tag{14}$$

When $x_1(t)$ is near $\pm\pi/2$, the nonlinear equations can be simplified as

$$\dot{x}_1(t) = x_2(t),$$
$$\dot{x}_2(t) = \frac{2gx_1(t)/\pi - a\beta u(t)}{4l/3 - aml\beta^2} \quad (15)$$

Notice that when $x_1 = \pm\pi/2$, the system is uncontrollable. Hence, we approximate the system by the following two-rule fuzzy model:

$Rule\,1:\ \ IF\ x_1(t)\ is\ about\ 0,\ \ THEN\ \ \ \dot{x}(t) = A_1 x(t) + B_1 u(t)$

$Rule\,2:\ \ IF\ x_1(t)\ is\ about\ \pm\pi/2(|x_1| < \pi/2),\ \ THEN\ \ \ \dot{x}(t) = A_2 x(t) + B_2 u(t)$

Here,

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4l/3 - aml} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix} \quad (16)$$

Membership functions for Rules 1 and 2 are shown in Figure:

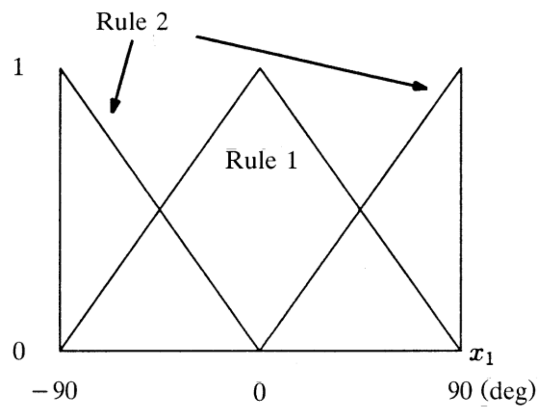


Fig. 1 Membership functions $M_1(z_1(t))$ *and* $M_2(z_1(t))$

Thus the open loop system is represented as:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))\{A_i x(t) + B_i u(t)\} \quad (17)$$

Here u(t) is taken as unit step function. And $h_i(z(t))$ represents the degree of Membership function where M_1 and M_2 are the Member Functions. M_1 for Rule 1 and M_2 for Rule 2.

$$h_1(x(t)) = M_1(x_1(t))$$
$$h_2(x(t)) = M_2(x_1(t))$$

Now fuzzy control rule corresponding to PDC control Law is as follows:

$Rule\,1:\quad IF\ \ x_1(t)\ is\ about\ 0,\quad THEN\qquad u(t) = -F_1 x(t)$

$Rule\,2:\quad IF\ \ x_1(t)\ is\ about\ \pm\pi/2(|x_1| < \pi/2),\quad THEN\qquad u(t) = -F_2 x(t)$

Now aggregating the local linear gains to obtain to a control input for the close loop nonlinear model:

$$u(t) = -\sum_{i=1}^{2} h_i(x_1(t))F_i x(t) \ = -h_1(x_1(t))F_1 x(t) - h_2(x_1(t))F_2 x(t) \qquad (18)$$

Now putting the above value of control input in the open loop equation we obtain the Takagi Sugeno close loop fuzzy system.

$$\dot{x}(t) = \sum_{i=1}^{2}\ \sum_{j=1}^{2} h_i(x(t))h_j(x(t))\{A_i - B_i F_j\}x(t) \qquad (19)$$

MATLAB Simulink Model:

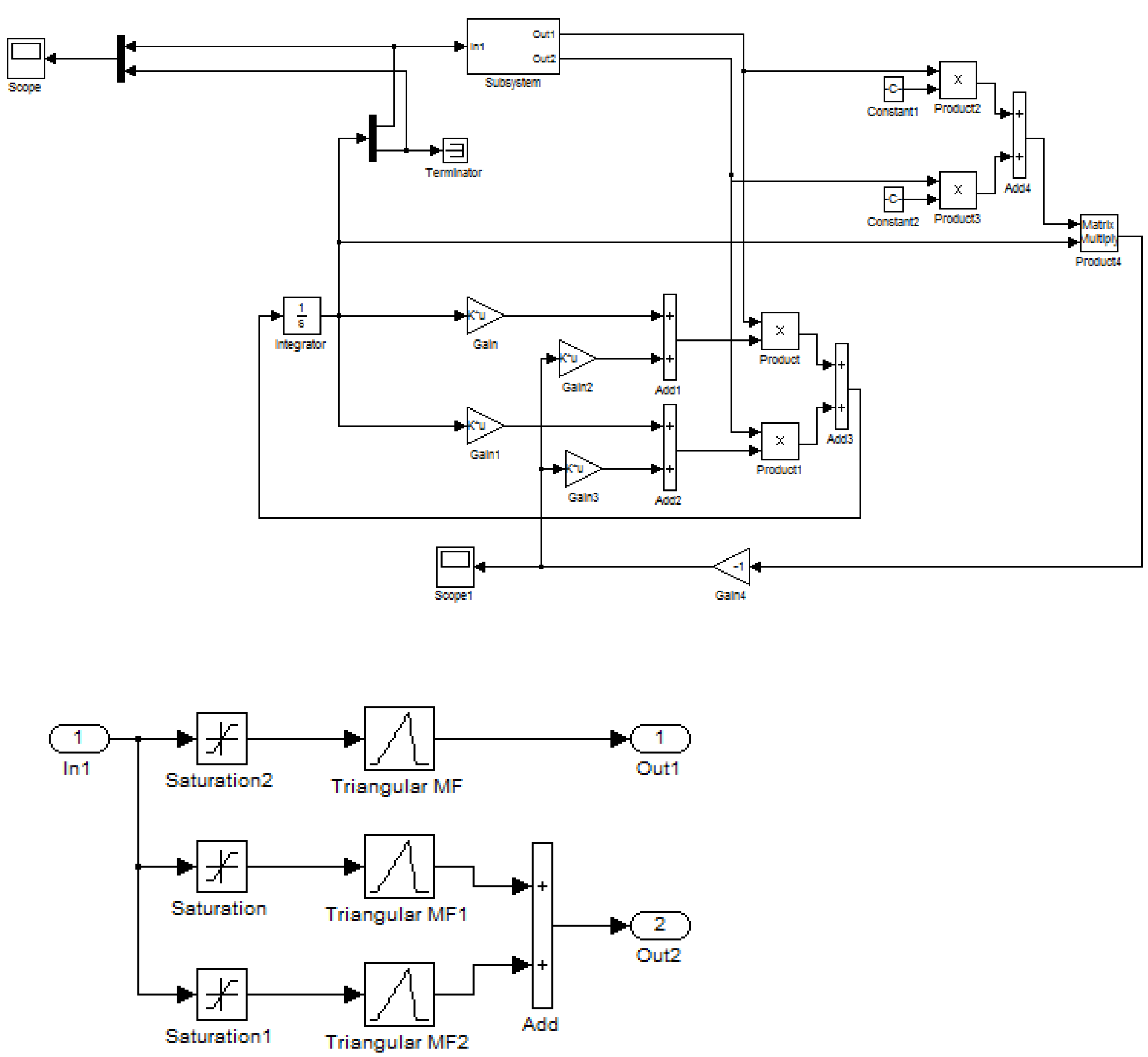


Fig. 2 (a) Simulink implementation of close loop 2 Rule Inverted Pendulum. (b) Fuzzy Operation Block

VI. SIMULATION RESULTS

The simulation are done on the MATLAB platform and solution to the LMIs are obtained using CVX [8] solver.

Here angular position and control input of the inverted pendulum are taken as the outputs from the simulink block. Values of various parameters used are m=2 Kg, M=8 Kg, 2l=1m, g=9.8 m/s^2. Here 3 cases have been taken. In case 1 simple feedback controller gains are obtained by solving LMIs in (10), in case 2 and 3 Decay rate is introduced and corresponding controller gains are obtained by solving LMIs in (11) and in case 4 constraint on control input is introduced and associated controller gains are obtained by solving LMIs in (12). Note that all the outputs are taken at an initial angular position angle of 1 radian.

Case 1: System response when simple state feedback gain are applied.

F1=[-104.5687 -3.4057], F2=[-69.0274 -3.4087]

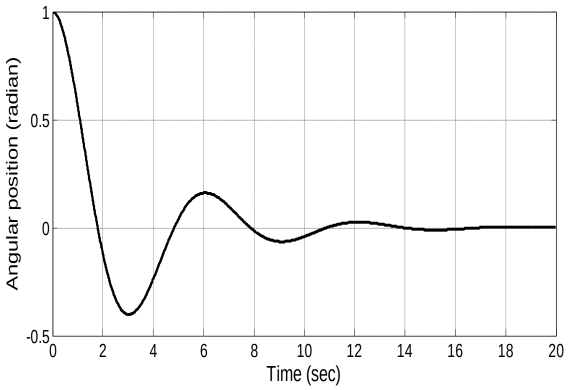


Fig. 3 Angular position response for inverted pendulum.

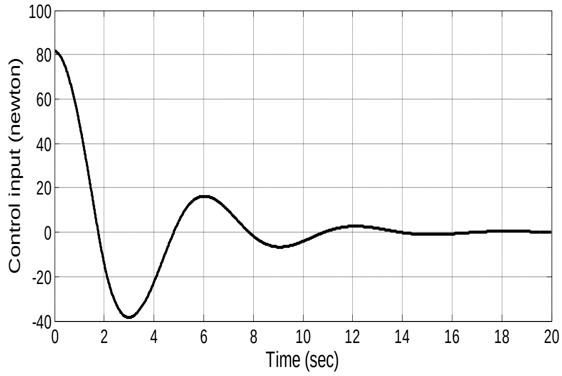


Fig. 4 Control input response for inverted pendulum

Case 2: System response when decay rate of 1 is introduced then corresponding gains are:

F1=[-143.4662 -24.1904] , F2=[-107.9668 -24.2163] alpha=1

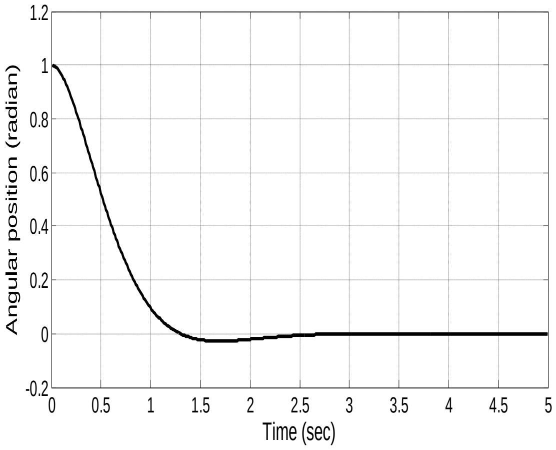


Fig. 5 Angular position response for inverted pendulum. with decay rate=1

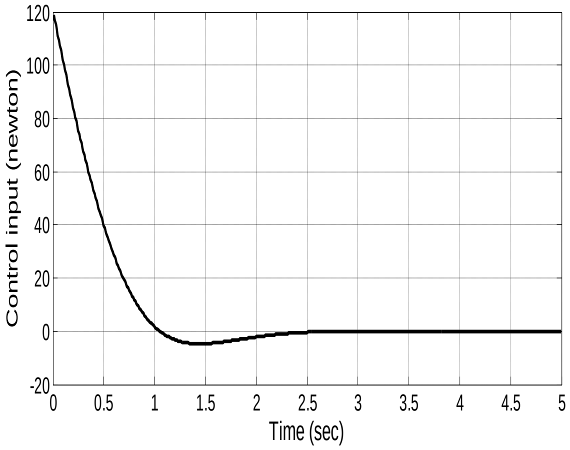
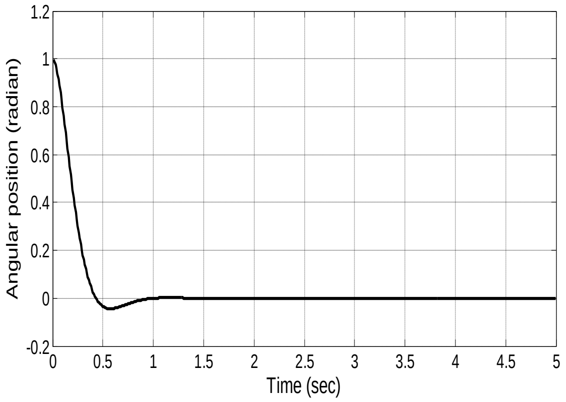


Fig. 6 Control input response for inverted pendulum with decay rate=1

Case 3: System response when decay rate of 5 is introduced the corresponding gains are:

F1=[-435.8022 -61.8046], F2=[-400.5157 -61.8569].



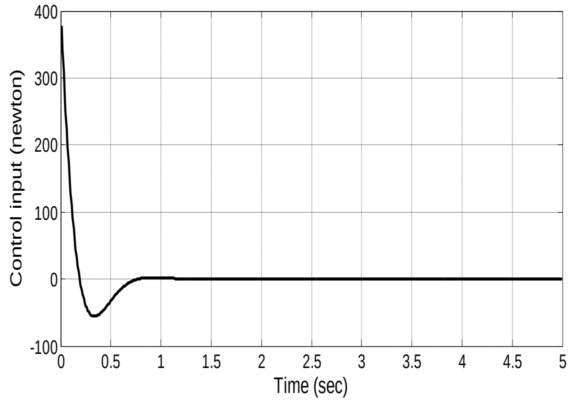
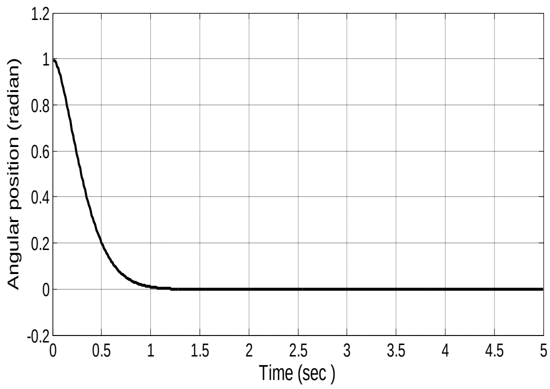


Fig. 7 Angular position response for inverted pendulum. with decay rate=5.

Fig. 8 Control input response for inverted pendulum with decay rate=1

Case 4: System response when along with decay rate constraint on control input is introduced then the corresponding gains are:

F1=[-287.3622 -60.7650], F2=[-219.8868 -55.2849].



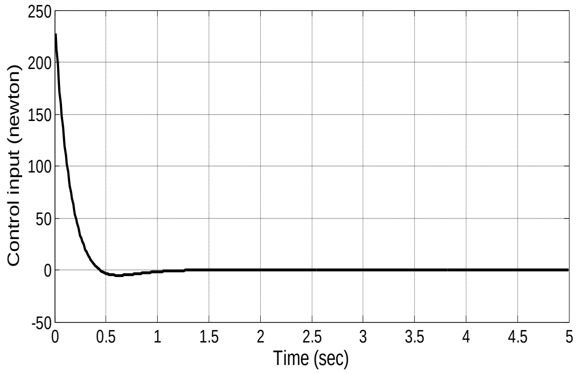


Fig. 7 Angular position response for inverted pendulum. with decay rate=5 and u=350.

Fig. 8 Control input response for inverted pendulum with decay rate=5 and u=350.

VII. CONCLUSIONS

In this paper, modeling and control of two rule inverted pendulum system based on T-S fuzzy approach is shown. Stability condition of this system can be handled as a LMI problem which can be solved using CVX solver and the response is taken from MATLAB simulink. It can be seen from case 1 that although the inverted pendulum system has stabilized but the response is very slow and oscillatory. To improve the system response decay rate is introduced, first with the case 2 when decay rate is 1 and then in case 3 when decay rate is 5. It has been observed that with the increase in decay rate the response of the system becomes much faster but with the tradeoff that control input also increases simultaneously. Therefore in case 4 constraint on control input is introduced with the limit at 350 N. Simulation results demonstrates the effectiveness of this approach.

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