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RADIATION EFFECT ON MHD OSCILLATORY FLOW THROUGH A POROUS MEDIUM WITH THE EFFECT OF SUCTION/INJECTION

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ABSTRACT:
In this paper, we investigate the unsteady oscillatory flow through a vertical channel with non-uniform wall temperature with the effect of suction/injection. The fluid is subjected to radiation effect and where the velocity slip at the lower plate is taken into consideration. Effects of the radiation parameters on temperature, skin friction, velocity profiles and the rate of heat transfer and we obtained the exact solutions of the dimensionless equations governing the fluid flow are discussed and computed graphically. Skin friction increases with the increase in injection on both channel plates.

KEY WORDS:
- ***Oscillatory flow;***
- ***Porous medium;***
- ***Magnetic field;***
- ***Fluid slip;***
- ***Suction/injection.***

1. INTRODUCTION:

The study of oscillatory flow of an electrically conducting fluid through a porous channel saturated with porous medium is important in many physiological flows and engineering applications such as magneto-hydrodynamic (MHD) generators, arterial blood flow, petroleum engineering and many more.

Many authors have studied the flow and heat transfer in oscillatory flow problems. Some of them are, Makinde and Mhone [1] investigated the combined effects of radiative heat transfer and magneto hydrodynamics on oscillatory flow in a channel filled with porous medium, and analysed by the assumption that the plates are impervious. Mahmood and Ali [2] investigated the effect of Navier slip on the free convective oscillatory flow through vertical channel with dissipative heat and periodic temperature. In addition, The Effect of slip condition on unsteady MHD oscillatory flow in a channel filled with porous medium in the presence of transverse magnetic field and radiative heat and mass transfer is studied by Nityananda Senapati and Rajendra Kumar Dhal [3]. Radiation and heat transfer effects on a MHD non – Newtonian unsteady flow in a porous medium with slip conditions are investigated by Gbadeyan and Dada [4]. The fluid is assumed not to absorb its own emitted radiation but that of the boundaries. The effect of buoyancy, heat source, thermal radiation and chemical reaction are taken into account embedded with slip boundary condition, varying temperature and concentration. The combine effect of a transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically third order fluid through a channel filled with saturated porous medium and non – uniform temperature is investigated by Hala Kahtanhamdi and Ahmed M. Abdulhadi [5]. Abdul-Hakeem and Sathiyanathan [6] presented analytical solution for two-dimensional oscillatory flow of an incompressible viscous fluid, through a highly porous medium bounded by an infinite vertical plate. Jha and Ajibade [7-9] reported some interesting results on the free convective oscillatory flows induced by time dependent boundary conditions. While Umavathi [10] studied the unsteady oscillatory flow and heat transfer in a horizontal composite channel. Nandeppanavar and Malkhed [11] studied the Hydro magnetic boundary layer Flow and heat transfer in a fluid over a continuously moving permeable stretching surface with Non uniform heat source/sink embedded in fluid saturated porous medium. Other interesting hydro magnetic oscillatory flow under different geometries can be found in [12] to [20].

After the survey of the literature, it is clearly observed that the slip flow of oscillatory hydro-magnetic fluid through a channel filled with saturated porous medium with the effect on suction/injection has not been investigated. Therefore in this paper we specifically extend the work done in [2], in the cold plate to include the effect of suction/injection. The remaining part of the paper is organized as follows: section 2 provides adequate information on the formulation and non-dimensionlization of the problem; section 3, presents the method of solution to the problem; in section4 results and discussions are presented and section 5 conclusion of the work.

2. MATHEMATICAL ANALYSIS:

Consider the unsteady laminar flow of an incompressible viscous electrically conducting fluid through a channel with slip at the cold plate. An external magnetic field is placed across the normal to the channel. It is assumed that the fluid has small electrical conductivity and the electro-magnetic force produced is also very small. The flow is subjected to suction at the cold wall and injection at the heated wall. We choose a Cartesian coordinate system(x', y') where x' lies along the centre of the channel, and y' is the distance measured in the normal section such that y'=a is the channels half width as shown in Fig.1 below.
Under the usual Bousinesq approximation the equations governing the flow as follows:

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{dP'}{dx'} + v\frac{\partial^2 u'}{\partial y'^2} - \frac{v}{K}u' - \frac{\sigma_e B_0^2}{\rho}u' + g\beta\left(T' - T_0\right), \tag{1}$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{K_f}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{4\alpha^2}{\rho C_p}(T - T_0) \tag{2}$$

With the boundary conditions $T=T_1$

$$u' = \frac{\sqrt{K}}{\alpha_s}\frac{du'}{dy'}\ ,\ T = T_0\ on\ y' = 0\ , \tag{3}$$

$$u' = 0, T' = T_1\ on\ y' = a. \tag{4}$$

Where t'- time, u'- axial velocity,v_0-constant horizontal velocity,ρ-fluid density,P'-fluid pressure, v-kinematic viscosity, K-porous permeability,
σ_e- Electrical conductivity,B_0-magnetic field intensity, g-gravitational acceleration, β-volumetric expansion,C_p-is the specific heat at constant pressure, α- is the term due to thermal radiation, k represents the thermal conductivity, T' fluid temperature andT_0 referenced fluid temperature.

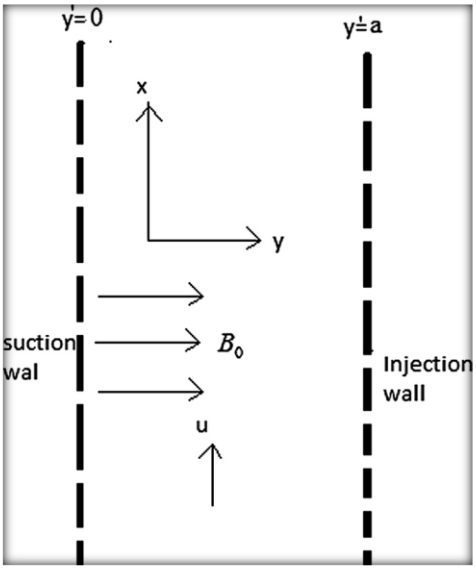

Fig. 1 Geometry of the problem

Introducing the dimensionless parameters and variables given in (5)

$$(x, y) = \frac{(x'y')}{h}\ , u = \frac{hu'}{h^2}\ , t = \frac{vt'}{h^2}\ , p = \frac{h^2 p'}{\rho v^2}\ ,$$
$$Gr = \frac{g\beta(T_1 - T_0)h^3}{v^2}\ , Pr = \frac{\rho C_p v}{k},$$
$$\theta = \frac{T - T_0}{T_1 - T_0}\ , N = \frac{4\alpha^2 h^2}{\rho C_p v}\ , \gamma = \frac{\sqrt{K}}{\alpha_s h}\ , Ha^2 = \frac{\sigma_e B_0^2 h^2}{\rho v}$$
$$Da = \frac{K}{h^2}\ , s = \frac{v_0 h}{v}\ , \tag{5}$$

We obtain the dimensionless equations (6) and (7)

$$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \left(Ha^2 + \frac{1}{Da}\right)u + Gr\theta \tag{6}$$

$$\frac{\partial \theta}{\partial t} - s\frac{\partial \theta}{\partial v} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + N\theta \tag{7}$$

With the appropriate boundary conditions (8) and (9)

$$u = \gamma \frac{du}{dy}, \theta = 0 \ \text{ on } y = 0 \tag{8}$$

$$u = 0, \theta = 1 \ \text{ on } \ y = 1 \tag{9}$$

In equations (6)-(9), Da is the Darcy parameter, s is the suction/injection parameter, Ha^2 is Hartmann's number, Gr is the Grashof number, Pr is the Prandtl number, N is the thermal radiation parameter and γ is the Navier slip parameter.

3. METHODS OF SOLUTION

As shown in [1, 2], we assume that an oscillatory pressure gradient, such that the solutions of the dimensionless equations
(6)-(9) is in the following form.

$$-\frac{dP}{dx} = \lambda e^{i\omega t}, u(t,y) = u_0(y)e^{i\omega t}, \theta(t,y) = \theta_0(y)e^{i\omega t} \tag{10}$$

Where λ is any positive constant, and ω is the frequency of oscillation. In view of (10), Equations (6)-(9) reduced to a boundary-valued problem in the following form.

$$u_0'' + su_0' - \left(Ha^2 + \frac{1}{Da} + i\omega\right)u_0 = -\lambda - Gr\theta_0 ;$$

$$u_0(0) = \gamma u_0'(0), \ \ u(1) = 0 \tag{11}$$

$$\theta_0'' + sPr\theta_0' + (\delta - i\omega)Pr\theta_0 = 0 ; \qquad \theta_0(0) = 0, \theta_0(1) = 1 \tag{12}$$

The exact solution of the (12) becomes

$$\theta(t,y) = (A_0 e^{m_1 y} + B_0 e^{m_2 y})e^{i\omega t} \tag{13}$$

As a result, the rate of heat transfer is given by

$$Nu = \frac{\partial\theta}{\partial y} = (A_0 m_1 e^{m_1 y} + B_0 m_2 e^{m_2 y})e^{i\omega t} \tag{14}$$

While the exact solution of (11) is

$$u(t,y) = \{A_1 e^{m_3 y} + B_1 e^{m_4 y} + Q_0 + Q_1 e^{m_1 y} + Q_2 e^{m_2 y}\}e^{i\omega t} \tag{15}$$

and the shear stress is given by the relation

$$S_f = \frac{\partial u}{\partial y} = (A_1 m_3 e^{m_3 y} + B_1 m_4 e^{m_4 y} + m_1 Q_1 e^{m_1 y} + m_2 Q_2 e^{m_2 y})e^{i\omega t} \tag{16}$$

All the constants are defined as follows

$$m_1 = \frac{-s\,Pr + \sqrt{(s\,Pr)^2 - 4Pr(\delta - i\omega)}}{2}, \quad m_2 = \frac{-s\,Pr - \sqrt{(s\,Pr)^2 - 4Pr(\delta - i\omega)}}{2}, \quad m_3 = \frac{-s + \sqrt{s^2 + 4(H^2 + 1/Da + i\omega)}}{2}, \quad m_4 = \frac{-s - \sqrt{s^2 + 4(H^2 + 1/Da + i\omega)}}{2}$$

$$Q_0 = \frac{\lambda}{H^2 + 1/Da + i\omega}, \qquad Q_1 = \frac{-GrA_0}{m_1^2 + sm_1 - (H^2 + 1/Da + i\omega)}, \quad Q_2 = \frac{-GrB_0}{m_2^2 + sm_2 - (H^2 + 1/Da + i\omega)}$$

$$A_0 = \frac{-1}{e^{m_2} - e^{m_1}}, A_1 = \frac{e^{m_4}(\gamma Q_1 m_1 + \gamma Q_2 m_2 - Q_0 - Q_1 - Q_2) - (1 - \gamma m_4)(-Q_0 - Q_1 e^{m_1} - Q_2 e^{m_2})}{e^{m_4}(1 - \gamma m_3) - e^{m_3}(1 - \gamma m_4)}$$

$$B_0 = \frac{1}{e^{m_2} - e^{m_1}}, B_1 = \frac{e^{m_3}(\gamma Q_1 m_1 + \gamma Q_2 m_2 - Q_0 - Q_1 - Q_2) - (1 - \gamma m_3)(-Q_0 - Q_1 e^{m_1} - Q_2 e^{m_2})}{e^{m_3}(1 - \gamma m_4) - e^{m_4}(1 - \gamma m_3)}$$

4. RESULTS AND DISCUSSION:

In this paper, the oscillatory hydro magnetic fluid flow through a permeable channel filled with a porous medium is investigated. The flow is due to increasing pressure gradient and free convection through a vertical channel. The temperature of the fluid within the channel with the effect of suction/injection is shown in Fig.2.

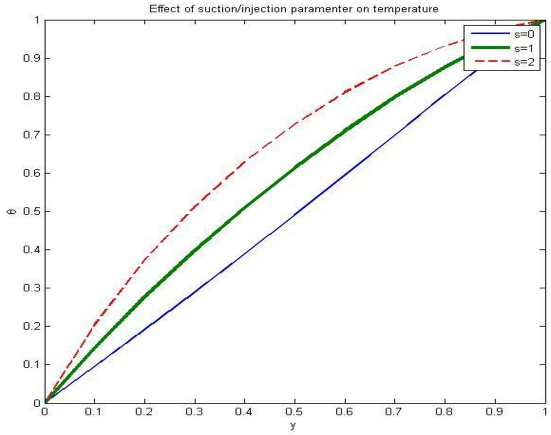

Figure 2 Effect of suction/injection parameter on fluid temperature
$\boldsymbol{\delta = 1, Pr = 1, \omega = \pi, t = 0, s = 0, s = 1, s = 2}$

It is observed from the plot, in the absence of suction/injection that fluid temperature is linearly distributed within the channel. As injection increases on the heated plate, fluid temperature increases within the channel and linearly observed at s = 0 has given way to concave distribution. As a result of the concavity with increase in the suction/injection parameter, the direction of the heat flow from the heated plate towards the cold plate.

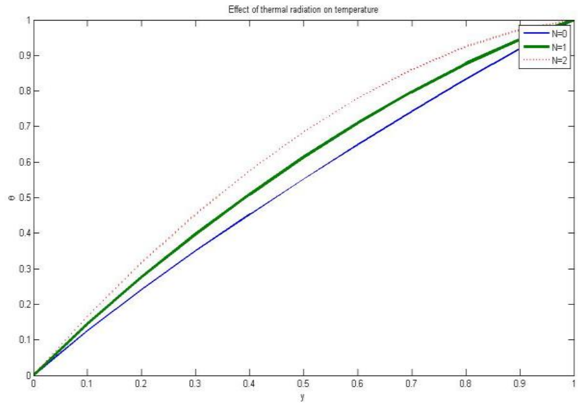

Figure 3 Effect of thermal radiation on fluid temperature
$\boldsymbol{Pr = 1, s = 1, \omega = \pi, t = 0, \delta = 0, \delta = 1, \delta = 2}$

As shown in Fig.3, that the fluid temperature seen to be increasing as the radiation parameter increases. This is because of the heat transfer from the heated wall to the fluid since the fluid absorbs its own radiations.
The plot of heat transfer rate across the channel is shown in Fig.4,

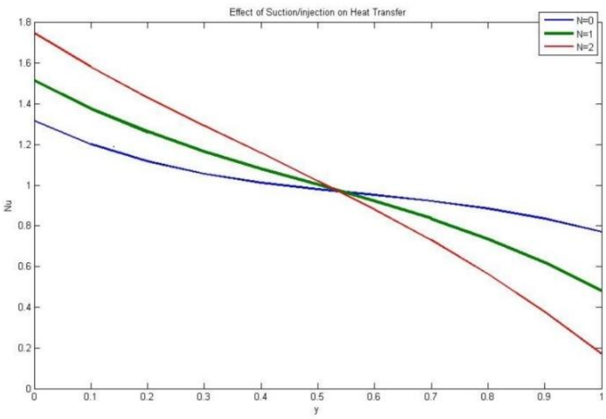

Figure 4 Effect of suction/injection parameter on the rate of heat transfer
$\boldsymbol{\delta = 1, Pr = 1, \omega = \pi, t = 0, s = 1, s = 2, s = 3}$

From the graph it is observed, in the fluid layer closer to the heated wall decreases the rate of heat transfer while the region close to the cold wall the rate of heat transfer increases. This is because of the reason that the, heat is transferred from the heated plate to the fluid and from the fluid to the cold plate.

In Fig.5, Due to internal heat generation it is observed that an increase in the thermal radiation increases the fluid velocity. Internal heat generation that enhances the fluid flow is shown. The fluid particles energized this is since the heat gained form the heated wall.

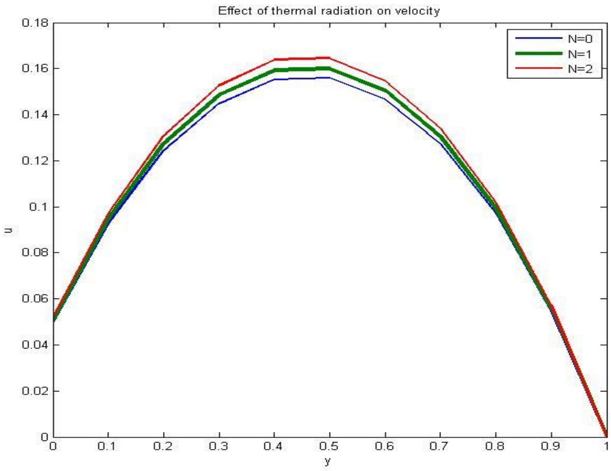

Figure 5 Effect of thermal radiation on fluid velocity

$\mathbf{Pr = 1, s = 1, \omega = \pi, Gr = 1, Da = 1, H = 1, \lambda = 1, \gamma = 0.1, t = 0, \delta = 0, \delta = 1, \delta = 2}$

Fig.6, presents the influence of suction/injection parameter of the fluid velocity. From the result it is shown that there is an increase in the fluid velocity towards the cold plate as the suction/injection parameter increases.

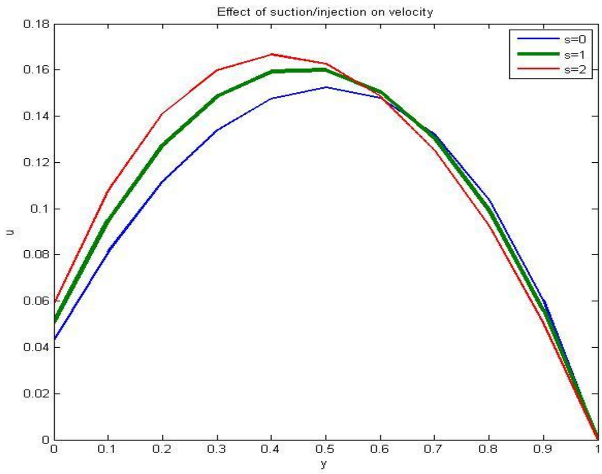

Figure6 Effect of suction/injection on fluid velocity

$\mathbf{s = 1, \delta = 1, \omega = \pi, Gr = 1, Da = 1, H = 1, \lambda = 1, \gamma = 0.1, t = 0, s = 0, s = 1, s = 2}$

Finally Fig.7, shows that the skin friction increases at both the walls that an increase in the suction/injection parameter.

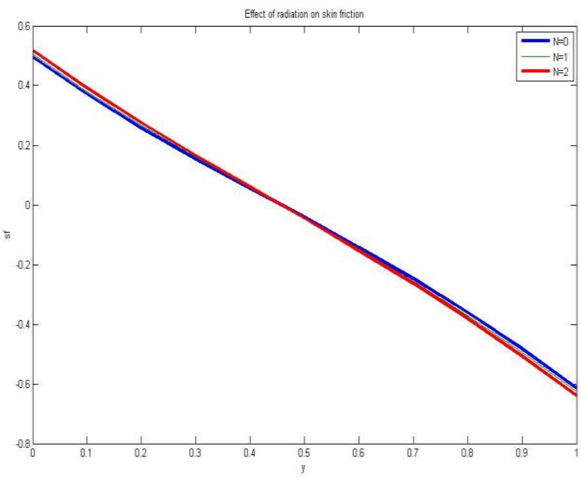

Figure 7 Effect of suction/injection on the skin friction across the channel

$\mathbf{Pr = 1, \delta = 1, \omega = \pi, Gr = 1, Da = 1, H = 1, \lambda = 1, \gamma = 0.1, t = 0, s = 1, s = 2, s = 3}$

5. CONCLUSION:

In this paper, the oscillatory flow of hydro magnetic fluid through a porous medium with the optically thin thermal radiation limit is investigated. The effect of radiation on temperature and velocity profile are presented and additionally the effect of the suction/injection parameter is also investigated.

- the flow velocity at the wall extensively enhanced,
- The temperature distribution of the fluid also elevated.
- The rate of heat transfer at the heated plate lessen and at the cold wall it raisen and
- The skin friction amplified on both channel plates.

REFERENCES:

[1] O.D. Makinde and P.Y. Mhone, Heat Transfer to MHD Oscillatory Flow in a Channel Filled with Porous Medium, Romanian. J. Phys. 50 (2005): 931 – 938.

[2] A. Mehmood, A. Ali, The effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel, Romanian J. Phys. 52 (2007) 85-91.

[3] Nityananda Senapati and Rajendra Kumar Dhal 2013; Effect of Slip Condition on Unsteady MHD Oscillatory Flow in a Channel Filled with Porous Medium with Heat Radiation and Mass Transfer, Int. J. Appl. Math. Stat. Sci. 2 (3): 11 – 20.

[4] Gbadeyan, J.A. and M.S. Dada 2013; On the Influence of Radiation and Heat Transfer on an Unsteady MHD non – Newtonian Fluid Flow with Slip in a Porous medium, Journ.Math. Res. 5 (3): 40 – 50.

[5] Hala Kahtanhamdi and Ahmed M. Abdulhadi 2012; the Effect of Slip Condition on Heat Transfer of MHD Oscillatory Third Order Fluid Flow in a Channel Filled with Porous Medium, Iragi Journal of Sci. 53 (2012): 887 – 893.

[6] A. K.Abdul Hakeem, K.Sathiyanathan, an analytic solution of an oscillatory flow through a porous medium with radiation effect, nonlinear Analysis: Hybrid Systems 3(2009) 288- 295.

[7] B. K Jha, Ajibade A. O. Free convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input. Int Commun Heat Mass Trans 36(2009), 624– 31.

[8] B. K. Jha and A. O. Ajibade, Free convective flow between vertical porous plates with periodic heat input Z. Angew. Math. Mech., (2010) 1 – 9

[9] B. K. Jha, A. O. Ajibade, Effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions, Commun Nonlinear Sci Numer Simulat 17 (2012) 1576–1587

[10] J.C. Umuvathi, A. J. Chamkha, A. Mateen, A. Al-Mudhaf, Nonlinear analysis: Modelling and control, 14, (2009) 397

[11]Abel, M. S., Nandeppanavar, M. M. and Malkhed, M. B. 2010b Hydro magnetic boundary layer Flow and heat transfer in a fluid over a continuously moving permeable stretching surface with Non uniform heat source/sink embedded in fluid saturated porous medium. Chem. Engng. Commun.197, 633–655. [5.1.9.9]

[12] Abou-zeid, M. Y. 2011 Magneto hydrodynamic boundary layer heat transfer to a stretching sheet including viscous dissipation and internal heat generation in a porous medium. J. Porous Media 14, 1007–1008. [5.1.9.9]

[13] Acharya, M., Dash, G. C. and Singh, L. P. 2000 Magnetic effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat. Indian J. Pure Appl. Math. 31, 1–18. [9.2.1]

[14] Agwu Nnanna, A. G., Haji-Sheikh, A. and Harris, K. T. 2004 Experimental study of local thermal Non-equilibrium phenomena during phase change in porous media. Int. J. Heat Mass Transfer 43, 4365–4375. [10.1.7]

[15] Ahmad, S. and Pop, I. 2010 Mixed convection boundary layer flow from a vertical plate embedded in a porous medium filled with Nano fluids. Int. Comm. Heat Mass Transfer 37, 987–991. [8.1.6]

[16] Aldabbagh, L. B. Y., Manesh, H. F. and Mohamad A. A. 2008 Unsteady natural Convection inside a porous enclosure heated from the side. J. Porous Media 11, 73–83.

[17] Raja Sekhar, K., G.V. Ramana Peddy and B.D.C.N. Prasad 2012; Chemically Reacting On MHD Oscillatory Slip Flow in a Planar Channel with Varying Temperature and Concentration, Adv. Appl. Sci. Res. 3 (5): 2652 – 2659.

[18] Devika, B., P.V. Satya Narayana an S. Venkataramana 2013; MHD Oscillatory Flow of a Visco Elastic Fluid in a Porous Channel with Chemical Reaction, Int. J. Engr. Sci. Inv. 2 (2): 26 – 35.

[19] Gital, A.Y. and M. Abdul Hameed 2013; Mixed Convection Flow for Unsteady Oscillatory MHD Second Grade Fluid in a Porous Channel with Heat generation, Int. J. Sci. Engr. Res. 4 (9) ISSN 2229 – 5518.

[20] Umavathi, C., A.J. Chankha, A. Mateen and A. Al – Mudhaf 2009; Unsteady Oscillatory Flow and Heat Transfer in a Horizontal Composite Porous medium Channel, Nonlinear Analysis: Modelling and Control, 14 (3): 397 – 425.