International Journal of Technical Innovation in Modern Engineering & Science (IJTIMES)
Impact Factor: 5.22 (SJIF-2017), e-ISSN: 2455-2585
Volume 4, Issue 12, December-2018

FLOW OF A DUSTY VISCOUS FLUID BETWEEN TWO PARALLEL PLATES UNDER THE INFLUENCE OF MAGNETIC FIELD AND GRAVITY

[1] Bhavna Singh, [2] Ajit Paul

[1] *Assistant Professor, Department of Mathematics and Statistics, SHUATS Allahabad*
[2] *Professor, Head, Department of Mathematics and Statistics, SHUATS Allahabad*

***Abstract:** A dusty viscous fluid is considered to be flowing between two parallel plates and it is consider that the particles and the fluid motion is effected by the gravity and the magnetic field, the magnetic field is replaced by the Hartmann number and the gravity is by the Froude number and it is seen that the Froude number is only in the unsteady part of the motion.*

The flow between parallel plates is affected by many reasons the effects of gravity, viscosity subjected to the inertial forces of the flow. The study of the flow of the dusty viscous fluid between parallel plates is of great utility because its implications can be made in many fields. In this paper the effect of gravity and magnetic field is being taken in consideration.

We are considering here the flow of a dusty viscous fluid between two parallel plates distance d apart from each other. We assume that the dust particles are sufficiently large to be attracted by the gravity and let electromagnetic field be applied so the transformed equations will be

(1) $$\frac{\partial q}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{KN}{\rho}\left(q_p - q\right) - \sigma\frac{B_0^2 u}{\rho} - g$$

(2) $$\frac{\partial q_p}{\partial t} = \frac{K}{m}\left(q - q_p\right) - g$$

Let us introduce the following non dimensional quantities

(3) $$q_p^{\,*} = \frac{q_p}{U},\ p^* = \frac{p}{\rho U^2},\ x^* = \frac{x}{L},\ y^* = \frac{y}{L},\ t^* = \frac{tU}{L},\ M = B_0 L\sqrt{\frac{\sigma}{\mu}},\ f = \frac{mN}{\rho}$$

$$K = \frac{m}{\tau},\ \tau = \frac{L\tau^*}{U},\ Fr = \frac{U^2}{gL}$$

Using the non-dimensional quantities of equation (3) in equation (1) and (2)

(4) $$\frac{\partial q^*}{\partial t*} = -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2 q^*}{\partial y^{*2}} + \frac{f}{\tau^*}\left(q_p * - q*\right) - M^2 q^* - \frac{1}{Fr}$$

(5) $$\frac{\partial q_p^{\,*}}{\partial t^*} = \frac{1}{\tau^*}\left(q^* - q_p^{\,*}\right) - \frac{1}{Fr}$$

After dropping the *the equations (4) and (5) so transformed will be

(6) $$\frac{\partial q}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 q}{\partial y^2} + \frac{f}{\tau}\left(q_p - q\right) - M^2 q - \frac{1}{Fr}$$

(7) $$\frac{\partial q_p}{\partial t} = \frac{1}{\tau}\left(q - q_p\right) - \frac{1}{Fr}$$

Equations (6) and (7) are the transformed equation with non-dimensional form where M^2 is Hartmann number which is a ratio of the electrostatic force to the viscous force and Fr is the Froude number which is the ratio of the inertia force to the gravitational force. Now assuming that

(8) $$q = q_0(y) + q_1(y)e^{i\omega t} \text{ and } \frac{\partial p}{\partial x}$$

Where the first term signifies the steady part of the flow and the second term signifies the unsteady part of the fluid flow. The equations can be separated for the steady and unsteady parts as follows

(9) $$\left(\frac{1}{\tau}+i\alpha\right)\frac{d^2q_1}{dy^2}-\left\{\frac{M^2}{\tau}-\alpha^2+i\alpha\left(M^2+\frac{f}{\tau}+\frac{1}{\tau}\right)\right\}q_1=B\left(i\alpha-\frac{1}{\tau}\right)$$

(10) $$\frac{d^2q_0}{dy^2}-M^2q_0=A+(f+1)\frac{1}{Fr}$$

Solution of the above equations are given as below

(11) $$\frac{d^2q_1}{dy^2}-(\eta_1+i\eta_2)^2u_1=\eta_3$$

Where $\eta_1=\dfrac{\alpha^2\tau-M^2+\alpha^2\left(M^2\tau^2+f\tau+\tau\right)}{1+\alpha^2\tau^2}$

$\eta_2=\dfrac{\alpha\left(\alpha^2\tau^2+f+1\right)}{1+\alpha^2\tau^2}$ and $\eta_3=\dfrac{(1-i\alpha\tau)^2}{1+\alpha^2\tau^2}$

(7.12) $$q_1(y)=C_1e^{\eta_1y}\left(\cos\eta_2y+i\sin\eta_2y\right)+C_2e^{-\eta_1y}\left(\cos\eta_2y-i\sin\eta_2y\right)-\frac{\eta_3}{(\eta_1+i\eta_2)^2}$$

Now using the boundary conditions $q_1(0)=\sin\lambda_1t$ and $q_1(d)=\sin\lambda_2t$

(7.13) $$q_1(y)=\left\{(\sin\lambda_1t+\xi)-\frac{(\sin\lambda_1t+\xi)e^{\eta_1d}\left(\cos\eta_2d-i\sin\eta_2d\right)-\sin\lambda_2t}{2\left(\cos\eta_2d\cosh\eta_1d-i\sin\eta_2d\sinh\eta_1d\right)}\right\}e^{(\eta_1-i\eta_2)y}$$
$$+\left\{\frac{(\sin\lambda_1t+\xi)e^{\eta_1d}(\cos\eta_2d-i\sin\eta_2d)-\sin\lambda_2t}{2(\cos\eta_2d\cosh\eta_1d-i\sin\eta_2d\sinh\eta_1d)}\right\}e^{(-\eta_1+i\eta_2)y}-(\sin\lambda_1t+\xi)$$

Where $\xi=\dfrac{\eta_3}{(\eta_1+i\eta_2)^2}$

Now $\dfrac{d^2q_0}{dy^2}-M^2q_0=A+(f+1)\dfrac{1}{Fr}$

Will give the value of

$$u_0(y)=C_3e^{My}+C_4e^{-My}-\frac{AFr+f+1}{M^2Fr}$$

Now using the boundary conditions $q_0(0)=0$ and $q_0(d)=0$ in equation (13)

$$q_0(y)=\frac{AFr+f+1}{M^2Fr}\left\{1-(e^{Md}-1)2SechMd\right\}e^{My}+\frac{AFr+f+1}{M^2Fr}\left\{(e^{Md}-1)2SechMd\right\}e^{-My}-\frac{AFr+f+1}{M^2Fr}$$

Now using $q(y)=q_0(y)+q_1(y)e^{i\alpha t}$

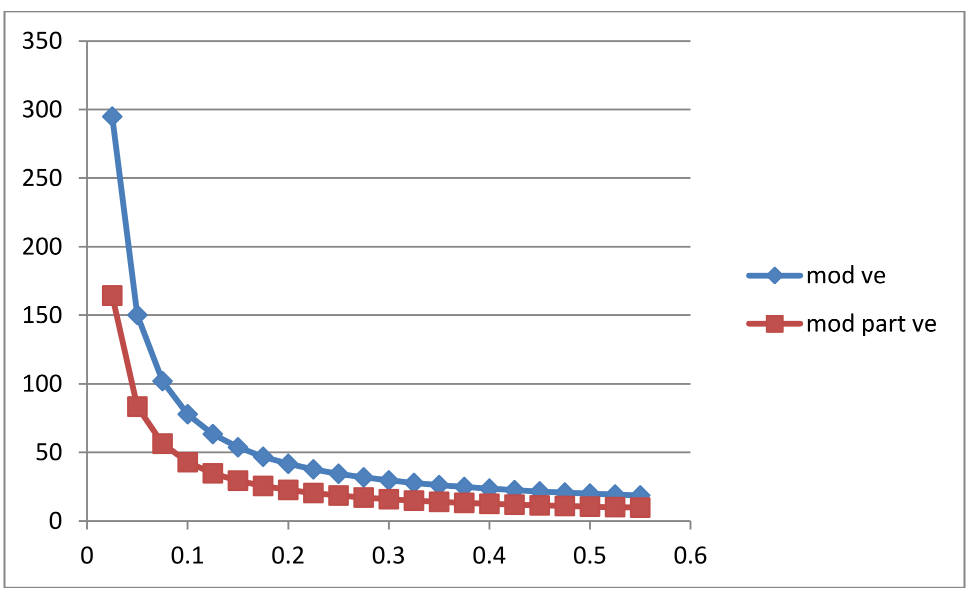
$$q(y,t)=\frac{AFr+f+1}{M^2Fr}\begin{bmatrix}\left\{1-(e^{Md}-1)2SechMd\right\}e^{My}\\+\left\{(e^{Md}-1)2SechMd\right\}e^{-My}-1\end{bmatrix}+$$
$$\begin{bmatrix}\left\{\sin\lambda_1t+\xi-\dfrac{(\sin\lambda_1t+\xi)e^{\eta_1d}\left(\cos\eta_2d-iSin\eta_2d\right)-\sin\lambda_2t}{2\left(Cos\eta_2dCos\eta_1d-iSin\eta_2dSinh\eta_1d\right)}\right\}e^{(\eta_1-i\eta_2)y}+\\\left\{\dfrac{(\sin\lambda_1t+\xi)e^{\eta_1d}\left(Cos\eta_2d-iSin\eta_2d\right)-\sin\lambda_2t}{2\left(Cos\eta_2dCosh\eta_1d-iSin\eta_2dSinh\eta_1d\right)}\right\}e^{(-\eta_1+i\eta_2)y}-(\sin\lambda_1t+\xi)\end{bmatrix}e^{i\alpha t}$$

Using equation above the equation of the velocity of the dust particle will be
Now using the boundary conditions v=0at t=0 v will become

$$q_p = \left[\frac{AFr + f + 1}{M^2 Fr}\left\{\begin{matrix}\{1-(e^{Md}-1)2SechMd\}e^{My} \\ +\{(e^{Md}-1)2SechMd\}e^{-My} - 1\end{matrix}\right\} - \frac{\tau}{Fr}\right] +$$

$$\left[\begin{matrix}\left\{e^{(\eta_1 - i\eta_2)y} - 1 - \frac{e^{\eta_1 d}(\cos\eta_2 d - iSin\,\eta_2 d)\left(e^{(\eta_1 - i\eta_2)y} - e^{(-\eta_1 + i\eta_2)y}\right)}{2(Cos\,\eta_2 dCos\,\eta_1 d - iSin\,\eta_2 dSinh\,\eta_1 d)}\right\}\frac{\left(\lambda_1^2\tau^2 - \alpha^2\tau^2 + 1 - 2\alpha\tau i\right)}{\left(\lambda_1^2\tau^2 - \alpha^2\tau^2 + 1\right)^2 + 4\alpha^2\tau^2}\left\{\begin{matrix}(\alpha\tau i + 1)\sin\lambda_1 t \\ -\lambda_1\cos\lambda_1 t\end{matrix}\right\} \\ + \left\{e^{(\eta_1 - i\eta_2)y} - 1 - \frac{e^{\eta_1 d}(\cos\eta_2 d - iSin\,\eta_2 d)\left(e^{(\eta_1 - i\eta_2)y} - e^{(-\eta_1 + i\eta_2)y}\right)}{2(Cos\,\eta_2 dCos\,\eta_1 d - iSin\,\eta_2 dSinh\,\eta_1 d)}\right\}\frac{\xi}{i\alpha\tau + 1} + \\ \left\{e^{(\eta_1 - i\eta_2)y} - e^{(-\eta_1 + i\eta_2)y}\right\}\frac{\left(\lambda_2^2\tau^2 - \alpha^2\tau^2 + 1 - 2\alpha\tau i\right)}{\left(\lambda_2^2\tau^2 - \alpha^2\tau^2 + 1\right)^2 + 4\alpha^2\tau^2}\{(\alpha\tau i + 1)\sin\lambda_2 t - \lambda_2\cos\lambda_2 t\}\end{matrix}\right]e^{i\alpha t}$$

$$+\left[\begin{matrix}\left\{e^{(\eta_1 - i\eta_2)y} - 1 - \frac{e^{\eta_1 d}(\cos\eta_2 d - iSin\,\eta_2 d)\left(e^{(\eta_1 - i\eta_2)y} - e^{(-\eta_1 + i\eta_2)y}\right)}{2(Cos\,\eta_2 dCos\,\eta_1 d - iSin\,\eta_2 dSinh\,\eta_1 d)}\right\}\frac{\tau\lambda_1\left(\lambda_1^2\tau^2 - \alpha^2\tau^2 + 1 - 2\alpha\tau i\right)}{\left(\lambda_1^2\tau^2 - \alpha^2\tau^2 + 1\right)^2 + 4\alpha^2\tau^2} + \\ \left\{e^{(\eta_1 - i\eta_2)y} - 1 - \frac{e^{\eta_1 d}(\cos\eta_2 d - iSin\,\eta_2 d)\left(e^{(\eta_1 - i\eta_2)y} - e^{(-\eta_1 + i\eta_2)y}\right)}{2(Cos\,\eta_2 dCos\,\eta_1 d - iSin\,\eta_2 dSinh\,\eta_1 d)}\right\}\frac{\xi}{i\alpha\tau + 1} + \\ \left\{e^{(\eta_1 - i\eta_2)y} - e^{(-\eta_1 + i\eta_2)y}\right\}\frac{\tau\lambda_2\left(\lambda_2^2\tau^2 - \alpha^2\tau^2 + 1 - 2\alpha\tau i\right)}{\left(\lambda_2^2\tau^2 - \alpha^2\tau^2 + 1\right)^2 + 4\alpha^2\tau^2} - \left\{\frac{AFr + f + 1}{M^2 Fr}\left(\begin{matrix}\{1-(e^{Md}-1)2SechMd\}e^{My} \\ +\{(e^{Md}-1)2SechMd\}e^{-My} - 1\end{matrix}\right) - \frac{\tau}{Fr}\right\}\end{matrix}\right]e^{-\frac{t}{\tau}}$$



Graph between Froud number and velocity of fluid and dust particles

The velocity of the particle phase and fluid phase are given on the y axis and the Froude number is on the x axis it is clear from the graph that when the Froude number is between zero and .1 the velocity of the particle phase and fluid phase has decreased exponentially and as the Froude number has reached to .1 and after that there is a steady decrease in the velocity with respect to the Froude number.

References

Suraju Olusegun Ajadi, A note on the unsteady flow of dusty viscous fluid between two parallel plates
J. Appl. Math. & Computing Vol. 18(2005), No. 1 - 2, pp. 393 – 403

G. K. Ganguly and S. Lahiri, Oscillatory motion of dusty viscous incompressible fluid between two parallel plates, Indian journal of theoretical Physics,41(2) (1992), 91–95.

V. M. Soundalgekar and Bhat, J. P.,Laminar convection through porous medium between two vertical parallel plates, Bull. Cal. Math. Soc.82 (1990), 93–98.