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STATIC ANALYSIS OF ELECTROSTATICALLY ACTUATED MEMS BASED MICROCANTILVER BEAM USING THE NON-LINEAR DISTRIBUTED PARAMETER MODEL

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Abstract

Static Analysis of a MEMS (Micro electro mechanical systems) based micro-cantilever beam using the distributed parameter modeling. The pull-in voltage of the micro-cantilever beam is obtained and the same is compared with results obtained by M.Younis and the analytic method The pull-in voltage thus obtained provided an accurate resemblance with the other approaches The simulation results were obtained by solving the equations in the MATLAB. The method of Weighted Galerkin's approach was used to solve the differential equation and both linear and non-linear results were obtained. The method predicted the pull in voltage higher than predicted by the reduced order model. The result obtained for the displacement at pull-in was also compared with the reduced order model and analytical method and result thus obtained turned out be in the middle of the those predicted by the reduced order model and the analytic method.

KEYWORDS

MEMS, Pull-in voltage; Weighted Galerkin method; Reduced order model.

1 | INTRODUCTION

Several decades have passed by since the discovery and development of micro-electro-mechanical systems(MEMS).This technology has reached a level of maturity that ,today, several MEMS devices are being used in our every-day life ranging from accelerometers and pressure sensors in cars, rado-frequency (RF) switches, microphones in cell phones. The emerging field of micro and nano-electromechanical (MEMS and NEMS) oscillators has fueled a renaissance in the field of resonant sensors and actuators with an unending flow of producing smaller and better electromechanical devices while providing a closely coupled link between the physical, chemical and biological worlds. These systems have gained a wide theoretical interest and practical application in the field of sensors and actuators and have recently been adopted in valuable analytic instruments. In contrast to their macro-counterparts such as Quartz-crystal balances, surface acoustic waves, or flexural plates they perform with increased functionality and complexity for various chemical and biological sensing applications They are further used for a variety of different sensing purposes and offer unique possibilities by extending the dynamic range and ultimate sensitivity of several orders of magnitude above those of their macro-components including conventional Quartz crystal oscillators.

Simulation tools for MEMS represent essential and urgent needs for designers and researchers to advance the technology for next levels. In the early days of MEMS developments, the majority of the research has been directed towards the methods of fabrication with little presented on modeling and simulation. Hence researchers had to rely on either simple analytical formulas based on crude approximations or on complicated Finite Element models using software that are not geared for MEMS..The outcome of these was the simulation results that contradict or are in disagreement with the experimental data. This has deepened the problem even more since researchers did not count much on their model's prediction and relied more on experimental testing.

Hence a trial and error approach had been adopted in which a device is designed: prototypes are fabricated and then tested, and based on the test results the design is modified again, retested and so on. The result has been a long development cycle, which is costly and time consuming. The need for modeling and simulation tools thus becomes justified as a means of reducing the design time and cost as the reliable simulation tools can effectively reduce the design cycle from weeks and months to days. To allow for more progressive design goals and to optimize the performance of existing devices, simulation tools may become indispensable.

Micro cantilevers are the basic MEMS structures which can be used as both sensors and actuators. The actuation principle is based upon measurement of change in deflection upon external stimulus. The external stimulus can be electrostatic, magnetic, piezo-electric, thermal ,etc. The effect of this external stimulus has been widely analyzed by the researchers and particularly interesting case has been that of Pull-in phenomenon. The phenomenon of pull-in is very important in the study of MEMS. For example the radio frequency (RF) MEMS switches should be actuated above pull-in voltage, whereas the MEMS sensors should be actuated below Pull-in voltage. MEMS actuators and sensors are characterized by the high actuation voltages and efforts to reduce it has been the primary focus of researchers .The effect of change in dimensions of micro cantilever have been shown to have profound effect on the pull-in voltage For example increasing the common area between cantilever and electrode decreases the pull in voltage and increasing the gap between cantilever and electrode increases the pull in voltage[1]. A study of static and dynamic analysis of a fixed-fixed beam and a cantilever beam to the dc and step dc voltage has been done by the Razezdah et al[2].The introduction of the design corrective coefficients independent of the beam material and the geometric properties results in a closed form relationship between static pull-in voltage of the lumped model and static & dynamic pull- in voltages of the distributed models, and takes into account the residual stresses, axial force and damping effects. The eigenvalue problem describing the vibration of the microbeam around its statically deflected position is solved numerically for the natural frequencies and mode shapes [3]. It shows that the ratio of the width of the air gap to the micro beam thickness can be tuned to extend the domain of the linear relationship between the dc polarization voltage and the fundamental natural frequency. This fact and the ability of the nonlinear model to accurately predict the natural frequencies for any dc polarization voltage allow designers to use a wider range of dc polarization voltages. The static deflection of fixed-fixed beam was calculated and maximum non dimensional deflections of up to 0.39 near the pull-in were found which is significantly above the traditionally used stability limit of $W_{max}= 1/3$. The static and dynamic behavior of a micro cantilever, with relatively large gap to beam-length ratios, under electrostatic actuation has been studied [7] with special emphasis on the nonlinear effects due to geometry, electric forces, and inertial terms In case there is large gap between deformable conductor and ground plane, it is essential to consider higher order corrections of electrostatic forces during the formulation of the model. In the present work, it has been shown that results are much improved when higher order terms are taken into account during static and dynamic analysis.

2. MODELLING AND SIMULATION DETAILS.

We choose a rectangular micro cantilever made of silicon as shown in fig (1) of specific dimensions and formulate its distributed parameter model. We use the galarakin's weighted residual method to calculate the static deflection of the micro beam. The pull in voltage is calculated on the basis of this model and compared with that obtained from the reduced order model as calculated by M.younis[14].The material of the beam is silicon. The dimensions of the beam are given as :

Density **ρ =2332 kg/m3** , Young's modulus **E =160.0 GPa** , Length **l =100.0 μ m** ,

Breadth **w =10 .0 μm** , Height **t=1 μm,** Gap **do=1 μm**

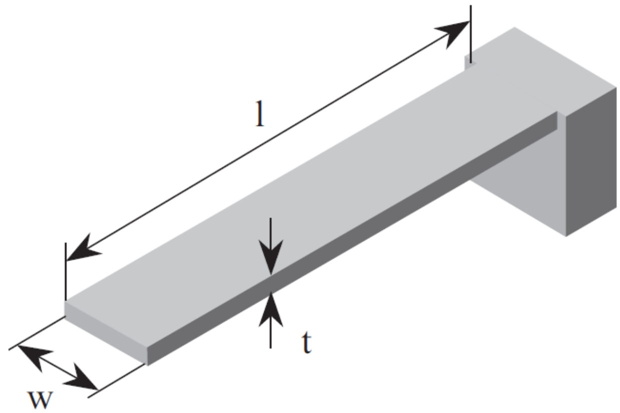

FIGURE 1: Constant cross section micro cantilever beam

2.1 Electrostatic actuation and detection

Electrostatic transduction is the most common actuation and sensing method in MEMS because of its simplicity and high efficiency. Two of the classical successful MEMS devices have been relying on this method: the Analog Devices accelerometers for airbag deployments, which use capacitive detection to sense the motion, and the micro mirror in the digital mirror display DMD for projection displays by Texas Instruments, which relies on electrostatic actuation. Other examples of micro devices employing this method include microphones, pressure sensors, temperature sensors, RF switches, band-pass filters, and resonators. MEMS devices utilizing electrostatic transduction are also called electrostatic MEMS. Electrostatic transduction relies on simple capacitors of parallel plate electrodes

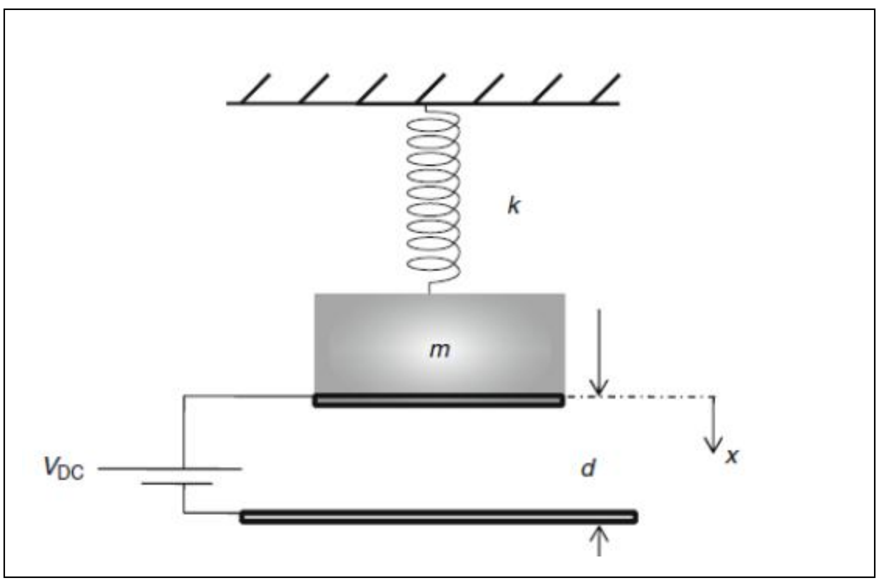

FIGURE 2: A model of parallel plate capacitor spring mass system

In this case, the electrostatic force acting on the upper electrode assuming the gap to be filled with air is expressed as

$$F = \frac{\mathcal{E}_o A V^2}{2(d-x)^2} \qquad 1$$

Where $\mathcal{E}_o$ is the permittivity of free space, $\mathcal{E}_o$ =**8.854 e-12 C/V**

A restoring force of the structure *kx* is induced to oppose the electrostatic force, which leads to a new equilibrium position of the structure. Figure 4b shows a free body diagram of the structure (the effect of weight is neglected because in microstructures it is very small compared to the actuation force). Hence, the equation governing the equilibrium position *x* can be written as

$$KX = \frac{\mathcal{E}_o A V^2}{2(d-x)^2} \qquad 2$$

Equation **2** is a cubic equation in *x* with three possible solutions. It turns out that one of the three solutions predicts $x > d$, and hence it is nonphysical and discarded (the structure cannot penetrate and move below the lower electrode). Of the other two physical solutions, one represents unstable solution, meaning that practically the structure cannot hold into this position So we are left with one physical solution ($x < d$) that is stable, which represents the "real" deflection of the upper electrode in response to the DC load. It turns out that corresponding to every value of voltage there are two equilibrium positions ,one of them is stable and another is unstable . The pull in voltage can simply be calculated by substituting **x=d/3** in equation **2** to obtain

$$V_{PULL} = \sqrt{\frac{8Kd^3}{27\mathcal{E}_o A}} \qquad 3$$

2.2 Static distributed parameter modeling

By referring to figure **1**, we write the Euler- Bernoulli beam equation for a micro cantilever with the specifications given paragraph 2 neglecting the fringing field at the edges as

$$EI\frac{d^4w}{dx^4} = \frac{\mathcal{E}_o A V^2}{2L(d-w)^2} \qquad 4$$

E is the young's modulus of the beam

I is the moment of inertia of the beam

w is the deflection at any x

d is the initial gap between beam and the electrode

The above equation is non-dimensionalised by making the following substitutions

wo=w/d, **xo=x/l** so as to obtain

$$\frac{d^4w_o}{dx^4} = B(1+2w_o) \qquad 5$$

$$B = 6\frac{\mathcal{E}_o V^2 L^4}{Eh^3 d^3} = \alpha V^2 \qquad 6$$

$$\alpha = 6\frac{\mathcal{E}_o L^4}{Eh^3 d^3} \qquad 7$$

Where we have expanded the nonlinear term in the denominator using Taylor expansion series and retaining only the first term .

The above equation is solved by assuming a solution in the form of a polynomial function of the fourth order given below. We omit the knot with w and x in the subsequent calculations

$$w = a_o + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 \qquad 8$$

By imposing the boundary conditions of the micro cantiliver, the following equation is obtained

$$w = a_4(6x^2 - 4x^3 + x^4) \qquad 9$$

The residue of equation **5** is calculated by substituting the assumed solution into **5** and is written as

$$R_d = 24a_4 - B(1+2d) \qquad 10$$

The above residue is minimized by the weighted galerikin approach. In this method we choose the weighting function in order to find the unknown constants .We need as many weighting functions as the number of unknown constants. The residue is minimized in the weighted integral sense in the following way

$$\int_0^1 R_d = 0 \qquad 11$$

Where **w(x)** is the properly chosen weighting functions .In the weighted galarekin approach, the weighting functions are chosen to be the same as the trial functions so that upon the introduction of the weighting function and execution of integral we obtain the value of the constant $\mathbf{a_4}$.This value of $\mathbf{a_4}$ is then substituted in the **9** to obtain the deflection profile of the beam as

$$w(x) = \frac{378\,B}{9072-1456B}(6x^2 - 4x^3 + x^4) \qquad 12$$

The deflection at the tip of the micro cantilever is obtained by substituting x=1 in the above equation so as to obtain

$$w(1) = \frac{1134\,B}{9072-1456B} \qquad 13$$

The denominator becomes zero at B=6.23077, corresponding to the voltage of 13.1 V. However pull in occurs much below this value as will be shown by incorporating the non linear term in the forcing function.

We write equation 4 by retaining one nonlinear term in the Taylor series of forcing function. The new equation becomes

$$\frac{d^4 w_o}{dx^4} = B(1 + 2w_o + 3w_o^2) \qquad 14$$

When the above equation is solved by the weighted galerkin's approach , two values of constant $\mathbf{a_4}$ are obtained as below

$$a_4 = \frac{58968 - 9464B + \sqrt{N}}{63072B} \qquad 15$$

$$a_4 = \frac{58968 - 9464B - \sqrt{N}}{63072B} \qquad 16$$

$$N = -220368512B^2 - 1116146304B - 3477225024$$

The deflection profile of the beam is then obtained as

$$w(x) = \frac{58968 - 9464B - \sqrt{N}}{63072B}(6x^2 - 4x^3 + x^4) \qquad 17$$

One of these values of $\mathbf{a_4}$ predicts that the deflection decreases with the increase in the voltage and hence is not acceptable. The other solution predicts the deflection profile which is consistent with the observation. The pull in voltage is obtained by putting the value of N=0 which gives the value of pull in voltage as

$$V_{Pull} = \sqrt{\frac{2.13}{\alpha}} \qquad 18$$

2.3 Results of distributed parameter modeling

The plot of equation **12** is shown in figure **3** .The plot shows the tip deflection of the micro cantilever based on the linear model where only one of the terms in the forcing function is retained from its Taylor series expansion.

The deflection profile of the beam is shown in figure **4** for various values of $\boldsymbol{\alpha V^2}$.This is obtained by plotting the equation for specified values of B. Only the stable solution is retained in this plot Finally the beam tip deflection is plotted with the voltage as shown in figure **6** .The plot is that of equation **17** and shows both unstable and stable solutions. The plot shows the effect of the first order non linearity. It can be inferred that the effect of nonlinearity is significant only after **6V**.The graph shows the two equilibrium solutions corresponding to each voltage. As the voltage increases the two solutions approach each other and coalesce at a certain voltage which is the pull in voltage .This is an example of the saddle node bifurcation.

We compare the results of our analysis with that of Younis [11], where the results are obtained by the reduced order methods. The pull in voltage obtained by the reduced order method is approximately given by the following equation

$$V_{Pull} = \sqrt{\frac{1.72}{\alpha}}$$

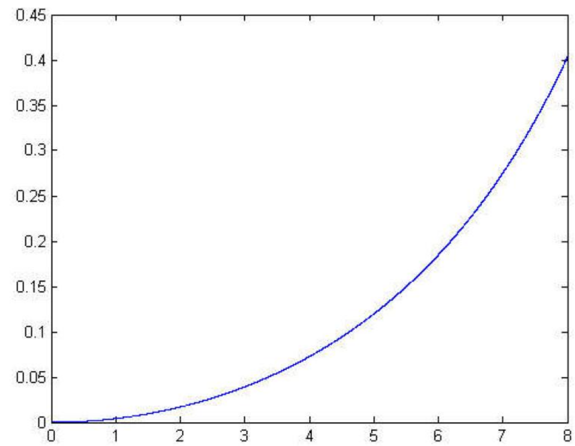

FIGURE 3: The tip deflection as a function of voltage based on linear model

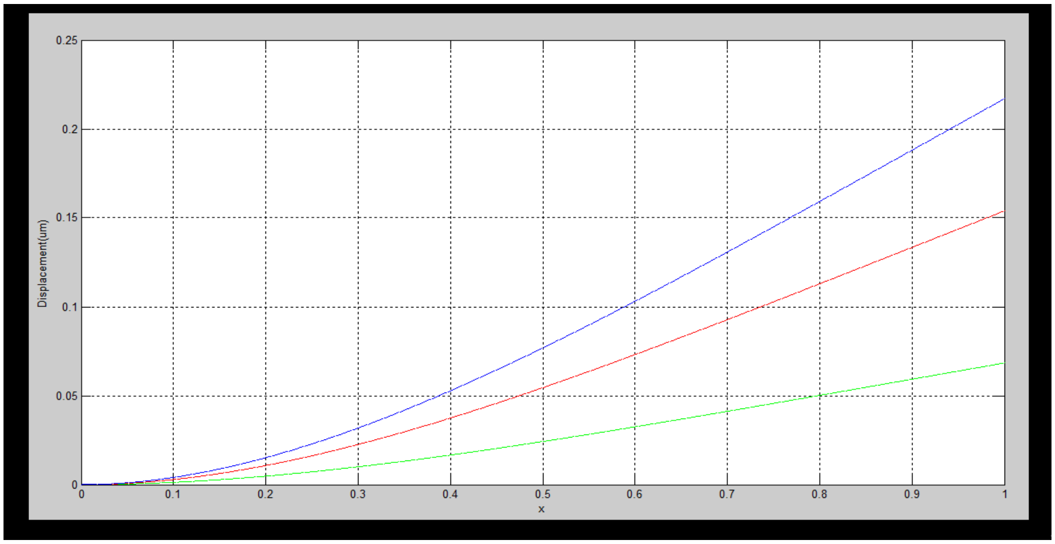

FIGURE 4 : The deflection profile of the beam for various values of αV^2 for αV^2=0.2(green);1(red) and 1.29(blue)

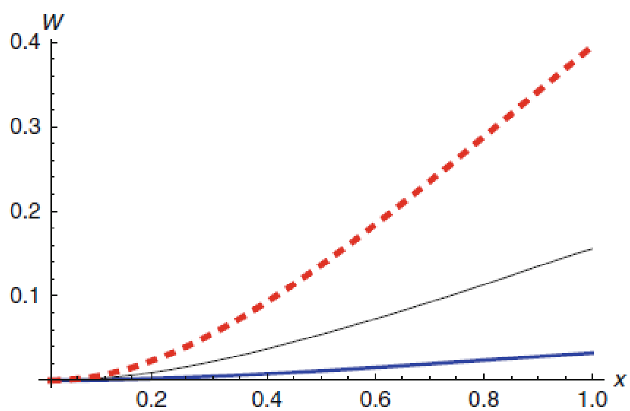

FIGURE 5: Deflection profile obtained by M .younus for various values of αV^2 for αV^2=0.2(green);1(blue) and 1.29(red)

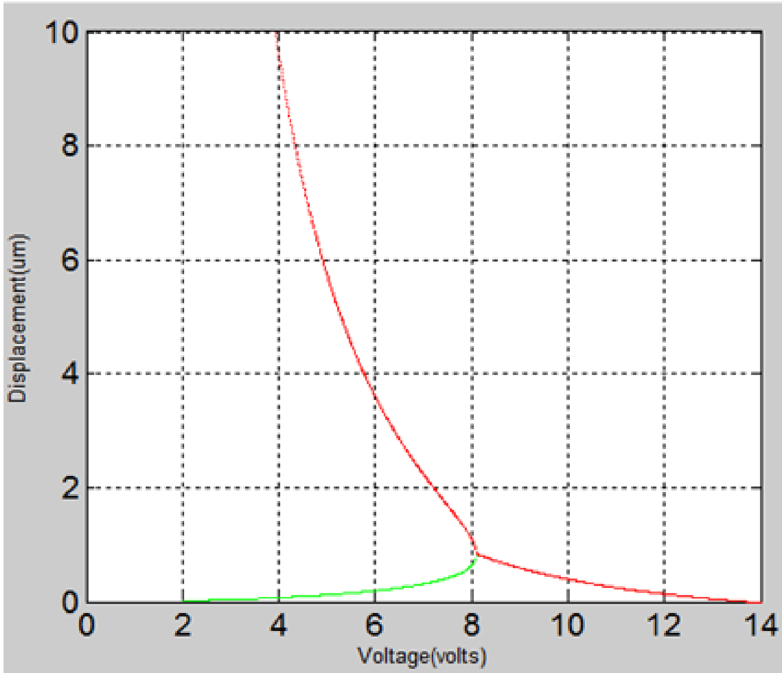

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FIGURE 6 :Tip deflection as a function of voltage (nonlinear model).Green line shows the stable and the red shows unstable solutions

We see from figure **4** and figure **5** that error between our results and that of younis increases with the increase in the voltage. This is because the reduced order model predicts more accurate result because it also takes the higher order terms of the forcing function into the consideration. That is why the pull in voltage predicted by the reduced order model is also lower than the pull in that we calculated. By taking the higher order terms it turns out the displacement at any section along the micro cantilever is greater than our results so that for a particular voltage the tip deflection of the micro cantilever is larger as predicted by the reduced order model than that predicted by the weighted residual method that we employed

3. Summary

In this manuscript, the pull in voltage of the micro cantilever beam is obtained using the weighted Galerkan approach and the corresponding tip deflection is obtained using both linear and non-linear approach. In the non-linear approach the non linearity up to only the quadratic term is retained. The corresponding results are compared with the one obtained by M.younis[11] whose results are based on reduced order model. The pull in voltage as obtained by the weighted Galerkan method has an error of 0.08% with that of the pull in obtained by the reduced order model. The result of the reduced model is more accurate as it takes the non linearity of higher order by retaining cubic and fourth order forcing functions.

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