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AN EXTENSION OF ANCIENT INDIAN METHOD OF CALCULATING CUBE ROOT

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ABSTRACT

In this paper firstly we have tried our best to make attempts to extend Bhāskarācāya's Method of finding Cube root of a perfect cubic number to 5^{th} root of any integer and then applied to compute n^{th} root of any positive integer.

KEYWORDS: Ancient Mathematics, Vedic Mathematics, Cube root, Number Theory, Hindu Mathematics.

1. INTRODUCTION

An attempt has been made to give a chronological description of methods for obtaining the real root of an integer given by the Ancient Hindu Mathematician. Bhāskara II, in his book Lilāvati gave the Method to obtain the cube root of a perfect cubic number.

It is obvious that ancient Indian mathematician knew the cube root technique Bhāskara's method is indeed slightly different from his predecessors.

2. DEFINITION

A Number is said to be a perfect number if it is expressible in the form of x^n where x and n both are positive integers and n > 1.

Now we first consider the following table which will make the calculations at various steps a little easier if one remembers this table.

End of Roots of Perfect Numbers
According To Their Unit Places

Number	Unit places in different powers														
	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	4	8	6	2	4	8	6	2	4	8	6	2	4	8
3	3	9	7	1	3	9	7	1	3	9	7	1	3	9	7
4	4	6	4	6	4	6	4	6	4	6	4	6	4	6	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	9	3	1	7	3	1	7	9	3	1	7	9	3	3
8	8	4	2	6	8	4	2	6	8	4	2	6	8	4	2
9	9	1	9	1	9	1	9	1	9	1	9	1	9	1	9

Note that:

1. For 5^{th}, 9^{th}, 13^{th}, 17^{th} etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9, end respectively in 1, 2, 3, 4, 5, 6, 7, 8 & 9.
2. For 6^{th}, 10^{th}, 14^{th}, 18^{th} etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9 end respectively in 1, 4, 9, 6, 5, 6, 9, 4 & 1.

3. For 7^{th}, 11^{th}, 15^{th}, 19^{th} etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9 end respectively in 1, 8, 7, 4, 5, 6, 3, 2 & 9.
4. For 8^{th}, 12^{th}, 16^{th}, 20^{th} etc. powers of 1, 2, 3, 4, 5, 6, 7, 8 & 9 end respectively in 1, 6, 1, 6, 5, 6, 1, 6 & 1

Applying these operations over perfect number, we can find out the n^{th} root immediately.

3. 1^{st} 5^{th} ROOT METHOD

Note that: Fifth power of numbers from 1 to 9 are respectively 1, 32, 243, 1024, 3125, 7776, 16807, 32768 and 59049.

Step I : Put a vertical line on the unit place and four dots on four digits behind it (i.e. tens, hundreds, thousands, ten thousands). Repeat the process till all the digits are over. Then we are determined that total number of vertical lines gives total numbers of digits in its fifth root.

Step II : From the last vertical line at extreme left subtract the greatest possible 5^{th} power of any number(say a).

Step III : Form II pada by taking down the reminder and the next digit, divide it by $5a^4$ to get quotient b (where $b \leq 9$)

Step IV : Form the III pada by taking down the reminder and the next digit. Subtract $10a^3b^2$ from III pada, $10a^2b^3$ from IV pada, $5ab^4$ from V pada and b^5 from VI pada respectively. If subtraction of $10a^3b^2$ is not possible reduce to lowest number until subtraction becomes possible and the next subtraction are done with this value of b.

Step V : If more digits are left, divide VII pada by $5(10a + b)^4$ to get quotient c (where $c \leq 9$). Subtract $10(10a+b)^3c^2$ from VIII pada, $10(10a+b)^2c^3$ from IX pada, $5(10a+b)c^4$ from X pada and c^5 from XI pada respectively. If subtraction of $10(10a+b)^3c^2$ is not possible, c is reduced and adjusted in the same way as b.

Step VI : If more digits are left divide the next pada XII by $5(100a+10b+c)^4$ to get quotient d (where $d \leq 9$) and adjusted in the same way as b & c. Subtract $10(100a+10b+c)^3d^2$, $5(100a + 10b + c)^2d^3$, $5(100a+10b+c)d^4$ & d^5 from next padas are respectively.

If more digits are left the process may be extended in the same way.

4. RESULTS

(i) For the number having two vertical lines, the root is 10a+b, (where b is the unit place for perfect number only).

(ii) For three vertical lines, the root is 100a+10b+c, (where c is the unit place for perfect number only).

(iii) For four vertical lines, the root is 1000a+100b+10c+d, (where d is the unit place for perfect number only) and so on.

Example 1: Find the 5th root of 14348907

Let the given number is perfect number.

$a^5 = 2^5$

$5a^4 = 5.2^4$

$10a^3b^2 = 10:2^3.7^2$

$= 3920$

$10a^2b^3 = 10;2^2.7^3$

$= 13720$

$5ab^4 = 5.2.7^4$

$= 24010$

$b^5 = 7^5$

$= 16807$

. . 1 1
14348907
<u>32</u>
= 80) 1114 (7=b
<u>560</u>
5548
<u>3920</u>
16289
<u>13720</u>
25690
<u>24010</u>
16807
<u>16807</u>
x

2 = a
Here unit place is 7.
So that quotient is 7
and the root is 27.

Hence fifth root of 14348907 is 27.

Example 2: Find the 5th root of 4747561509943.

$a^5 = 3^5$

$5a^4 = 5.3^4$

$10a^3b^2 = 10:3^3.4^2$

$= 4320$

$10a^2b^3 = 10;3^2.4^3$

$= 5760$

$5ab^4 = 5.3.4^4$

$= 3840$

$b^5 = 4^5$

$= 1024$

$5(10a+b)^3 = 5(34)^4$

$10(10a+b)^3c^2 = 10(34)^3.3^2$

$= 3537360$

$10(10a+b)^2c^3 = 10(34)^2.3^3$

$= 312120$

$5(10a+b).c^4 = 5(34)^3.3^4$

$= 13770$

$c^5 = 3^5$

. . 1 . . . . 1 1
4747561509943
<u>243</u>
= 405) 2317 (4=b
<u>1620</u>
6975
<u>4320</u>
26556
<u>5760</u>
207961
<u>3340</u>
2041215
<u>1024</u>
= 6681680)204 01910 (3 =c
<u>20045040</u>
3568709
<u>3537360</u>
313499
<u>312120</u>
13794
<u>13770</u>
243
<u>243</u>
x

3 = a

Hence the 5th root of 4747561509943 is 343

Example 3: Find the 5th root of 14693280768

$$a^5 = 1^5$$
$$5a^4 = 5.1^4$$
$$10a^3b^2 = 10:1^3.(08)^3 = 640$$
$$10a^2b^3 = 10(a)^2\ (08)^3 = 5120$$
$$5ab^4 = 5.1.(08)^4 = 20480$$
$$b^5 = (08)^5 = 32762$$

```
    1 . . . . 1. . . . . 1
   146932880768
   1
= 5) 46 (08=b
   40
    693
    640
     5328
     5120
      20807
      20480
        32768
        32768
          x
```

1 = a

Here 4 is not divided by 5 So that put 0 as quotient and divides 46. Hence the root is 108. Within two steps we can find the root, further calculations are only for verifcation.

Hence the 5th root of 14693280768 is 108.

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