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PERISTALTIC TRANSPORT OF A CONDUCTING FLUID IN AN ASYMMETRIC CHANNEL

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Abstract:

Peristaltic is an inherent property of many syncytial smooth muscle tubes which occurs due to stimulation. A major industrial application of this principle is in the design of roller pumps to eliminate contamination of pumping fluids with the pumping machinery. The lubrication – theory model by Jaffrin & Shapiro et al(1971) is applicable globally for pumping characteristics at a small Reynolds number. Srivastava & Agarwal (1980) modelled the blood as an electrically conducting fluid and investigated the oscillatory flow. Depending on the perturbation solution of Mishra & Ramachandra Rao(2003), we have studied in this paper the Peristaltic transport of a viscous conducting fluid in an asymmetric channel as shown in different physical models. Finally we conclude that the time averaged flow rate increases with the decreasing phase shift.

Key words:

Distensible tube, Syncytial muscle Tubes, Sinusoidal waves, Small Reynolds number, Arteriosclerosis, Impulsive Magnetic Field (IMF).

Introduction:

Peristaltic pumping is the process of fluid transport arising from the progression of contraction waves along a distensible tube. Peristalsis induces propulsive and mixing movements and pumps the fluid against the pressure rise. Physiologically, peristalsis is an inherent property of many syncytial smooth muscle tubes which occurs due to stimulation. Stimulation at any point can cause a contractile ring to appear in the circular muscle of the gut, and this ring then spreads along the tube. In this way, peristalsis occurs in the gastrointestinal tract, the bile ducts, and other glandular ducts throughout the body, the uterus, and many other smooth muscle tubes of the body. A major industrial application of this principle is in the design of roller pumps, which are used in pumping fluids without being contaminated due to the contact with the pumping machinery.

The accuracy of the fluid mechanics of peristaltic transport has been confirmed experimentally by Latham (1966) and Weinberg, Eckstein and Shapiro (1971). The earliest models of peristaltic pumping are based on the assumption of trains of periodic sinusoidal waves in infinitely long two-dimensional channels or ax symmetric tubes (Shapiro 1967; Fung &Yih 1968; Yih & Fung 1969; Shapiro et al. 1969)[3]. These models which were applied primarily to characterize the basic fluid mechanics of pumping process, fall into two classes: (1) the model developed by Fung & Yih which is restricted to small peristaltic wave amplitudes but has no restrictions on Reynolds number; and (2) the lubrication-theory model introduced by Shapiro et al. (1969) in which effects of fluid inertial and wall curvature are neglected but no restrictions are placed on wave amplitude. A complete review of peristaltic transport is given by Jaffrin and Shapiro (1971)[4]. The lubrication-theory model is applicable globally in the limit of totally occluding peristaltic waves and is found to be a reasonably accurate approximation of global pumping characteristics at a small Reynolds number and wall curvature, Jaffrin (1973); Tatabatake & Ayukawa (1982). A numerical investigation of peristaltic waves in circular tubes was discussed by Xiao and Damodaran (2002).

The magneto hydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids, (e.g., the blood flow in arteries). The effect of magnetic field on blood flow was studied by Sud et al. (1977) and it is observed that the effect of suitable magnetic field accelerates the speed of blood. [9]Srivastava and Agrawal (1980) and Prasad and Ramacharyulu (1981) by considering the blood as an electrically conducting fluid and constitutes a suspension of red cell in plasma. Also, [1]Agrawal and Anwaruddin (1984) studied the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as blood pump in carrying out cardiac operations. Li et al. (1994) have used an impulsive magnetic field in the combined therapy of patients with stone fragments in the upper urinary tract. It was found that the impulsive Magnetic field (IMF) activates the impulsive activity of the ureteral smooth muscles in 100% of cases. Nonlinear peristaltic transport of MHD flow through a porous medium was studied by Mekheimer and Al-Arabi (2003). Mekheimer (2004) studied the peristaltic transport of blood under effect of a magnetic field in non uniform channels. Some of the physiological systems in human body cannot be modeled by a symmetrical channel, especially the sagittal cross section of the uterus. [2]Eytan and Elad (1999) and Eytan et al. (2001) have studied the intra uterine fluid in the sagittal cross section of the uterus by a asymmetric channel under lubrication approach. Recently, Mishra and Ramachandra Rao (2003) developed the flow in an asymmetric channel generated by peristaltic waves propagating on the walls. [5]Mishra and Ramachandra Rao (2003) obtained a perturbation solution for the problem of peristaltic flow of a viscous Newtonian fluid in an asymmetric channel. In view of these, we modeled the peristaltic transport of a conducting fluid in an asymmetric channel.

The aim of the present study in this chapter is to study the MHD peristaltic flow in a two-dimensional asymmetric channel under the assumptions of long wavelength and low Reynolds number in a wave frame of reference. The effects of phase shift and Hartmann number on the pumping characteristics are discussed in detail.

Mathematical formulation and Solution:

We consider the peristaltic transport of a viscous conducting fluid in an asymmetric channel with flexible walls with asymmetry being generated by the propagation of waves on the channel walls travelling with same speed c but with different amplitudes and phases. We assume that a uniform magnetic field strength B_0 is applied in the transverse direction to the direction of the flow (i. e., along the direction of the y-axis) and the induced magnetic field is assumed to be negligible. Fig.1. shows the physical model of the asymmetric channel.

The channel walls are given by

$$Y = H_1(X,t) = a_1 + b_1 \cos\frac{2\pi}{\lambda}(X - ct) \qquad \text{(upper wall)} \quad (1a)$$

$$Y = H_2(X,t) = -a_2 - b_2\cos\left(\frac{2\pi}{\lambda}(X-ct)+\theta\right) \quad \text{(lower wall)} \quad (1b)$$

where b_1, b_2 are amplitudes of the waves, λ is the wavelength, $a_1 + a_2$ is the width of the channel, θ is the phase difference ($0\le\theta\le\pi$) and t is the time.

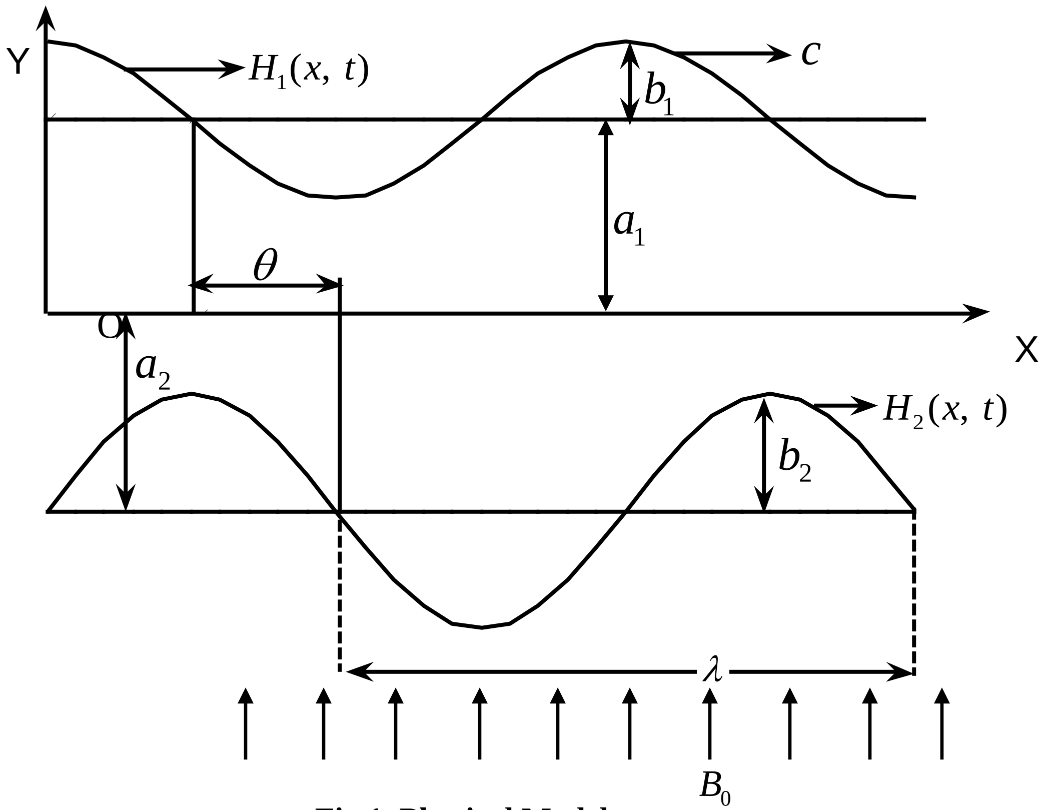

Fig.1. Physical Model

We introduce a wave frame of reference (x, y) moving with velocity c in which the motion becomes independent of time when the channel length is an integral multiple of the wavelength and the pressure difference at the ends of the channel is a constant (Shapiro et al., (1969)). The transformation from the fixed frame of reference (X, Y) to the wave frame of reference (x, y) is given by

$$x = X - ct,\; y = Y,\; u = U - c,\; v = V \;\text{ and }\; p(x) = P(X, t),$$

where (u, v) and (U, V) are the velocity components, p and P are pressures in the wave and fixed frames of reference, respectively.

The equations governing the flow in wave frame of reference are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma_e B_0^2}{\rho}u, \quad (3)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \tag{4}$$

where σ_e is the electrical conductivity of the fluid, ρ is the density and μ is the viscosity of the fluid.

Introducing the following non-dimensional variables

$$\bar{x}=\frac{x}{\lambda},\bar{y}=\frac{y}{a_1},\bar{u}=\frac{u}{c},\bar{v}=\frac{v}{c\delta},\delta=\frac{a_1}{\lambda},d=\frac{a_2}{a_1}$$

$$\bar{p}=\frac{pa_1^2}{\mu c\lambda},h_1=\frac{H_1}{a_1},h_2=\frac{H_2}{a_1},\phi_1=\frac{b_1}{a_1},\phi_2=\frac{b_2}{a_1}.$$

in the governing equations (1 - 4), and dropping the bars, we get

$$h_1=1+\phi_1\cos 2\pi x,\ h_2=-d-\phi_2\cos\left(2\pi x+\theta\right) \tag{5}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$\operatorname{Re}\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-M^2u, \tag{7}$$

$$\operatorname{Re}\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \tag{8}$$

where $\operatorname{Re}=\frac{\rho a_1 c}{\mu}$ is the Reynolds number and $M=B_0 a_1\sqrt{\frac{\sigma_e}{\mu}}$ is the Hartmann number.

Using long wavelength (i.e., $\delta<<1$) and negligible inertia (i.e., $\operatorname{Re}\to 0$) approximations, we have $\frac{\partial p}{\partial y}=0$, $\frac{\partial^2 u}{\partial y^2}-M^2u=P$. (9)

where $P=\frac{dp}{dx}$.

The corresponding non-dimensional boundary conditions are given as

$u=-1$ at $y=h_1$ and $y=h_2$ (10)

Solving equation (9) using the boundary conditions (10), we get

$$u=c_1\cosh My+c_2\sinh My-P/M^2 \tag{11}$$

where $c_1=\frac{\left(-1+P/M^2\right)\left[\sinh Mh_2-\sinh Mh_1\right]}{\left[\cosh Mh_1\sinh Mh_2-\cosh Mh_2\sinh Mh_1\right]}$ and

$$c_2=\frac{\left(-1+P/M^2\right)\left[\cosh Mh_1-\cosh Mh_2\right]}{\left[\cosh Mh_1\sinh Mh_2-\cosh Mh_2\sinh Mh_1\right]}.$$

The volume flow rate in the wave frame is given as

$$q=\int_{h_2}^{h_1} u\,dy$$

$$=\frac{c_1}{M}\left(\sinh Mh_1-\sinh Mh_2\right)+\frac{c_2}{M}\left(\cosh Mh_1-\cosh Mh_2\right) -\frac{P}{M^2}\left(h_1-h_2\right). \quad (12)$$

From (12), we have

$$P=\frac{dp}{dx}=\frac{qM^3D_1+D_2M^2}{D_2-\left(h_1-h_2\right)MD_1} \quad (13)$$

where

$D_1=\cosh Mh_1\sinh Mh_2-\cosh Mh_2\sinh Mh_1$ and

$D_2=\left(\cosh Mh_1-\cosh Mh_2\right)^2-\left(\sinh Mh_1-\sinh Mh_2\right)^2$

The instantaneous flux at any axial station is given by

$$Q(x,t)=\int_{h_2}^{h_1}(u+1)dy=q+h_1-h_2 \quad (14)$$

The average volume flow rate over one wave period (T= λ/c) of the peristaltic wave is defined as

$$\overline{Q}=\frac{1}{T}\int_0^T Q\,dt=\frac{1}{T}\int_0^T(q+h_1-h_2)dt=q+1+d. \quad (15)$$

The pressure rise over one wave length of the peristaltic wave is given by

$$\Delta p=\int_0^1\frac{dp}{dx}dx=\int_0^1\frac{qM^3D_1+D_2M^2}{D_2-\left(h_1-h_2\right)MD_1}dx$$

$$=\int_0^1\frac{\left(\overline{Q}-1-d\right)M^3D_1+D_2M^2}{D_2-\left(h_1-h_2\right)MD_1}=\overline{Q}I_1+I_2 \quad (16)$$

where $I_1=\int_0^1\frac{M^3D_1}{D_2-\left(h_1-h_2\right)MD_1}dx$ and $I_2=\int_0^1\frac{-(1+d)M^3D_1+D_2M^2}{D_2-\left(h_1-h_2\right)MD_1}dx$.

The equation (16) can be rewritten as

$$\overline{Q}=\frac{\Delta p-I_2}{I_1}. \quad (17)$$

Discussion of the results:

Using equation (11) we have plotted the variation of axial velocity u with y at $x=0.25$ with $\phi_1=0.7,\phi_2=1.2,d=2,\frac{dp}{dx}=-0.5$, $\theta=0$ and for different values of Hartmann number M as shown in Fig. 2(i). The velocity profiles are parabolas. The maximum velocity occurs at the centre of the channel and increases as M increases. This is due to peristalsis. Fig. 2(ii) shows the variation of axial velocity u with y at

$x = 0.25$ with $\phi_1 = 0.7, \phi_2 = 1.2, d = 2, \frac{dp}{dx} = 0$, $\theta = 0$ and for different values of M. In this case also the maximum velocity increases with the increment in M.

The variation of axial velocity u with y at $x = 0.25$ with $\phi_1 = 0.7$, $\phi_2 = 1.2$, (i) $\frac{dp}{dx} = 0.5$, $d = 2$, $\theta = 0$ and for different values of M as depicted in Fig.2 (iii) As M increases the maximum velocity increases. Further for small values of M (≈ 0.5) , the flow takes place in the reverse direction. Fig.3 shows the variation of axial velocity u with y for different values of Hartmann number M with $\phi_1 = 0.7, \phi_2 = 1.2, d = 2,$, $\theta = \frac{\pi}{4}$ and for (i) $\frac{dp}{dx} = -0.5$, (ii) $\frac{dp}{dx} = 0$, (iii) $\frac{dp}{dx} = 0.5$.

The maximum velocity increases for adverse pressure gradient, zero pressure gradient and favourable pressure gradient. For small values of M (0.5), the flow takes place in the reverse direction when $\frac{dp}{dx} > 0$. The variation of maximum velocity u with y at $x = 0.25$ for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2, d = 2,$ $\theta = \frac{\pi}{2}$ and for (i) $\frac{dp}{dx} = -0.5$ (ii) $\frac{dp}{dx} = 0$ (iii) $\frac{dp}{dx} = 0.5$ as shown in Fig.4.

As M increases the maximum velocity increases. For small values of M , the flow takes place in the reverse direction when $\frac{dp}{dx} > 0$.

Further as the phase shift increases the velocity decreases. The variation of pressure rise Δp with time averaged volume flow rate $\bar{Q}$ for different phase shifts with $\phi_1 = 0.7, \phi_2 = 1.2, d = 2$ and for (i) $M = 0.5$ and (ii) $M = 1$ as shown in Fig.5.

It is observed that in the pumping region and free pumping region as phase shift θ increases the time averaged flow rate as well as pressure rise both decrease. An interesting observation here is that in co-pumping region $\bar{Q}$ increases with phase shift θ for an appropriately chosen $\Delta p (< 0)$. Further as M increases the time averaged volume flow rate as well as pressure rise both increase.

When the amplitudes of the peristaltic waves are same, we observe the same phenomena as in the case of that for different amplitudes of the peristaltic waves as shown in Fig.6. $\Delta p > 0$, we observed that the time averaged flow rate $\bar{Q}$ increases with the decreasing phase shift θ. But below certain value of Δp (≈ -0.5) the opposite behaviour is observed. For a given $\bar{Q}$ (approximately <1.8) Δp increases with the decreasing phase shift θ.

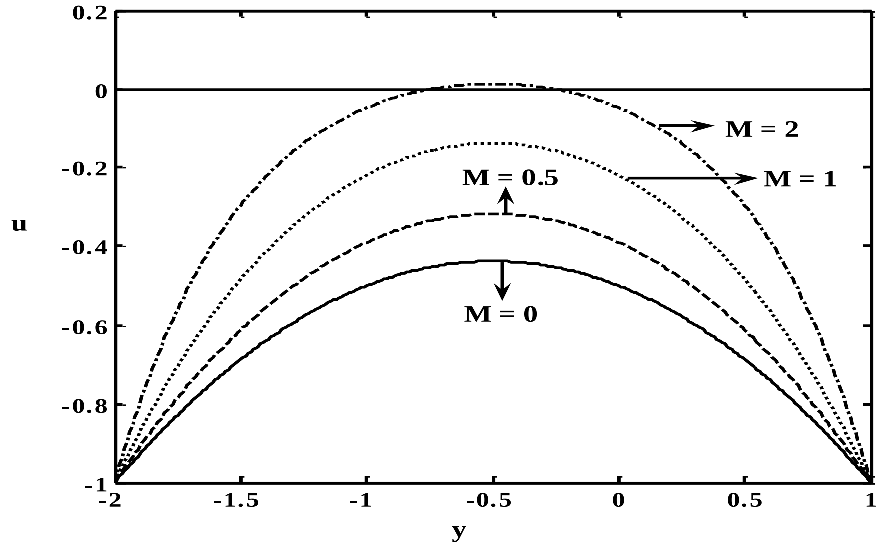

Fig.2(i). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, $x = 0.25$ and $\theta = 0$ for $\frac{dp}{dx} = -0.5$.

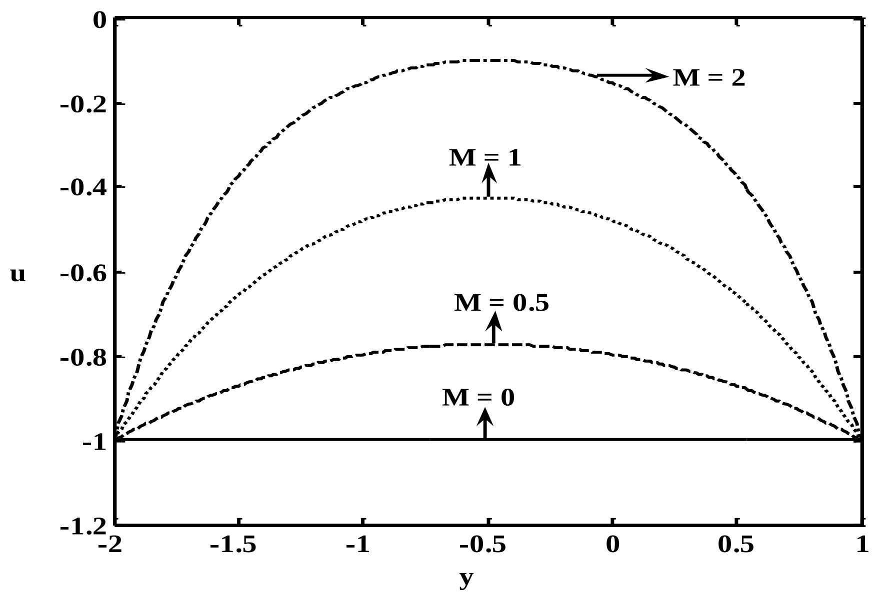

Fig.2(ii). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, $x = 0.25$ and $\theta = 0$ for $\frac{dp}{dx} = 0$.

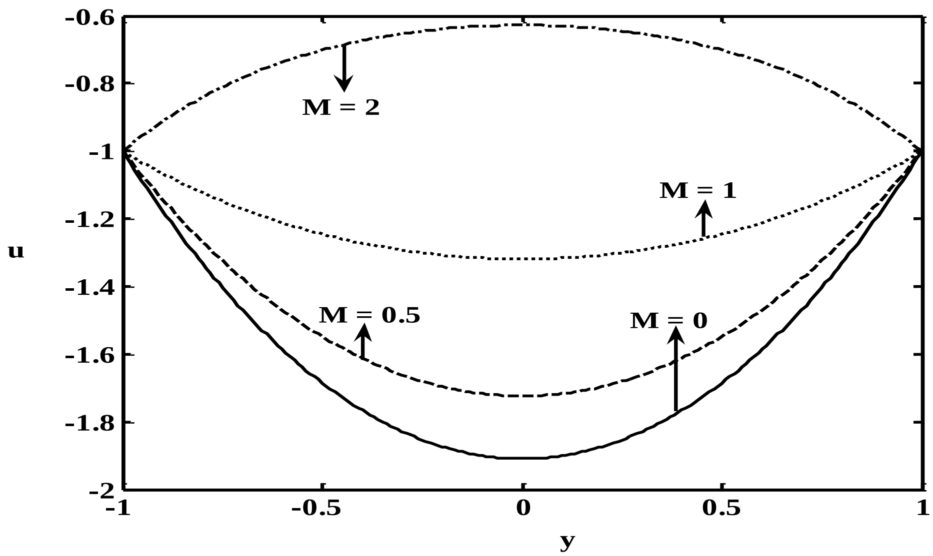

Fig.2(iii). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, $x = 0.25$ and $\theta = 0$ for $\frac{dp}{dx} = 0.5$.

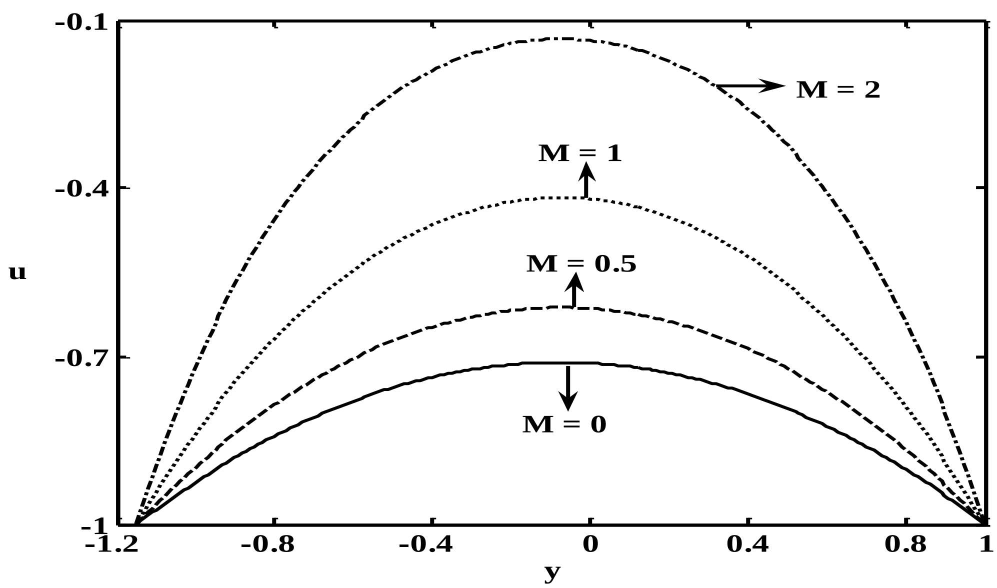

Fig.3(i). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, $x = 0.25$ and $\theta = \pi/4$ for $\frac{dp}{dx} = -0.5$

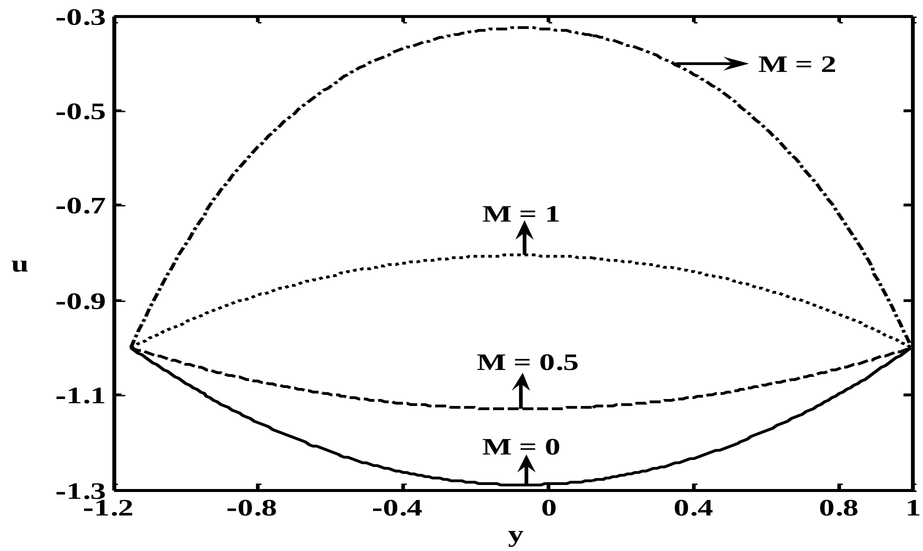

Fig.3(ii). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, $x = 0.25$ and $\theta = \pi/4$ for $\frac{dp}{dx} = 0$.

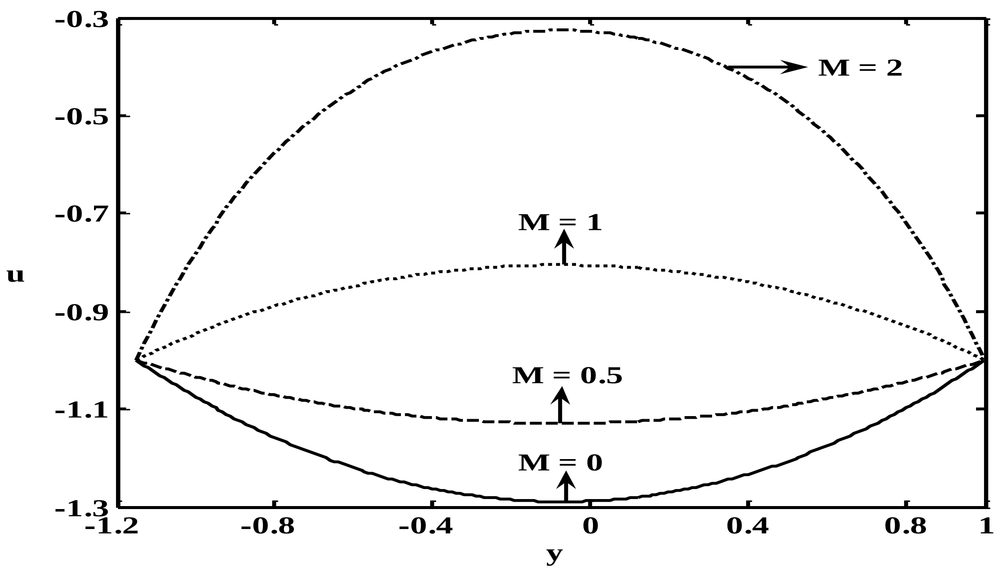

Fig.3(iii). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, $x = 0.25$ and $\theta = \pi/4$ for $\frac{dp}{dx} = 0.5$

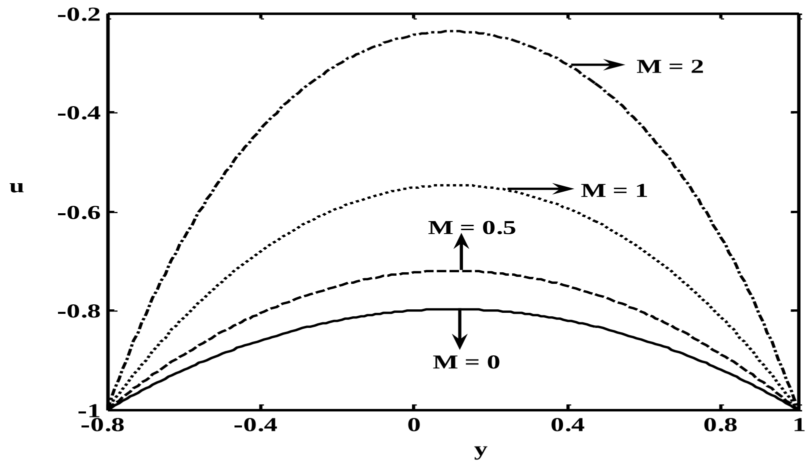

Fig.4(i). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, $x = 0.25$ and $\theta = \pi/2$ for $\frac{dp}{dx} = -0.5$.

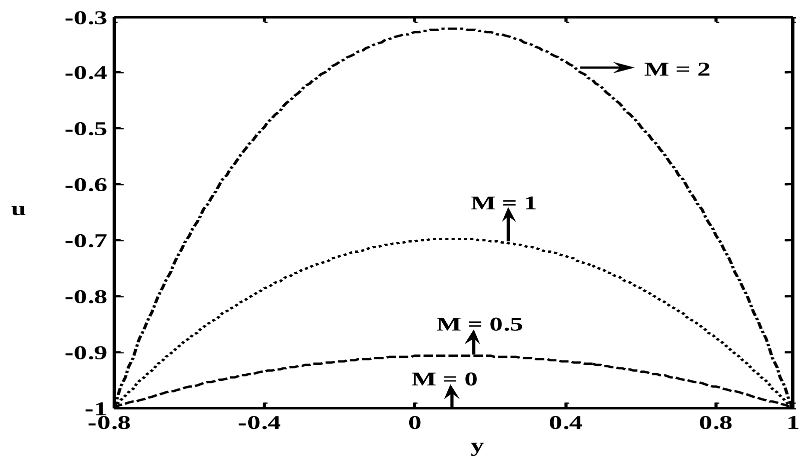

Fig.4(ii). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2$, x = 0.25 and $\theta = \pi/2$ for $\frac{dp}{dx} = 0$.

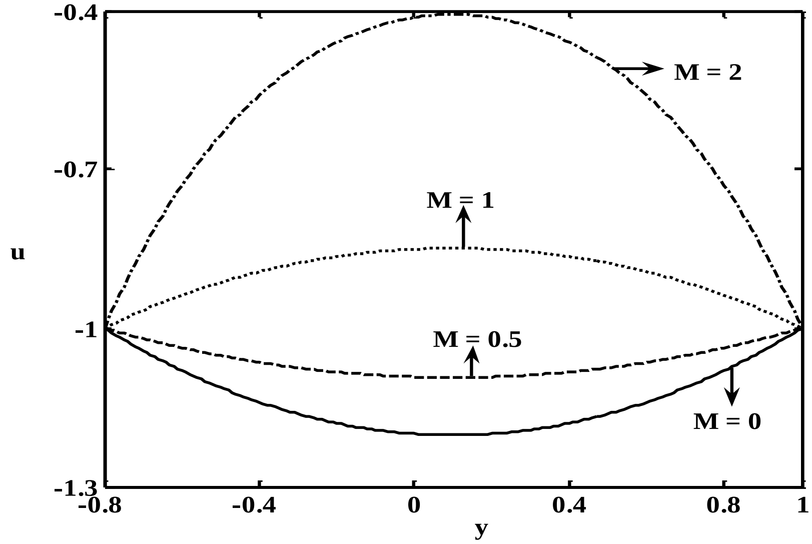

Fig.4(iii). The variation of velocity u with y for different values of M with $\phi_1 = 0.7, \phi_2 = 1.2$, $d = 2,$ $x = 0.25$ and $\theta = \pi/2$ for $\frac{dp}{dx} = 0.5$

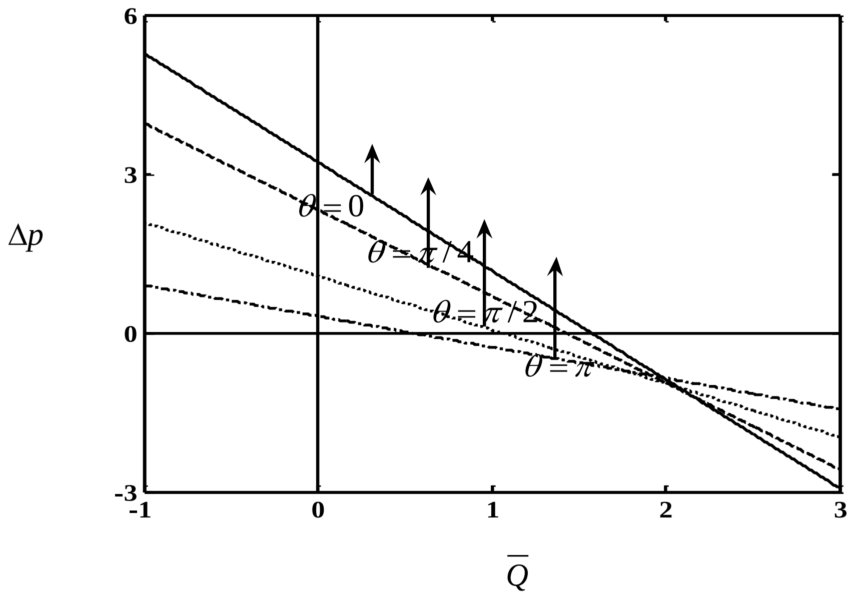

Fig.5(i). The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different phase shifts with $d = 2, \phi_1 = 0.7, \phi_2 = 1.2$ and $M = 0.5$.

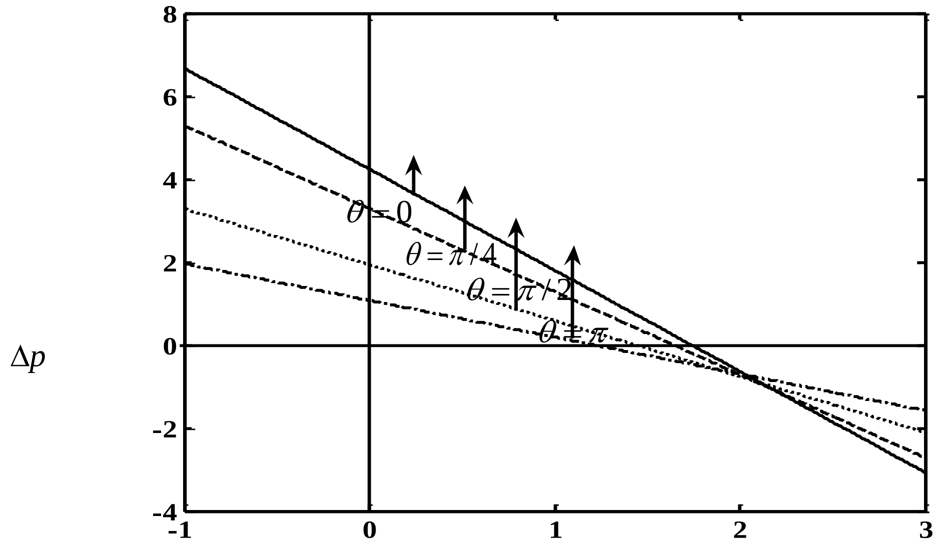

Fig.5(ii). The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different phase shifts with $d = 2, \phi_1 = 0.7, \phi_2 = 1.2$ and M = 1.

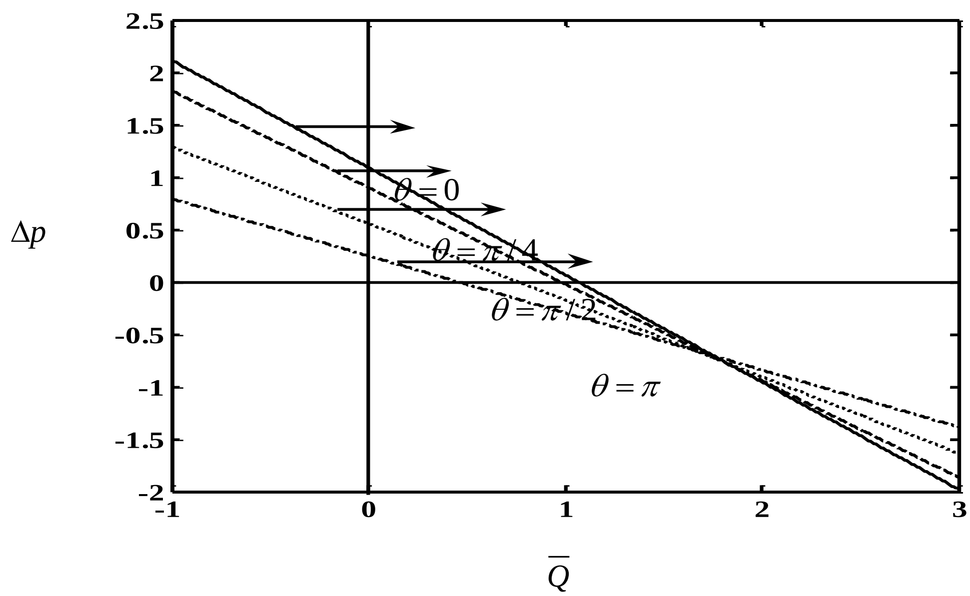

Fig.6(i). The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different phase shifts with $d = 2, \phi_1 = 0.7, \phi_2 = 0.7$ and M = 0.5.

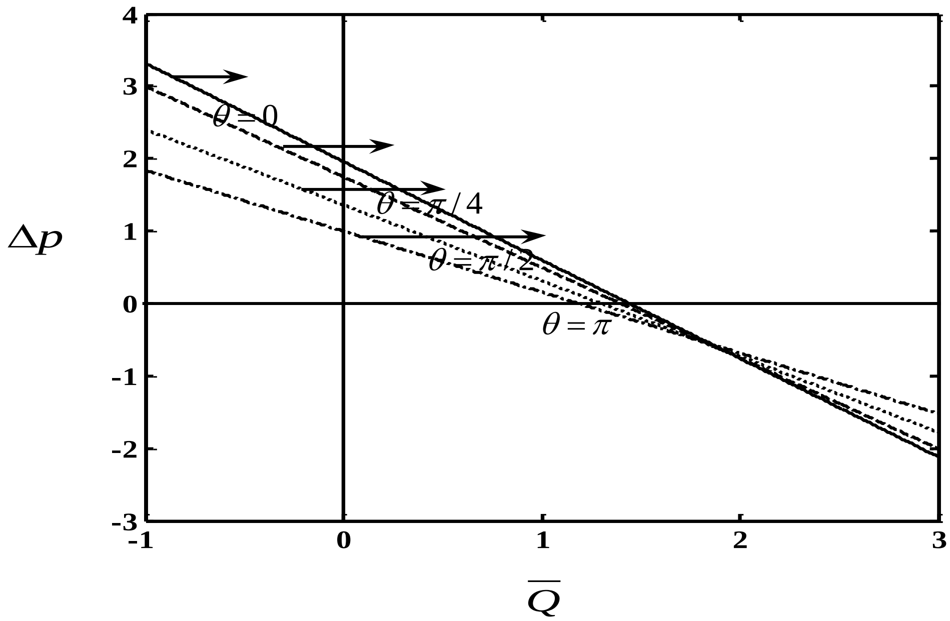

Fig.6(ii). The variation of pressure rise Δp with time-averaged volume flow rate $\overline{Q}$ for different phase shifts with $d = 2, \phi_1 = 0.7, \phi_2 = 0.7$ and M = 1.

Conclusion:

The study of the peristaltic transport of blood under effect of magnetic field in non- uniform channels, particularly in an "asymmetric channel" is of much importance and will be helpful for future researchers in this field. Some physical models are shown in graph form are also of much help.

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