International Journal of Technical Innovation in Modern Engineering & Science (IJTIMES),(UGC APPROVED)
Impact Factor: 5.22 (SJIF-2017),e-ISSN:2455-2585
"International Conference on Modern Trends in Civil Engineering"(ICMTCE-2019)
(Towards Sustainable Development Goals)
Volume 5, Special Issue 02, Feb.-2019

MODELING OF GROUNDWATER USING TWO DIFFERENT APPROACHES

Rahul Soni[1], Padam Jee Omar[2]*, S. B. Dwivedi[1], P. K. S. Dikshit[1]

[1] *Department of Civil Engineering, IIT (BHU), Varanasi, India*
[2]*Assistant Professor, Motihari College of Engineering, Motihari, India*

Abstract:-
Water present below the earth surface in the pore spaces of the soil and the fractures present in the rocks are called groundwater. With the continuous increase in the water demand at industrial, agriculture and domestic level, groundwater availability in the area is falling. Since the 1960s, numerical modeling is often used for prediction of groundwater level and its condition in the concerning region. The aim of this paper is the use of numerical models to simulate the groundwater flow. Solutions to many issues related to groundwater flow require mathematical modeling. Commonly used models for solving groundwater flow equations - finite difference method (FDM) or finite element method (FEM) are at present proven tools of hydro-geologists. The most popular MODFLOW based FDM algorithm has been tested in thousands of cases and is now the standard in groundwater flow modeling. Apart from the known advantages of numerical modeling based on FDM or FEM methods, their disadvantage might be the need of extensive discretization of the modeled area and finding a compromise between the accuracy and complexity of the numerical model. In this paper, a steady state groundwater model was developed using FDM and AEM methods. Before conceptualization of the model, different input data was generated in a GIS environment whereas image classification exercise was performed to compute the land use map of the area. From the results, it was found that AEM does not require a fixed boundary condition and in FDM pumping wells are approximately located and averaged over the cell which becomes a cause of inaccurate location of wells which does not give an accurate result for that cell. This study can be very helpful for groundwater professionals in deciding the best suitable method for their study area.

Keywords: - Groundwater Model, AEM, FDM, Remote Sensing & GIS

Introduction:-

Water is essential to human life. Fresh water available for drinking is 2.5% of the total water on the Earth. Groundwater comprises 97% of the world's accessible fresh water Butts, K. H. (1997). Groundwater becomes a major source of drinking water because of high contamination in the surface water like Ganga River in Varanasi (Agrawal et al., 2010). The unplanned groundwater extraction and improper groundwater recharge are becoming a major cause of groundwater depletion in the different part of the country. Due to excessive depletion of groundwater, many problems such as land subsidence, shrinkage of aquifer arises (Chai et al. 2004). These problems in turn result to increased pumping cost, reduction in stream base flow, changes in groundwater flow pattern at the local or regional level, failure of established wells and risk for human life and infrastructures.
For effective groundwater management researchers developed many groundwater models and modeling methods. There are two main groundwater models which based on analytic element method and grid-based numerical method. Grid-based numerical method consists of a finite difference method and the finite element method of modeling. Numerical method of modeling required extensive data input, and the accuracy of numerical models depends upon the model input data, the size of space and time discretization (the greater the size of the discretization steps, greater are the possible error), and the numerical method used to solve the model equations. The key function of Analytic element model is that they do not require the discretization of the internal model domain into cells or elements as in case of numerical method (Wang & Anderson 1995, Anderson et al. 2015).
The Analytic element model is defined by "analytic elements" representing line sources and sinks such as rivers and drains or specified head and specified flow boundaries. Wells are also represented as points, and recharge and aquifer properties can be defined on polygons (Strack 1998, Hunt et al. 1998, Bakker et al. 1999, Lange 1999, Moorman 1999).
In the grid-based methods, pumping wells are approximately located and averaged over the cell of a grid which becomes a cause of inaccurate location of wells. Particularly groundwater modeling for the large region where coarse grids are used, it becomes the cause of considerable error in solutions (Kumar Pradeep and Kumar Anil, 2014).

Fundamental Equation of Groundwater Flow

Steady State Condition

Darcy, 1856 gives the law that flow velocity through a porous medium is directly proportional to the hydraulic gradient. If a head loss occurs h_1 to h_2 in small interval l_2 to l_1. Then discharge velocity is given by Darcy law

$$v = -K\frac{\partial h}{\partial l} \tag{1.1}$$

Where the K is aquifer conductivity.

Specific discharge
$$v = \frac{Q}{A} \tag{1.2}$$

The specific discharge has units of velocity and is also known as the *Darcy velocity*. It should be noted that the Darcy velocity is an artificial velocity as it corresponds to the total quantity of water flowing divided by the cross-sectional area. The average velocity, v_w of the water can, therefore, be calculated as the Darcy velocity divided by the effective porosity, n_e.

$$v_w = v/n_e \tag{1.3}$$

The three-dimensional generalization of Darcy's law assumes that the one-dimensional form (Equation 1.1) is true for the three principal components of flow *x*, *y* and *z*:

$$v_x = -K_x\frac{\partial h}{\partial x} \qquad v_y = -K_y\frac{\partial h}{\partial y} \qquad v_z = -K_z\frac{\partial h}{\partial z} \tag{1.4}$$

Let us consider the flow into and out of the elemental cube shown in Figure no. 1.

Derive a mass balance for the cube by summing the results from each component direction. The flow rate through each face is the product of the flow rate through the face per unit area and the area of the face. Consider the cube in Figure no. 1, the flow through the right face can be expressed as

$$(v_x - \frac{\partial v_x}{\partial x}\frac{dx}{2})dy\,dz \tag{1.5}$$

Similarly, the rate of flow leaving the element across the same face is given by the expression

$$(v_x + \frac{\partial v_x}{\partial x}\frac{dx}{2})dy\,dz$$

Therefore, the net volume of flow entering the element due to flow in the x-direction can be expressed as:

$$(\frac{\partial v_x}{\partial x})dxdydz$$

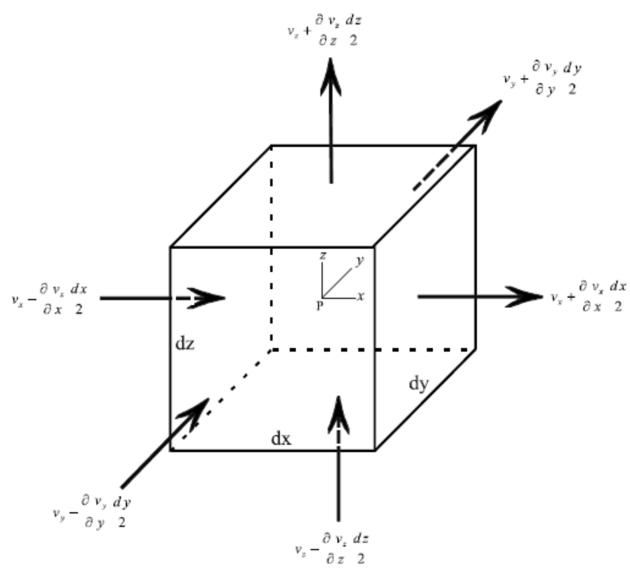

Figure no. 1 Flow for an elementary volume of fluid

Similar expressions can be derived for the other principle directions, y, and z so that an expression for the total volume of water entering the element can be expressed as

$$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial x} + \frac{\partial v_z}{\partial x}\right) dx\,dydz \tag{1.6}$$

Under steady state condition equation, no (1.6) must be zero

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial x} + \frac{\partial v_z}{\partial x} = 0$$

This is the continuity equation for steady state condition.
By differentiating Darcy's law (Equations 1.4) with respect to *x*, *y*, and *z* and adding by the continuity equation (Equation 1.6), we obtain a single second-order partial differential equation:

$$\frac{\partial}{\partial x}\left(-K_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(-K_y\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(-K_z\frac{\partial h}{\partial z}\right)=0 \tag{1.7}$$

This is the general equation of three-dimensional steady state flows.
If we assume the aquifer is homogeneous and isotropic, the value of K is independent of x, y, z and above equation is written as:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0 \tag{1.8}$$

This equation is known as *Laplace's equation* which is governing equation of groundwater flow through the homogeneous and isotropic medium under steady state condition.

Method of Groundwater Modeling

The Analytic Element Method (AEM)

The analytic element method was introduced by O. D. L Strack. The analytic element method was developed for the mathematical modeling of the flow of groundwater and was originally intended for problems of regional flow. The method is based upon the superposition of analytic functions and might be considered as an extension of classical models of regional flow based on the superposition of elementary functions. The AEM is a computational method based upon the superposition of analytical expression to represent the two-dimensional vector fields. The analytic element models can superimpose hundreds of exact analytic solutions to solve groundwater flow problems and are capable of simulating streams, lakes, and complex boundary conditions. Each analytic element can simulate different types of geo-hydrological features (Extraction wells, rivers, infiltration areas, aquifer non-homogeneities, and other features). River is represented by line-sink, well by point-sink, non-homogeneity by line-doublet and recharge by area-sink.
The basic assumptions involve in AEM are flow must be steady, Horizontal layers with a constant thickness, no vertical recharge from leakage, Homogenous and isotropic soil.
In the AEM, groundwater flow is often expressed regarding complex potential Ω $[L^3 T^{-1}]$ as

$$\Omega = \Phi + i\Psi$$

Where, discharge potential Φ $[L^3 T^{-1}]$ and the stream function Ψ $[L^3 T^{-1}]$ fulfill the Cauchy-Riemann condition therefore, Φ and Ψ may be represented as real and imaginary parts of an analytical function Ω = Ω (z) of the complex variable $z = x + iy$, defined in the flow domain.

Finite Difference Method (FDM)

In recent years, a numerical model is become extremely important in groundwater simulation, mainly for making a prediction and improve the process of understanding. There is two methods of numerical model:

1. Finite difference method
2. Finite element method

One of the most important numerical methods for modeling of groundwater flow is the finite difference method, which is used for building, and then, solving partial differential groundwater equations.
The steady-state flow is described by a form of Laplace equation. It is a flow in which all the flow properties (velocity, pressure, etc.) remain constant concerning time. The groundwater flow equation is often derived for a small representative elemental volume where the properties of the medium are assumed to be effectively constant. A mass balance is obtained on the water flowing in and out of this small volume along with Darcy's law to arrive at the transient groundwater flow equation.
The flow equation is based on the continuity equation

Inflow – Outflow = change of storage

For a small portion of an aquifer, it can be restated as:

Subsurface sum + net flow = change in storage

Combining Darcy's law with this continuity equation yields the general form of the equation describing the transient flow

$$T\,\frac{\partial 2_h}{\partial x^2} = -q$$

Where: h is the groundwater potential [L]
T is the uniform aquifer transmissivity $[L^2/T]$
q is inflow per unit length $[L^3/L/T]$

There are many methods that can be used to approximate the groundwater head over a section of an aquifer. The straight-line approximation is the simplest method for this. Other methods such as Taylor's series or other mathematical functions provide more accurate approximations, but for my purposes, the straight-line method provides a good illustration of the fundamentals of finite difference approximation.

Result and Discussion:-

The comparison between two modeling method, the Analytic Element Method (AEM) and Finite Difference Method (FDM) may be performed in several ways, based on many aspects. Some of the following aspects are-

- Theoretical consideration to compare all the hydraulic and mathematical assumptions of these two methods,
- The comparison of the data requirement of these methods,
- Experiences in solving fictitious and real problems using both methods.

AEM provides a continuous groundwater surface while FDM provides a solution at a discrete point in the grid. FDM takes the influences as boundary conditions indirectly into consideration. For example, a river may be a given head boundary at certain grid points. AEM, on the other hand, describes them directly with such a potential that is the most suitable. So the same river turns to be a string of second order line sinks. To choose the most suited potential gives a modeler a high level of freedom. And if neither seems to be suitable, even a new one may also be defined to solve a particular problem. That gives the model a high level of flexibility in the development and use. On the other hand, FDM applies the general basic equation, so it should be suitable for almost any kind of problems.

The discharge potential applied by AEM makes it possible to handle confined and unconfined aquifers the same way, only the transformation to the piezometric head is different. This is a very useful feature, especially if the extent of confined and unconfined parts of a given aquifer is unknown, or subject to changes in case of different situations. On the other hand, the basic equation of FDM contains the term of transmissibility that is different in the two cases. Of course, there are several tools in FDM for the solution to this problem, but it needs a careful model development.

FDM requires given, fix boundaries around the area of interest. On each boundary, a properly defined boundary condition has to be given. But there may be such situations that finding a proper boundary is almost impossible. AEM requires no fix boundaries, but a stripe, the outer area.

In FDM need a grid which is usually rectangularly distributed over an entire area of interest, where in AEM consider all individual features influence the flow as elements. As FDM applies the basic equation at each grid point, all the aquifer parameters and other data have to be given at every point, or an algorithm is needed to calculate them. AEM also requires data connected to the elements and requires a general description of the aquifer. This means the same information in both cases, just presented in a different way.

As mentioned earlier, FDM requires well-defined boundary conditions all around, while the boundary conditions of AEM are connected to the elements, as control points. In the case of FDM, boundary conditions can be considered to interpolation from outer boundaries and in case of AEM, extrapolation from inner boundaries.

Developed AEM flow model is capable of generating the groundwater head for real field data and in some conditions found more efficient than FDM model. AEM based modeling is less complex and less time consuming and is capable of locating the optimal wells more precisely in the domain.

Conclusion:-

Though FDM usually requires a larger amount of data, this connected to grid points. So data processing means usually the transformation of geological and hydro-geological maps to a uniform data set. On the other hand, the data requirements of AEM are connected to the elements, and each type of elements may require a different way of data processing. The presentation of the computation results is usually the same in both cases, i.e., piezometric contours, flow patterns, specific discharges, etc. To obtain them from the grid values of FDM usually interpolation is needed. As AEM determines a continuous potential and stream function, so post-processing usually requires less effort. In short, there are some pros and cons of both the methods and usage depends on the types of problem.

References:-

[1]. Agrawal, A., Pandey, R. S., & Sharma, B. (2010). Water pollution with special reference to pesticide contamination in India. Journal of Water Resource and Protection, 2(05), 432.
[2]. Anderson, M. P., Woessner, W. W., & Hunt, R. J. (2015). Applied groundwater modeling: simulation of flow and advective transport. Academic press.
[3]. Bakker, Mark., Anderson Erik I., Olsthoorn T. N., and Strack Otto DL., (1999) "Regional groundwater modeling of the Yucca Mountain site using analytic elements." Journal of Hydrology 226, no. 3, 167-178.
[4]. Butts, K. H. (1997). The strategic importance of water. Parameters, 27(1), 65.
[5]. Chai, J. C., Shen, S. L., Zhu, H. H., & Zhang, X. L. (2004). Land subsidence due to groundwater drawdown in Shanghai. Geotechnique, 54(2), 143-147.

[6]. Hunt, R. J., Saad, D. A., and Chapel, D. M., (2003). "Numerical Simulation of Ground-Water Flow in La Crosse County, Wisconsin, and into Nearby Pools of the Mississippi River." Water-Resources Investigations Report 03–4154. USGS.

[7]. Kumar Pradeep G. N., and Kumar Anil P., (2014). "Development of Groundwater Flow Model using Visual MODFLOW" International Journal of Advanced Research, Volume 2, Issue 6, 649-656.

[8]. Lange, D. W. J., (1999). "A Cauchy boundary condition for the lumped interaction between an arbitrary number of surface waters and a regional aquifer." J. of Hydrology, 226 (3-4), 250-261.

[9]. Moorman, J. H. N., (1999). "Analytical element model analysis of the influence of different scenarios for the water level in a future retention basin". J. of Hydrology, Vol: 226(3-4); 144-151.

[10]. Strack O. D. L., (1998). "Principles of the analytic element method" Journal of Hydrology 226, 128–138.

[11]. Wang, H. F., & Anderson, M. P. (1995). Introduction to groundwater modeling: finite difference and finite element methods. Academic Press.