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EXCEL SOLVER USED AS A TOOL FOR GROUNDWATER MODELLING

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Abstract- The use of groundwater models is prevalent in the field of environmental hydrogeology. Groundwater flow, fate, and transport models have been applied to investigate a wide variety of hydro geological conditions. Groundwater flow models are used to calculate the rate and direction of movement of groundwater through aquifers and confining units in the subsurface. Fate and transport models estimate the concentration of a chemical in groundwater beginning at its point of introduction to the environment to locations down gradient of the source. Fate and transport models require the development of a calibrated groundwater flow model or, at a minimum, an accurate determination of the velocity and direction of groundwater flow that has been based on field data.

The present work presents a simple ground water modeling spreadsheet template developed in Excel with macros written in Visual Basic for Applications (VBA). It is based on Finite Difference method and can be used to model an aquifer of any shape and size. The given template can be expanded to build groundwater flow models of virtually unlimited number of nodes. The final modelled heads, at each node, can be output as XYZ file, which can be girded by any contouring software to produce contour maps and 3D surfaces. By using excel solver we can minimize the error, thus get the optimal solution. Mat lab coding was used to show the effect of pumping on water table.

Keywords- Excel solver, groundwater, model.

I. INTRODUCTION

Several types of models have been used to study groundwater flow system. These can be divided into three broad categories: analog model, mathematical models, and analytical and numerical models[1]. Hydro-geological studies usually involve Mathematical modelling of groundwater flow[2]. Such models consist of a set of differential equations, which govern the flow of groundwater. They have been in use since the late 1800s, but have found wide spread application with the increase in available computing power. In computer programming, these models are implemented using different approaches among which the finite difference and finite element methods are most common[3], [4].In both these methods a system of nodal points is superimposed over the problem domain. The difference between the two methods is in the distribution of nodes[5]. The finite difference nodes are in a regular grid order where node scan be block-centered or mesh-centered. The finite element methods, on the other hand, can have an irregular distribution of nodes, which are connected together to form triangular subareas called elements. Several commercial software applications for groundwater modelling exist[6]–[8]. To use these applications properly, the user needs to be fully trained. In this paper we present an Excel spreadsheet template for modelling groundwater which is very easy to use and requires no training. Due to the gridnature of Excel cells the finite difference method is used where each cell represents a grid node. Pre-programmed sample cells for interior nodes and no-flow boundary nodes for different sides and corners are given in the template. These cells are copied to the design region according to the requirements of the aquifer to be modelled. Boundary values for constant head nodes are defined and the iterative procedure is initiated to get the flow model.

II. METHODOLOGY

2.1 *Types of Groundwater Flow Models[9]–[11]*:

1. Analytical Models (Exp and ERF functions)
   – 1-D solution, Ogata and Banks (1961)
   – 2-D solution, Wilson and Miller (1978)
   – 3-D solutions, Domenico & Schwartz (1990)

2. Numerical Models (Solved over a grid - FDE)
   – Flow-only models in 3-D (MODFLOW)
   – MODPATH - allows tracking of particles in 2-D placed in flow field produced from MODFLOW.

2.1.1 *Modelling using Excel Spread sheet:*
Several commercial software applications for groundwater modelling exist[8], [12]. To use these applications properly, we need to be fully trained. In this paper an Excel spreadsheet template for modelling groundwater, which is very easy to use and requires no training, is presented. Due to the grid nature of Excel cells the finite difference method is used where each cell represents a grid node. Pre-programmed sample cells for interior nodes and no-flow boundary nodes for different sides and corners are given in the template. These cells are copied to the design region according to the requirements of the aquifer to be modelled. Boundary values for constant head nodes are defined and the iterative procedure is initiated to get the flow model.

2.1.2 *Mathematical background:*
The physics of groundwater flow in two dimensions is defined by Darcy's lawas: -

$$q_x = -k\frac{\partial h}{\partial x}\ ;\ q_y = -k\frac{\partial h}{\partial y}\ ;\ q_z = -k\frac{\partial h}{\partial z} \quad (1)$$

Where q_x, q_y, $q_z$are the specific discharge in the *x*, *y*, *z* directions, *K* is the Hydraulic Conductivity, and *h* is the head which is the function of all three space coordinates and therefore represented by partial derivate[13], [14].
The second important law is the continuity or conservation which for steady state conditions states that the amount of water flowing into a representative elemental volume must be equal to the amount flowing out. Mathematically it can be expressed by the continuity equation as follows:

$$\frac{\partial q_x}{\partial_x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial_z} = 0 \quad (2)$$

Now combining the above two equations and assuming *K* to be independent of *x*,*y*,*z* for a homogeneous and isotropic. The finite difference approximation to Laplace's equation [15] for such a grid is given by aquifer we get a single second-order partial differential equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (3)$$

This is Laplace's equation which governs the flow of groundwater through an isotropic, homogeneous aquifer under steady state conditions. This equation simply states that the sum of partial derivates of head (*h*) with respect to *x*, *y*, and *z* is zero. The solution of Laplace's equation requires specification of boundary conditions, such as Dirichlet conditions and Neumann conditions [16]. As currently we are dealing with groundwater flow in two dimensions, the Laplace's equation reduces to:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad (4)$$

2.1.3 *Finite Difference Scheme:*

1. Finite difference approximations involve applying Taylor's expansions to the equations (flow and transport) and approximating the derivatives in the equation.

2.1.3.1 *Taylor's Expansion:*

A. First-order approximation is *f(xi+1) = f(xi) + f′(xi)(xi+1 – xi)*Straight line projection to next point.
B. Second-order approximation captures curvature

f(xi+1) = f(xi) + f'(xi)(xi+1 – xi) +(f''(xi) (xi+1 – xi)2)/2!

Taylor's Expansion

$$h(x+\Delta x) = h(x) + \Delta x h'(x) + \frac{\Delta x^2}{2} h''(x) + \Lambda$$

$$h(x-\Delta x) = h(x) - \Delta x h'(x) + \frac{\Delta x^2}{2} h''(x) + \Lambda$$

Where *h′(x)* is the first derivative and *h″(x)* is the second derivative and so on.
Taylor's Expansion for Second Derivative

$$h''(x) = \left(\frac{d^2 h}{dx^2}\right)_x \approx \frac{1}{\Delta x^2}\{h(x+\Delta x) - 2h(x) + h(x-\Delta x)\}$$

Approximation to Laplace's Equation: -

$$h''(x_i, y_j)_x + h''(x_i, y_j)_y \approx$$

$$\frac{1}{\Delta x^2}\left[\{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}\} + \{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}\}\right] =$$

$$\frac{1}{\Delta x^2}\left[h_{i-1,j} + h_{i,j-1} - 4h_{i,j} + h_{i+1,j} + h_{i,j+1}\right] = 0 \quad (5)$$

Summing terms and solving for $h_{i,j}$ gives:

$$h_{i,j} = \frac{1}{4}\{h_{i-1,j} + h_{i,j-1} + h_{i+1,j} + h_{i,j+1}\} \quad (6)$$

Using finite difference scheme simulation of groundwater flow (steady):-

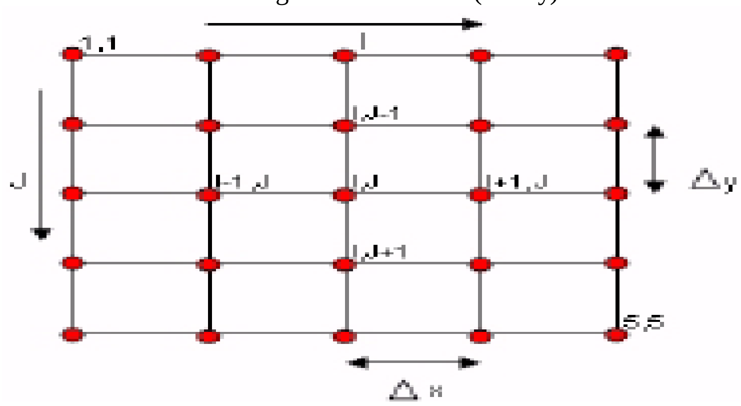

Fig. 1 Finite difference grid of nodes. (Δx=Δy)

III. WORKING THEORY

To design a new model we simply need to copy and paste the required sample cells into the design region of the template [Fig. 2] according to the shape, size, and type of boundaries that define the aquifer. The values for constant head nodes at the boundaries are also defined.

3.1 Steady flow: Governing Equation,

$$R + \frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) = 0 \quad (7)$$

By finite difference

If $K_x = K_y$(Homogeneous and isotropic soil) and Δx=Δy then,

$$h_{i,j} = \frac{1}{4}\left[(h_{i+1,j} + h_{i,j+1} + h_{i-1,j} + h_{i,j-1}) + R\Delta x^2 / K\right] \quad (8)$$

R= volumetric flux per unit volume being pumped.
We also consider the no flow condition, that is, any head difference above and below the specified cell. To design a new model the user simply needs to copy and paste the required sample cells into the design region of the template.

3.2 *Unsteady state:* Governing Equation:

$$\frac{\partial}{\partial x}\left(K_{xx} \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz} \frac{\partial h}{\partial z}\right) - W = S_s \frac{\partial h}{\partial t} \quad (9)$$

$Ss = \Delta V/(\Delta x \Delta y \Delta z \Delta h)$;

where *Kxx*, *Kyy*, and *Kzz* are defined as the hydraulic conductivity along the *x*, *y*, and *z* coordinate axis, *h* represents the potentiometric head, *W* is the volumetric flux per unit volume being pumped, *Ss* is the specific storage of the porous material and *t* is time.

By finite difference:

$$h_{i,j}^{n} = h_{i,j}^{n-1} + c(h_{i+1,j}^{n-1} + h_{i,j+1}^{n-1} - 4h_{i,j}^{n-1} + h_{i-1,j}^{n-1} + h_{i,j-1}^{n-1}) + R\Delta t / S_S \quad (10)$$

Where;

$$R = -\frac{Q}{\Delta x^2} \quad \text{and} \quad c = \frac{K\Delta t}{S_S \Delta y^2}$$

Convergence criteria:

- Convergence for a flow code requires that the change in the solution at each point be less than a specified target, called the convergence criterion, or sometimes epsilon ε.
- If ε is too large, convergence will occur before a solution is reached.
- Here we take $\varepsilon = 0.001$.

3.3 *Working with Excel and Excel Solver:*

3.3.1 Head distribution in Excel grid:

	1	3	4	5	6	7	8	9	10	
	99	99	99	99	99	99	99	99	99	
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
100	99	99	99	99	99	99	99	99	99	99
	99	99	99	99	99	99	99	99	99	

No flow boundary condition

Head distribution in grid cell

Fig. 2 The head distribution in the grid.

When $t = 0$ (steady state) we minimize the error by Excel solver, result shows the variation of head for minimization, that is,

Minimize $f(h_{i,j})$: $$h_{i,j} = \frac{1}{4}\left[(h_{i+1,j} + h_{i,j+1} + h_{i-1,j} + h_{i,j-1}) + R\Delta x^2 / K\right]$$

Subject to: $\varepsilon \le .001$

IV. Results

(A) When $t = 0$, that is, steady flow:

	98.98	98	97.3	96.76	96.6	96.7	97.05	97.5	98	98.5	
100	98.98	98	97.3	96.76	96.6	96.7	97.05	97.5	98	98.5	99
100	98.9	97.9	97	96.42	96.3	96.5	96.94	97.4	97.9	98.5	99
100	98.75	97.5	96.4	95.62	95.7	96.2	96.75	97.3	97.9	98.4	99
100	98.58	97.1	95.5	93.93	94.8	95.7	96.54	97.2	97.8	98.4	99
100	98.49	96.8	94.4	89.86	93.7	95.4	96.45	97.2	97.8	98.4	99
100	98.61	97.1	95.5	94.02	94.9	95.8	96.62	97.3	97.9	98.5	99
100	98.82	97.6	96.6	95.81	95.9	96.4	96.9	97.4	98	98.5	99
100	99	98.1	97.3	96.74	96.6	96.8	97.19	97.6	98.1	98.5	99
100	99.13	98.3	97.7	97.24	97.1	97.2	97.39	97.7	98.1	98.6	99
100	99.2	98.5	97.9	97.47	97.3	97.3	97.5	97.8	98.2	98.6	99
	99.2	98.5	97.9	97.47	97.3	97.3	97.5	97.8	98.2	98.6	

Fig. 3 Variation of head after error minimization

Error

0.001	1E-03	0.001	0.001	0.001	1E-03	1E-03	1E-03	0.001	1E-03	0.01
0.001	1E-03	1E-03	1E-03	0.001	1E-03	0.001	1E-03	0.001	0.001	0.01
0.001	1E-03	0.001	1E-03	1E-03	0.001	0.001	0.001	0.001	0.001	0.01
0.001	0.001	8E-04	1E-03	0.001	0.001	0.001	1E-03	1E-03	1E-03	0.0098
0.001	0.001	0.001	5E-06	1E-03	0.001	1E-03	0.001	0.001	0.001	0.009
0.001	0.001	0.001	1E-03	1E-03	1E-03	0.001	0.001	0.001	0.001	0.01
0.001	0.001	1E-03	0.001	1E-03	1E-03	1E-03	1E-03	1E-03	1E-03	0.01
0.001	1E-03	0.001	1E-03	0.001	1E-03	0.001	1E-03	1E-03	0.001	0.01
0.001	1E-03	1E-03	1E-03	1E-03	0.001	1E-03	0.001	0.001	0.001	0.01
0.001	0.001	0.001	1E-03	1E-03	0.001	1E-03	0.001	1E-03	1E-03	0.01
										0.0989

Fig. 4 Variation of error in steady flow

(B) When t = 1,that is, unsteady flow:

	99	99	99	99	99	99	99	99	99	99	
100	100	99.8	100	99.8	100	99.7	99.61	99.4	99.2	99.1	99
100	100	100	100	100	100	100	100	99.7	99.4	99.2	99
100	99.37	100	100	100	101	100	100.3	99.8	99.5	99.2	99
100	99.1	99.9	100	100	100	101	100.4	99.9	99.6	99.3	99
100	99.04	99.9	100	100	100	100	100.4	100	99.6	99.3	99
100	99.11	99.9	100	100	100	100	100.3	99.9	99.6	99.3	99
100	99.29	100	100	100	100	100	100.2	99.8	99.5	99.2	99
100	99.97	100	100	100	100	100	99.87	99.6	99.4	99.2	99
100	99.83	99.8	100	99.8	100	99.7	99.57	99.4	99.3	99.1	99
100	99.57	99.4	99	99.4	99	99.3	99.28	99.2	99.1	99.1	99
	99	99	99	99	99	99	99	99	99	99	

Fig. 5 Variation of head after error minimization

Error

0.001	1E-03	1E-03	0.001	0.001	0.001	1E-03	0.001	1E-03	0.001	0.01
0.001	1E-03	0.001	1E-03	1E-03	0.001	0.001	1E-03	1E-03	1E-03	0.01
0.001	1E-03	1E-03	0.001	1E-03	0.001	1E-03	1E-03	0.001	1E-03	0.01
0.001	1E-03	0.001	1E-03	0.001	0.001	0.001	0.001	0.001	1E-03	0.01
0.0002	0.001	1E-03	1E-03	1E-03	0.001	1E-03	0.001	0.001	0.001	0.0092
4E-05	0.001	1E-03	1E-03	1E-03	1E-03	0.001	0.001	0.001	0.001	0.009
0.001	0.001	1E-03	1E-03	1E-03	1E-03	1E-03	1E-03	1E-03	0.001	0.01
0.001	1E-03	0.001	1E-03	1E-03	1E-03	1E-03	1E-03	1E-03	0.001	0.01
0.001	1E-03	0.001	0.001	1E-03	1E-03	0.001	1E-03	0.001	0.001	0.01
0.001	1E-03	1E-03	0.001	1E-03	0.001	1E-03	0.001	1E-03	0.001	0.01
										0.0982

Fig. 6 Variation of error

(A) When t = 2,that is,unsteady flow:

	99	99	99	99	99	99	99	99	99	99	
100	99.84	99.8	100	99.8	100	99.7	99.58	99.3	99.2	99.1	99
100	100.2	100	100	100	100	100	99.98	99.6	99.4	99.2	99
100	100.4	101	101	101	101	101	100.2	99.8	99.5	99.2	99
100	100.5	101	101	101	101	101	100.4	99.9	99.5	99.3	99
100	100.5	101	101	101	101	101	100.4	99.9	99.6	99.3	99
100	100.5	101	101	101	101	101	100.4	99.9	99.5	99.3	99
100	100.4	101	101	101	101	101	100.2	99.8	99.5	99.2	99
100	100.3	100	100	100	100	100	99.88	99.6	99.4	99.2	99
100	99.95	99.9	100	99.9	100	99.7	99.55	99.4	99.2	99.1	99
100	99.61	99.5	99	99.4	99	99.3	99.26	99.2	99.1	99.1	99
	99	99	99	99	99	99	99	99	99	99	

Fig. 7 Variation of head after error minimization in Unsteady flow

Error

	0.001	1E-03	0.001	0.001	1E-03	1E-03	1E-03	0.001	1E-03	0.009
0.0008	0.001	0.001	1E-03	1E-03	1E-03	1E-03	0.001	0.001	1E-03	0.0098
0.0003	1E-03	1E-03	0.001	0.001	1E-03	0.001	1E-03	1E-03	0.001	0.0093
0.0003	1E-03	1E-03	1E-03	0.001	1E-03	0.001	1E-03	0.001	1E-03	0.0093
0.0008	1E-03	1E-03	1E-03	1E-03	0.001	0.001	1E-03	0.001	1E-03	0.0098
0.0004	0.001	0.001	1E-03	1E-03	1E-03	0.001	0.001	1E-03	0.001	0.0094
0.0006	0.001	1E-03	1E-03	0.001	1E-03	0.001	1E-03	0.001	0.001	0.0096
0.0005	1E-03	0.001	0.001	0.001	1E-03	1E-03	1E-03	0.001	1E-03	0.0095
7E-05	0.001	1E-03	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.0091
5E-06	0.001	1E-03	0.001	0.001	0.001	0.001	0.001	0.001	1E-03	0.009
										0.0938

Fig. 8 Variation of error in unsteady flow

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	-5000	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Fig. 9 Effect of pumping on head

OTHER DATA:
T_x = 500 m^2
(Δx* Δy) = 100 ×100m
P = $1000 m^3$/ day
Ss = 0.015
Q = 5000 m^3/day
Variation of head = contour mapping (using Mat lab)
We plot the variation of head in graphical form known as contour drawing, which is shown in Figure 5.

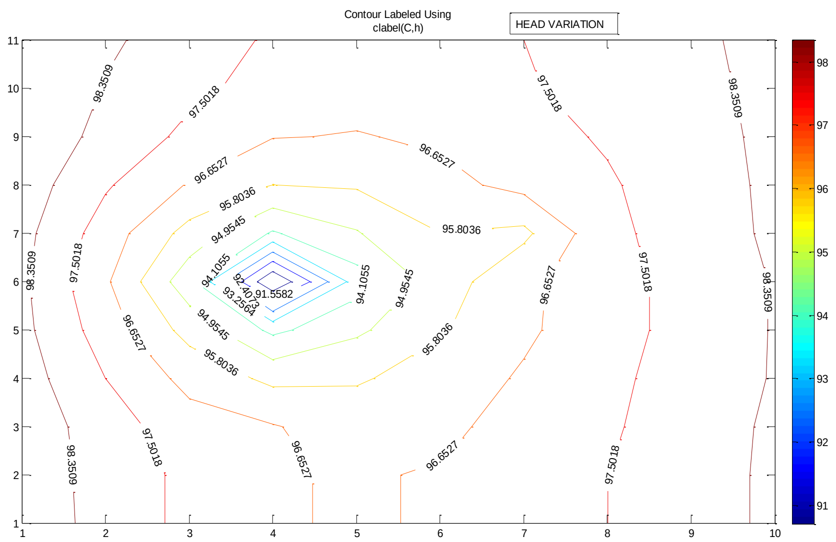

Fig. 10 Variation of head after pumping

V. CONCLUSIONS

It is concluded that Excel-based spreadsheet modeling can be effectively utilized to develop the equipotential surface features of the groundwater flow regimes. Directions of groundwater flows are monitored by assigning the vector lines at right angle to these equipotential surfaces. Groundwater volume inactive storage can be ascertained from the complete flow-net based on Excel-based spreadsheet modeling. In addition it can also be used, along with contouring software, to generate small to large groundwater flow models. Numerical models are capable of solving the more complex equations that describe groundwater flow and solute transport. These equations generally describe multidimensional groundwater flow, solute transport, and chemical reactions. A model may be used to predict the pumping rate needed to capture a contaminant plume and to estimate the contaminant concentration of the extracted groundwater. Monitoring of hydraulic heads and contamination concentrations must be used to verify hydraulic containment and remediation of the contaminant plume.

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