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Design of FOPID Controller Based on Particle Swarm Optimization

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Abstract -This paper deals with the designing of Fractional Order Proportional Integral Derivative (FOPID) controller, also known as $PI^{\lambda}D^{\delta}$ Controller using Intelligent optimization method based on Particle Swarm Optimization (PSO). The FOPID controller is similar to conventional PID controller. But the only difference is that in FOPID Controller, the integral order (λ) and the derivative order (δ) are fractional. The FOPID and conventional PID controllers are mainly employed in the industrial applications to control the various parameters such as speed, pressure, flow, temperature etc. In order to design FOPID Controller, it is needed to optimize five parameters namely proportionality constant (Kp), integral constant(Ki), derivative constant(Kd), integral order (λ) and derivative order (δ) whereas in case of conventional PID controller, it just needed three parameters to optimize. Therefore, it is little bit complex to design FOPID controller but it is more challenging to design it compared to a classical PID controller. The FOPID controller also focuses on reducing the integral of squared error(ISE) and integral time square error(ITSE).

Keywords: Fractional Calculus, PID and FOPID controllers, Particle Swarm Optimization, ISE and ITSE errors.

I. INTRODUCTION

PID control is one of the oldest control strategies which is being widely used in many industrial applications due to its simple design, better performance including small settling time and low percentage overshoot for slow industrial processes. Therefore, it is more important to improve the quality and robustness of PID Controller. PID controller with fractional order of I and D is one of the methods that are used to improve the Conventional PID Controllers. In FOPID controller, in addition to proportional, integral and derivative constants (Kp, Ki, and Kd) it has two more additional parameters namely, the order of fractional integration (λ) and the order of fractional derivative (δ). Therefore, it has five parameters to find which made the design of FOPID controller more complex.

In order to get efficient and optimal controller, it is needed to find an optimal set of values for Kp, Ki, Kd, λ and δ. Many powerful intelligent searching methods are there to find an optimal solution. One such method is Particle Swarm Optimization (PSO) which is an evolutionary computation technique now-a-days. This technique combines evolutionary techniques and social psychology principles in socio-cognition human agents. This method mainly depends on the behaviors of living organisms such as fish schooling and bird flocking. It has better features compared to other soft computing techniques such as simple to understand, easy to implement, computationally efficient algorithm, more flexible and well-balanced mechanism to enhance global and local exploration abilities and finally it is more efficient than GA.

II. Fractional Calculus

Today, in most of the Industrial applications, fractional order systems are emerging which can be modeled by using Fractional Order Differential Equations. So, in order to solve these kind of equations, Fractional Calculus is used which is a branch of calculus that generalizes the derivative of a function to non-integer order, allowing calculations such as deriving a function to 1/2 order. Fractional Calculus is widely used in different fields with many applications in physics, engineering, mathematical biology, finance, life sciences, and optimal control.

The first appearance of fractional calculus is before three centuries prior. In 1695, Leibniz and L. Euler described the derivative of the order α=1/2 in 1730. It has been developed up to nowadays. The following are the great mathematicians, whom have provided important contributions up to the middle of 20 century, which includes P.S. Laplace (1812), J.B.J. Fourier (1822), N.H. Abel (1823-1826), J. Liouville (1832-1873), B. Riemann (1847), H. Holmgren (1865-67), A.K. Grunwald (1867-1872), A.V. Letnikov (1868-1872), H. Laurent (1884), P.A. Nekrassov (1888), A. Krug (1890), J. Hadamard (1892), O. Heaviside (1892-1912), S. Pincherle (1902), G.H. Hardy and J.E. Littlewood (1917-1928), H. Weyl (1917), P. Levy (1923), A. Marchaud (1927),H.T. Davis (1924-1936), A. Zygmund (1935-1945), E.R. Love (1938-1996), A. Erdelyi (1939-1965), H. Kober (1940), D.V. Widder (1941), M. Riesz (1949).

Fractional calculus is an efficient and successful tool for describing complex dynamical systems that cannot be well illustrated using ordinary differential and integral equations. There are several different ways to define fractional order differential and integral equations.

The generalized Differintegrator operator may be defined as

$${}_aD_t^{\alpha}f(t) = d^{\alpha}f(t)/[d(t-a)]^{\alpha}$$

where *a* is starting limitation, *t* is final limitation and α is fractional order. There are several mathematical definitions used for fractional differintegral. The most important of these definitions are:

a. Riemann-Liouville definition(RL)

$${}_aD_x^{-\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1}f(t)dt \quad , \qquad \alpha \in N$$

b. Caputo definition

$${}_a^CD_t^{\alpha}\, f(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^{n}(\tau)\,d\tau}{(t-\tau)^{\alpha-n+1}} \qquad n\text{-}1<\alpha<n$$

c. Grunwald-Letnikov definition

$${}_a^{GL}D_t^{\alpha}\, f(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left[\frac{t-a}{h}\right]}(-1)^j \binom{\alpha}{j} f(t-jh)$$

III. Stability of Fractional Order Systems

While designing a control system, the very fundamental but major requirement is stability.We can say particular integer order continuous-time linear time-invariant system as stable only if all the poles of the characteristic equation of a system lies in the left half of the S-plane. But for a fractional order system, the stability is not determined by only the locations of the poles in the left half side. It depends on the fractional order that becomes more complex.

The characteristic equation of a general linear fractional differential equation has the form

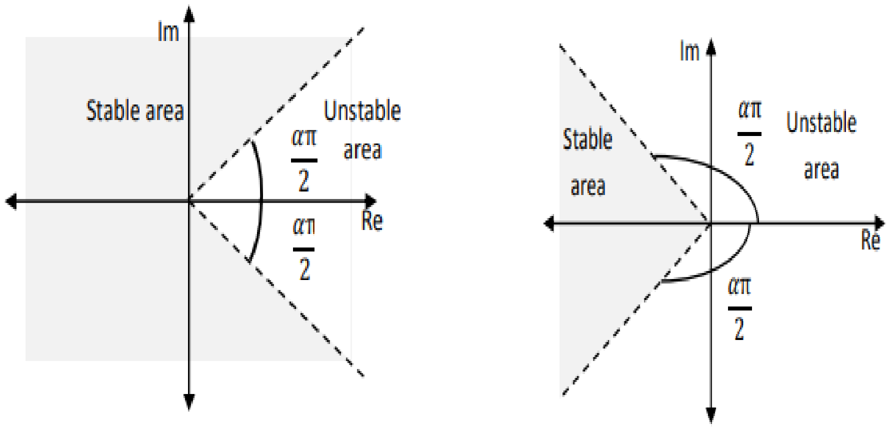
$$\sum_{i=0}^{j} a_i S^{\alpha_i} = 0$$

Where α_i is rational. The characteristic equation can be written as

$$\sum_{i=0}^{n} a_i S^{\frac{i}{m}} = 0$$

Where, m is integer, $\alpha = \frac{1}{m}$ and $a_i > 0$

Fig(1.a) shows the stability of fractional order system when 0 <α <1, the stability region is larger than integer order system. Fig(1.b) shows the stability of fractional order system when 1 <α <2, the stability region is less than integer order system.



0 <α<1 1 <α< 2

IV. FOPID Controller

The FOPID ($PI^{\lambda}D^{\delta}$) controller was proposed for the first time by Podlubny in 1999. It is an extension to classical PID Controller based on Fractional Calculus. From many decades, the conventional PID controllers are gaining much popularity in industries for process control applications because of their simple design and better performance in terms of percentage overshoot and settling time. Though conventional PID Controllers are the best, they are limited only to regular Integer Order systems. But nowadays, in the field of automatic control, most of the systems are Fractional order systems. So, this leads to introduction of FOPID Controllers with the main intention of controlling the Fractional Order Systems. The FOPID controller with closed loop system is shown in fig.2

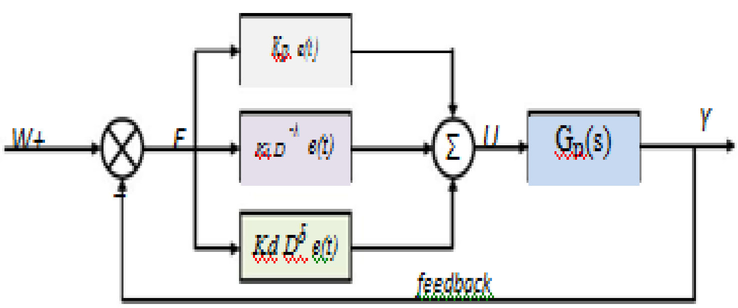


Fig. 2 FOPID Controller with general closed loop control system

The transfer function of FOPID Controller is given by

$$U(t) = K_p\, e(t) + K_i\, D^{-\lambda}\, e(t) + K_d D^{\delta}\, e(t)$$

a.

where λ and δ are positive real numbers, k_p is the proportionality gain constant, k_i is the integral gain constant, k_d is the derivativeconstant, i is the integration constant and d is the derivation constant.

By taking the Laplace Transform of above equation, the controller equation becomes

$$U(S) = K_p\, e(s) + K_i\, S^{-\lambda}\, e(s) + K_d S^{\delta}\, e(s)$$

Putting λ =0 and δ =0, we can obtain classical PID Controller. Similarly, if we put λ =0 and ki=0, we will obtain PD Controller and if we put δ =0 and k_d=0,

will obtain PI Controller. Like that, all the types of controllers are the particular cases of FOPID Controller. The conventional PID controller is represented by four points(P, PI, PD, PID), while the fractional PID controller is extended to plane. This extension leads to more flexibility and accuracy. The fig.3 below displays this concept clearly.

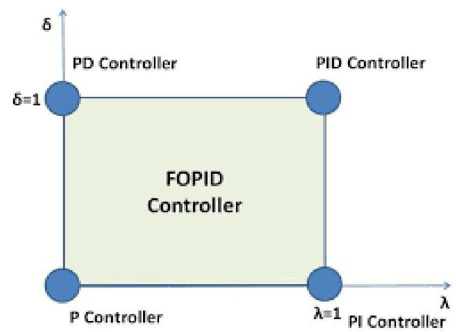


Fig. 3 Generalization of FOPID controller from point to plane

Due to the presence of more tuning Knobs in FOPID Controller, it can be expected that it will enhance the control performance of the systems.

IV. *Particle Swarm Optimization*

PSO is a Swarm intelligence meta-heuristic method which is inspired by the group behavior of Animals, for example bird flocks or fish schools. It is mainly a population based stochastic optimization technique similar to Genetic Algorithm. It means, it represents the state of algorithm by a population, which is iteratively modified until a termination criterion is satisfied. In PSO algorithm, the population $P=(p_1, p_2, ..., p_n)$ of the feasible solution is often called as Swarm. The feasible solutions $p_1, p_2, ..., p_n$ are called particles. This PSO technique was introduced in 1995 by Dr. Eberhart and Dr. Kennedy. This PSO technique can be applied to wide range of fields such as Function Optimization, model classification, neural networks training, data mining, signal processing, automatic adaption control, vague system control etc., because of its many advantages like it is so simple, easy to implement, it is derivative free, very efficient global search algorithm, very few algorithm parameters etc. In PSO, each particle has position and velocity that are both changing because of movement. The new position and velocity of particle can be calculated by two equations given below.

$$V_{id}^{k+1} = MV_{id}^{k} + C_1 * rand(\,) * (P_{bestid} - X_{id}^{k}) + C_2 * rand(\,) * (G_{bestid} - X_{id}^{k})$$

$$X_{id}^{k+1} = X_{id}^{k} + V_{id}^{k+1}$$

Wherei=1,2,3,....,N; where N is the number of particles in swarm.

d=1,2,3,....,D; where D is the number of dimensions of particle.

V and X are the velocity and positions of the particles respectively. P_{best} is the local best position of i^{th}particle ,G_{best} is the global best position of i^{th} particle. rand is the random number in range (0-1), C_1 ,C_2 are the regulating constants, M is the Inertia weight Constant.

In this paper, C_1 ,C_2 are taken as 2 and M as 0.75.

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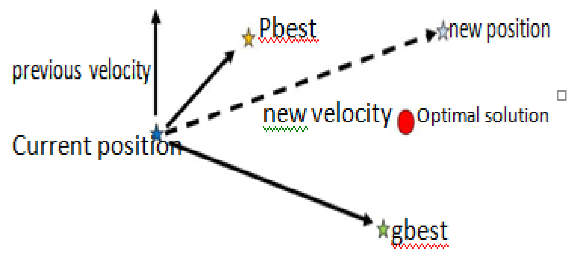
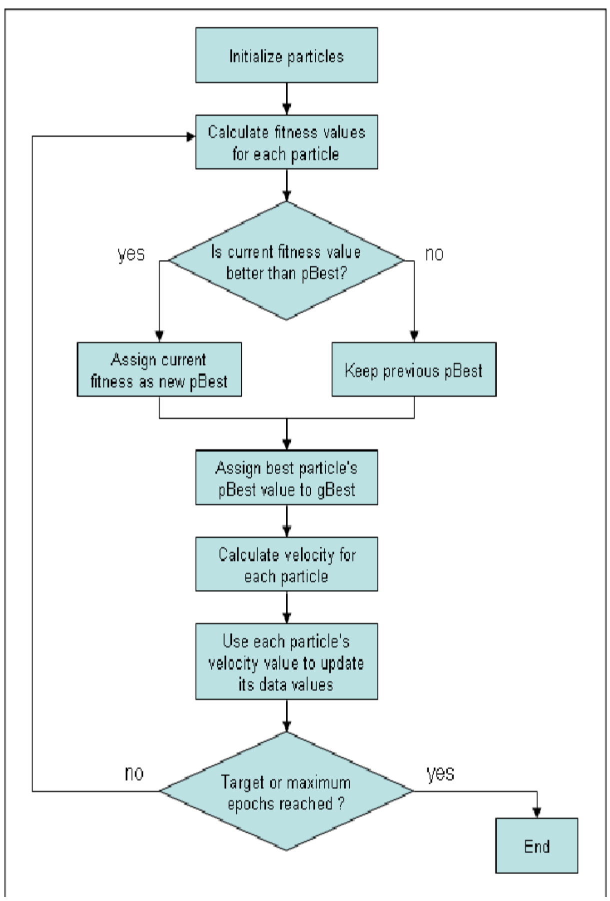


Fig. 4 Trajectory of Particle after Velocity Updating

The fig.4 shows the trajectory of solution. This trajectory depends on position and velocity equations of particle. The new position is the average of vectors.

The fig.5 given below shows the flowchart of PSO Algorithm



V. *Conclusion*

In this paper, by using Particle Swarm Optimization (PSO) algorithm, the optimal parameters of PID and FOPID controllers are determined. Moreover, in this paper, two comparisons are made. One is between the efficiencies of different optimization algorithms such as PSO algorithm and Genetic algorithm. The another one is between the capabilities of PID and FOPID controllers to minimize the performance criteria.PID and FOPID controllers are applied on three types of systems namely Stable, Non-minimum phase and Unstable systems and along with PID Constants, two performance indices ISE and ITSE are also

In general, the FOPID controllers give similar results for both PSO algorithm and Genetic algorithm.

But PSO algorithm is found to be the best one compared to Genetic algorithm because it needs less very number of iterations compared to Genetic algorithm to find the optimal parameters of PID controller so that, the computational time gets reduced when PSO technique is used. On the other side, the FOPID controller gives better performance criteria ISE or ITSE than conventional PID controllers.

The features of the proposed method are given below

(i) It is easy to implement

(ii) The problem is easy to formulate

(iii) For different problems, the entire procedure need not to be changed, only small part required to change

(iv) It allows the designer to find the global optimal parameters of the controller

(v) It allows the designer to use different performance criterions

(vi) The proposed method can be applied on different plants with promising results compared to other methods.

VI. *Illustrative Examples*

To illustrate the proposed design method of the FOPID controller, detailed examples are presented as well as the results of using FOPID are compared with the results of using conventional PID controller.

Example 1.

The third order open loop stable system is described as

$$G(s) = \frac{1}{(s+3)(s^2+2s+2)}$$

It is needed to find the optimal values of the conventional PID controller gains (K_p, K_i , and K_d) and then the optimal parameters of FOPID controller (K_p, K_i , K_d, λ and δ) in order to minimize the ISE and ITSE performance indices using PSO.

Table (I) shows the results of optimal values of PID and FOPID controllers for Example1.

Fig.(6) represents the step responses of the optimal PID and FOPID controller parameters.

Table I.The optimal parameters of PID and FOPID controllers for Example1

Criterion	Method	K_p	K_i	K_d	λ	δ	Value of criterion	Number of Iteration
ISE	PID with GA [27]	30.1960	100	100	1	1	0.126327	1000
	PID with PSO	30.6981	100	100	1	1	0.124561	150
	FOPID with PSO	100	100	100	0.335	1.7556	0.024006	150
ITSE	PID with GA [27]	60	100	100	1	1	0.026563	1000
	PID with PSO	60.3886	100	100	1	1	0.025774	150
	FOPID with PSO	96.7243	83.56	71.4112	0.882	1.3205	0.004452	150

Assume the input is unit step, the observation time T_{ob}=20 sec, the step size of simulation is H_s=0.001 sec. Maximum iterations are 150, the number of particles are 10 and the search space for K_p, K_i, and K_d is from 0 to 100 and the fraction orders λ and δ ranges from 0 to 2. The PSO parameters setting is C_1= 1.5, C_2=1.5 and *M* varies from 4 to 0.25.

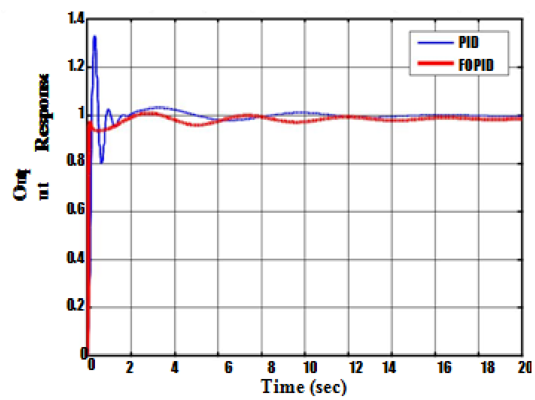


Fig. 6a ISE criterion

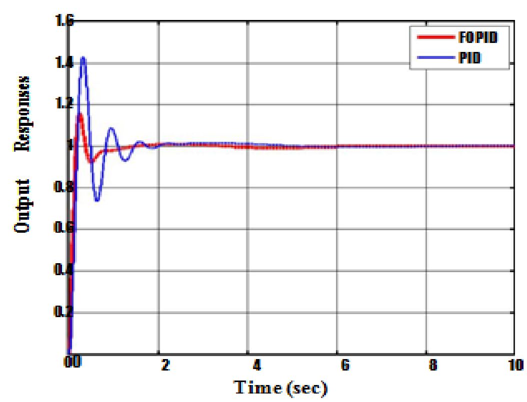


Fig. 6b ITSE Criterion

Fig.6The step responses of optimal PID and FOPID controllers for Example 1.

Example 2.

The third order open loop non-minimum phase system is described as

$$G(s) = \frac{(s-1)}{(s+1)(0.3s+1)^2}$$

It is needed to find the optimal values of the conventional PID controller gains and then the optimal gains and fractional orders of FOPID controller in order to minimize the ISE and ITSE criterion using proposed PSO. The input is assumed as unit step, the observation time is T_{ob}=20 sec, the step size of simulation is H_s=0.001 sec. the Maximum iterations are 150, the number of particles are 10 and the search space for Kp, Ki and K_d is from -1 to 1. The values of λ and δ ranges from 0 to 2. The PSO setting parameters are

C_1= 1.5, C_2=1.5 and *M* varies from 4 to 0.25.

Table (II) shows the results of optimal values of PID and FOPID controllers for Example2.

Fig7. represents the step responses of the optimal PID and optimal FOPID controllers.

Table II.The optimal parameters of PID and FOPID controllers for Example2.

Criterion	Method	Kp	Ki	Kd	λ	δ	Value of criterion	Pop size	Number of Iteration
ISE	PID with GA[27]	-0.6549	-0.4823	-0.3176	1	1	2.125728	30	1000
	PID with PSO	-0.6579	-0.4851	-0.3149	1	1	2.124423	10	150
	FOPID with PSO	-0.9832	-0.3885	-0.2406	1.124	1.379	2.0408	10	150
ITSE	PID with GA[27]	-0.8117	-0.5843	-0.3725	1	1	1.126229	30	1000
	PID with PSO	-0.8150	-0.5895	-0.3740	1	1	1.1242	10	150
	FOPID with PSO	-0.9924	-0.5978	-0.4200	0.979	1.185	1.0512	10	150

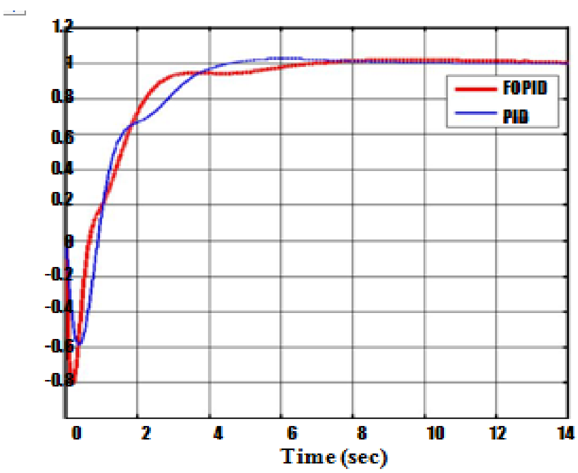


Fig.7a ISE criterion

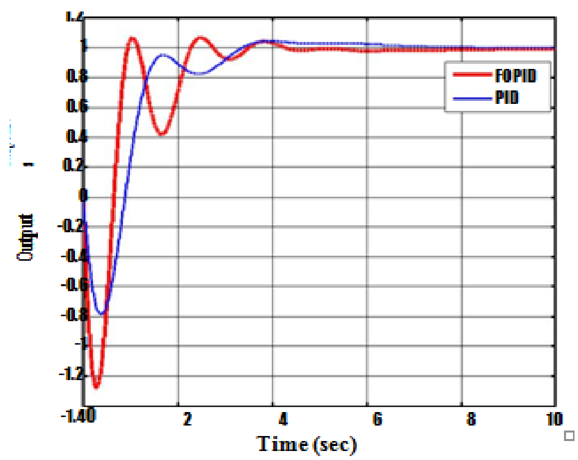


Fig.7b ITSE criterion

Fig.7The step responses of optimal PID and FOPID controllers for Example 2

Example 3.

The open loop unstable system with transfer function is

$$G(s) = \frac{(s+1)}{s(s^2-2s+2)}$$

It is needed to find the optimal values of the conventional PID controller gains and then the optimal gains and orders of FOPID controller in order to minimize the ISE or ITSE criterion using proposed PSO. The input is assumed as unit step, the observation time is T_{ob}=20 sec, the step size of simulation is H_s=0.001 sec. the Maximum iterations are 150, the number of particles are 10 and the search space for Kp, Ki and K_d is from -1 to 1. The values of λ and δ ranges from 0 to 2. The PSO setting parameters are C_1= 1.5, C_2=1.5 and *M* varies from 4 to 0.25.

Table (III) shows the results of optimal values of PID and FOPID controllers for Example(3).

Fig.(8) represents the step responses of the optimal PID and optimal FOPID Controllers.

Table III.The optimal parameters of PID and FOPID controllers for example3

Criterion	Method	Kp	Ki	Kd	λ	δ	Value of criterion	Number of Iteration
ISE	PID with GA[27]	10	0.83	10	1	1	0.0756	1000
	PID with PSO	10	0.806	10	1	1	0.0747	150
	FOPID with PSO	10	10	10	0	1.6082	0.01899	150
ITSE	PID with GA[27]	10	0.54901	10	1	1	0.03726	1000
	PID with PSO	10	0.5646	10	1	1	0.037124	150
	FOPID with PSO	10	10	10	0	1.2137	0.017424	150

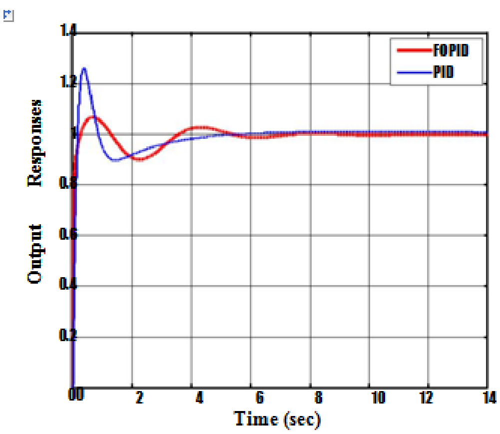


Fig. 8a ISE criterion

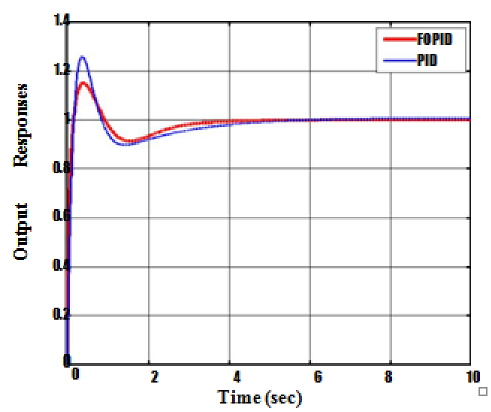


Fig.8b. ITSE criterion

Fig.8:The step responses of optimal PID and FOPID controllers for Example 3

References

[1] YuncanXue, Haibin Zhao and Qiwen Yang, "Self-tuning of PID Parameters Based on the Modified Particle Swarm Optimization," IEEE, 2008.

[2] C. Farges, J. Sabatier and M. Moze, "Fractional order polytopic systems: robust stability and stabilization," Springer, Advances in Difference Equations ,Sep. 2011.

[3] I. Radek M., "Stability of fractional-order systems with rational orders: A survey," Fractional Calculus &Applied Analysis, Vol. 12, No. 3, 2009, pp. 269-298.

[4] Zoran Vukic, OgnjenKuljaca, "Lectures on PID Controllers," April, 2002.

[5] Wenxing Xu, Z. Geng, Q. Zhu and XiangbaiGu, "A piecewise linear chaotic programming based robust hybrid particle swarm optimization," Elsevier, June 2012.

[6] http://introcs.cs.princeton.edu/java/assignments/collisions.html.

[7] M. Zamani, M. Karimi-Ghartemani, N. Sadati, "FOPID Controller Design for Robust Performance Using Particle Swarm Optimization.

[8] Almeida, Malinowska, and Torres, "A fractional calculus of variations for multiple integrals with application to vibrating string," American Institute of Physics, March 2010.

[9] R. Gorenflo and F. Mainardi, "Fractional Calculus: Integral and Differential Equations of Fractional Order," December 2000.

[10] Kambiz- A. Tehrani, T. Capitaine, L. Barrandon , M. Hamzaoui, S.M.R .Rafiei and A. Lebrun, "Current Control Design with a Fractional-Order PID for a Three-Level Inverter," IEEE, Oct. 2005.

[11] Duarte Pedro Mata de Oliveira Valério, "Fractional Robust System Control," Technical University of Lisbon , Oct. 2005.

[12] http://www.sosmath.com/calculus/improper/gamma/gamma.html

[13] B. Bonilla and J. J. Trujillo, "Fractional Order Continuity and Some Properties about Integr ability and Differentiability of Real Functions," Academic Press, April 1998.

[14] DeepyamanMaiti and Amit Konar, "Approximation of a Fractional Order System by an Integer Order Model Using Particle Swarm Optimization Technique,"IEEE, Control And Computer Vision In Robotics& Automation 2008.

[15] A.G. Radwan , A.M. Soliman, A.S. Elwakil, A. Sedeek, "On the stability of linear systems with fractional-order elements," Elsevier, Oct. 2007.

[16] M. Buslowicz, "Stability analysis of continuous-time linear systems consisting of n subsystems with different fractional orders," Bulletin of The Polish Academy of Sciences, Technical Sciences, Vol. 60, No. 2, 2012.