

MHD Micropolar Fluid and Heat Transfer with Radiation in a Permeable Channel using Differential Transform Method

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Abstract: Differential transform method (DTM) is employed to investigate the effect of hydromagnetic flow of incompressible micropolar fluid and heat transfer in a permeable channel with radiation subject to a chemical reaction. The similarity transformations are applied to reduce governing partial differential equation into nonlinear ordinary differential equations in dimensionless form. The effect of physical parameters such as Reynolds number, magnetic parameter, micro rotation/angular velocity, radiation parameter and Peclet number on the flow and heat transfer are analyzed. In the presence of magnetic, the effect of radiation parameter decreases the temperature and increasing by increasing values of Peclet number.

Keywords: MHD, Micropolar fluid, DTM, Peclet number, Radiation parameter.

1. INTRODUCTION

A micropolar fluid is the fluid with internal structures in which coupling between the spin of each particle and the macroscopic velocity field is taken into account. The theory of micropolar fluids was first introduced and formulated by Eringen [11]. This theory displays the effect of local rotary inertia and couple stress. This theory is expected to provide a mathematical model for the non-Newtonian fluid behavior observed in certain fluid such as exotic lubricants, polymeric fluid. Colloidal fluids, liquid crystals, dirty oils, animal blood, etc., which is more realistic and important from a technological point of view. The theory of thermos micropolar fluids was developed by Eringen [12], by extending his theory of micropolar fluid. Also in recent years some researchers investigated on micropolar fluids and their phenomenon.

Seddeek[30] investigated the flow of a magnetomicropolar fluid past a continuously moving plate. Such a fluid can be opted as a cooling liquid as its flow can be regulated by external magnetic field, which regulates heat transfer to some extent. The thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation is investigated by Ishak [18]. It is known that constant physical properties of the fluid may change with temperature and fluid viscosity. Elbarbary [10] studied the effect of variable viscosity on magneto-micropolar fluid flow in the presence of radiation.

The Mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media is presented by Lukaszewicz [20]. El-Arabawy [8] analyzed the problem of the effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation. Modather [23] have also considered the effect of chemical reaction on the heat and mass transfer of micropolar fluids in a saturated porous medium over an infinite moving permeable plate in presence of magnetic field.

Magyari and Chamkha [21] studied the combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. EL-Kabeir [9] discussed the problem of heat transfer in a micropolar fluid flow past a permeable continuous moving surface. Rashidi [28] have obtained analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation effect.

The radiative flows of an electrically conducting fluid with high temperature in the presence of a magnetic field are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas. The free convection flow through a porous medium bounded by a porous plate in the presence of thermal radiation was investigated by Raptis[27]. Makinde[22] examined the transient free convection along moving vertical porous plate in the presence of thermal radiation.

Ibrahim[17] discussed the effect of thermal radiation on mixed convection flow. On the other hand, the fluid can be treated as electrically conducting in the sense that it is ionized due to the high operating temperature. Accordingly, the influence of magnetic field on the fluid flow characteristics is important in the presence of thermal radiation. Das [7] investigated the effect of thermal radiation on fluid flow over a flat plate in the presence of magnetic field. Hayat [13] studied mixed convection MHD boundary layer flow through a porous medium in the presence of thermal radiation. The effects of thermal radiation on boundary layer flow and heat transfer toward a shrinking sheet were examined by Bhattacharyya and Layek [4]. Hayat [14] discussed stretched flow of Jeffrey fluid in the presence of thermal radiation.

Most of the engineering problems and industrial problems, especially some flow and heat transfer equations are nonlinear. Some of them were solved using the analytic methods such as perturbation method and homotopy perturbation

method Nayfeh [24], He [16]. Therefore, many different methods have recently introduced some ways to eliminate the small parameter. One of the semianalytic methods which do not require small parameters is the differential transformation method. Zhou [33] introduced first the concept of DTM and solved linear and nonlinear problems in electrical circuit problems.

In recent years, the DTM has been successfully employed to solve many types of nonlinear problems. Borhanifar and Abazari [5], Abazari and Borhanifar [1] used it for solving of the linear and non-linear problems. The magentic effect of Blasius equation with suction/blowing using differential transform method analyzed by Thiagarajan and Senthilkumar [31]. Khader and Megahed [19] analyzed the flow of a Newtonian fluid over an impermeable stretching sheet embedded in a porous medium with the power law surface velocity and variable thickness in the presence of thermal radiation. This method was successfully applied to various application problems Arikoglu and Ozkol[3], Peker [26], Yaghoobi and Torabi [32], Abazari and Kilicman [2], Odibat [25], Rashidi [28,29], Catal [6]. All of these successful applications verified the validity, effectiveness and flexibility of the DTM.

In this study, DTM is applied to find the approximate solution for the hydromagnetic micropolar flow and heat transfer of viscous incompressible and electrically conducting fluid over a porous channel in the presence of transverse constant magnetic field with radiation. Approximate solutions of the governing system of nonlinear ordinary differential equations were calculated in the form of DTM series with easily computable terms.

2. MATHEMATICAL ANALYSIS

Consider steady incompressible laminar flow of a

hydromagnetic fluid and heat transfer along a two dimensional with porous walls subject to a chemical reaction through which fluid is uniformly injected or removed with speed v_0 . The lower channel wall has a solute temperature T_1 while the upper wall has solute temperature T_2 . Let us consider the channel walls are parallel to the x-axis and located at $y = \pm h$. Based on these assumptions, the governing equations of continuity, momentum and energy are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{(\mu+k)}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} - \frac{\sigma B_0^2 u}{\rho}$$
(2)

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{(\mu+k)}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} - \frac{\sigma B_0^2 v}{\rho}$$
(3)

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = -\frac{\kappa}{\rho j} \left(2N + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) + \left(\frac{\mu_s}{\rho j} \right) \left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right)$$
(4)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_1}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}$$
(5)

where u and v are the velocity components along the x and y axes respectively, $\rho \square$ is the fluid density, μ is the dynamic viscosity, N is the angular or micro rotation velocity, P is the fluid pressure, T and c_p are the fluid temperature and specific heat at constant pressure respectively, k_1 is the thermal conductivity, j is the micro-inertia density, κ is a material parameter and $v_s = \left(\mu + \frac{k}{2}\right)j$ is the micro rotation viscosity.

Using the Rosseland approxiamtion, the radiative heat flux term q_r is given by $q_r = \frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$ (6)

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorbtion coefficient. Assuming that the differences in temperature within the flow are such that T^4 can be expressed as a linear combination of the temperature, we expand T^4 in Taylor's series about T_2 and neglecting higher order terms, we get $T^4 = 4T_2^3T - 3T_2^4$ (7) thus, we have

$$\frac{\partial q_r}{\partial y} = -\frac{-16T_2^3 \sigma^*}{3k^*} \frac{\partial^2 T}{\partial y^2} \tag{8}$$

The appropriate boundary conditions are

$$u = v = 0, \quad N = -s \frac{\partial u}{\partial y} \quad at \quad y = -h$$

$$u = \frac{v_0 x}{h}, \quad v = 0, \quad N = \frac{v_0 x}{h^2} \quad at \quad y = +h$$
(9)

where *s* is a boundary parameter and indicates the degree to which the microelements are free to rotate near the channel walls. The case s = 0 represents concentrated particle flows in which microelements close to the wall are unable to rotate. Other interesting particular cases that have been considered in the literature include s = 0.5 which represents weak concentrations and the vanishing of the antisymmetric part of the stress tensor and s = 1 which represents turbulent flow

Define a stream function as

$$u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x} \tag{10}$$

Introduce the following similarity transformations

$$\eta = \frac{y}{h}, \qquad \psi = -v_0 x f(\eta),$$

$$N = \frac{v_0 x}{h^2} g(\eta), \quad \theta(\eta) = \frac{T - T_2}{T_1 - T_2}$$
(11)

where $T_2 = T_1 - Ax$ with A as constant.

Obviously Eq. (1) is satisfied. Using Eqs. (10) and (11) in Eqs. (2) - (5) with boundary conditions Eq. (9), we obtain $(1+N_1)f^{N} - N_1g'' - \text{Re}(ff''' - f'f'' + Mf'') = 0$ (12)

$$N_{2}g'' + N_{1}(f''-2g) - N_{3}\operatorname{Re}(fg'-f'g) = 0$$
(13)
$$\left(1 + \frac{4}{3}R\right)\theta'' + Pe_{h}(f'\theta - f\theta') = 0$$
(14)

$$f(\eta) = f'(\eta) = g(\eta) = 0, \ \theta(\eta) = 1 \ at \ \eta = -1$$

$$f(\eta) = \theta(\eta) = 0, \ f'(\eta) = -1, \ g(\eta) = 1 \ at \ \eta = 1$$
(15)

The parameters of primary interest are the buoyancy ratio N, the Peclet numbers for the diffusion of heat Pe_h , radiation parameter R, the Reynolds number Re where for suction Re > 0 and for injection Re < 0 and the Grashof number Gr given by

$$N_{1} = \frac{\kappa}{\mu}, \qquad N_{2} = \frac{\nu_{s}}{\mu h^{2}}, \qquad N_{3} = \frac{j}{h^{2}},$$

$$Re = \frac{\nu_{0} h}{\nu}, \qquad Pr = \frac{\nu \rho c_{p}}{k_{1}}, \qquad Gr = \frac{g \beta_{r} A h^{4}}{\nu^{2}},$$

$$Pe_{h} = Pr Re, \qquad M = \frac{\sigma B_{0}^{2} h}{\rho \nu_{0}}, \qquad R = \frac{4 \sigma T_{2}^{3}}{k k_{1}}$$
(16)

where Pr is the Prandtl number, Ec is the Eckert number, Sc is the generalized Schmidt number, N_1 is the coupling parameter and N_2 is the spin gradient viscosity parameter. In technological processes, the parameter of particular interest is the local Nusselt number. It is defined as follows

$$Nu_{x} = \frac{q_{y=-h}x}{(T_{1} - T_{2})k_{1}} = -\theta'(-1)$$
(17)

where q'' is the local heat flux.

3. DIFFERENTIAL TRANSFORM METHOD

Consider a function u(x) which is analytic in a domain T and let $x = x_0$ represent any point in T. The function u(x) is then represented by a power series whose center is located at x_0 . The differential transform of the function u(x) is given by

$$U(k) = \frac{1}{k!} \left[\frac{d^k u(x)}{dx^k} \right]_{x=x_0}$$
(18)

where u(x) is the original function and U(k) is the transformed function. The inverse transformed is defined as follows

$$u(x) = \sum_{k=0}^{\infty} (x - x_0)^k U(k)$$
(19)

Combining Eqs. (18) and (19), we get

$$u(x) = \sum_{k=0}^{\infty} \frac{\left(x - x_0\right)^k}{k!} \left[\frac{d^k u(x)}{dx^k}\right]_{x = x_0}$$
(20)

Inspection of Eq. (20), indicates that the concept of differential transform is derived from Taylor series expansion. However, this method does not evaluate the derivatives symbolically. In actual applications, the function u(x) is expressed by a finite series and

Eq. (19) can be rewritten as follows:

$$u(x) \cong \sum_{k=0}^{m} (x - x_0)^k U(k)$$
(21)

which means that $u(x) \cong \sum_{k=m+1}^{\infty} (x - x_0)^k U(k)$ is negligibly small. Usually, the value of m is decided by convergence

of the series coefficients.

4. ANALYTICAL APPROXIMATIONS BY MEANS OF THE DTM

Taking the differential transform of Eqs.(12)-(14), we obtain

$$(1 + N_{1})(k + 1)(k + 2)(k + 3)(k + 4) F(k + 4) = -N_{1}(k + 1)(k + 2) G(k + 2)$$

$$- \operatorname{Re} \sum_{r=0}^{k} (k - r + 3)(k - r + 2)(k - r + 1) F(r) F(k - r + 3) = +\operatorname{Re} \sum_{r=0}^{k} (k - r + 2)(k - r + 1)(r + 1) F(r + 1) F(k - r + 2) = -\operatorname{Re} M(k + 1)(k + 2) F(k + 2) = 0 = N_{2}(k + 1)(k + 2) G(k + 2) = N_{3}(k + 1)(k + 2) F(k + 2) - 2G(k))$$

$$-N_{3} \operatorname{Re} \sum_{r=0}^{k} (k - r + 1) F(r) G(k - r + 1) = 0 = N_{3} \operatorname{Re} \sum_{r=0}^{k} (k - r + 1) G(r) F(k - r + 1) = 0$$

$$(22)$$

$$\begin{pmatrix} 1+\frac{4}{3}R \end{pmatrix} (k+1)(k+2) \ \Theta(k+2)$$

$$-Pe_{h} \sum_{r=0}^{k} (k-r+1) F(r) \Theta(k-r+1)$$

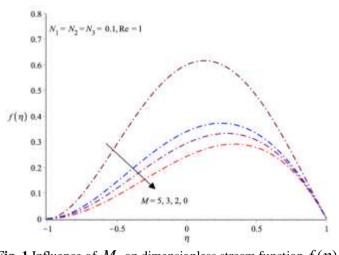
$$+Pe_{h} \sum_{r=0}^{k} (k-r+1) \Theta(r) F(k-r+1) = 0$$

$$(24)$$

where $F(k), G(k), \Theta(k)$ are the differential transform of $f(\eta)$, $g(\eta)$, $\theta(\eta)$ The transform of the boundary conditions are $F(0) = \alpha_0, F(1) = \alpha_1, F(2) = \alpha_2, F(3) = \alpha_3,$ (25) $G(0) = \beta_0, G(1) = \beta_1, \Theta(0) = \gamma_0, \Theta(1) = \gamma_1$

5. RESULTS AND DISCUSSION

In this work the DTM is applied to obtain a semianalytic solution of heat transfer equation of steady laminar viscous incompressible flow of a magnetohydrodynamic micropolar fluid along a two dimensional channel with porous walls with radiation. The effects of significant parameters such as Reynolds number, magnetic parameter, micro rotation/ angular velocity, radiation parameter, Peclet number, coupling parameter, spin gradient viscous parameter on the flow and heat characteristics are investigated.



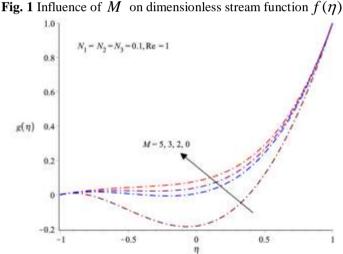
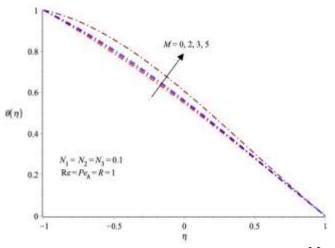


Fig. 2 Dimensionless micro rotation profiles for different M

Figs. 1-3 depicts the influence of magnetic field on the flow velocities, the micro rotation, and the temperature, respectively. From Figs. 1 and 3, it is clear that the effect of magnetic field is to increasing the velocity and temperature respectively. Illustrating the fact that the effect of magnetic field is to accelerate the velocity and temperature field. Fig. 2 demonstrates the plot of micro rotation profile $g(\eta)$ for different values of magnetic parameter M. It is observed that the magnetic parameter M increases, micro rotation $g(\eta)$ decreases. Fig. 3 portrays the plot of temperature profile) for different values of magnetic parameter M increases, temperature $g(\eta)$ decreases.

Fig. 4 demonstrates the plot of velocity field for N_1 coupling parameter. It is observed that the coupling parameter N_1 increases, velocity $f(\eta)$ increases. It is seen that the effect of N_1 is to accelerate the velocity.





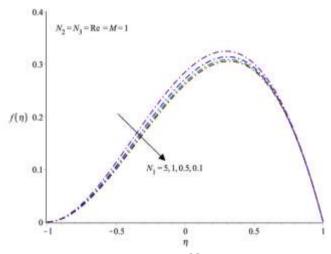


Fig. 4 Influence of coupling parameter N_1 on stream function profiles

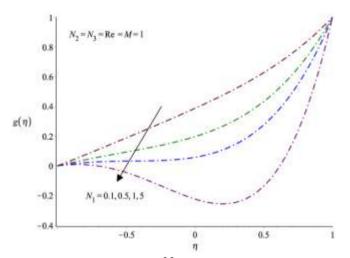
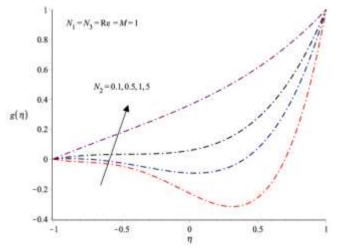
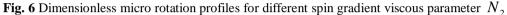


Fig. 5 Influence of coupling parameter N_1 on dimensionless micro rotation profiles

Figs. 5 and 6 displays the effect of N_1 and N_2 on the micro rotation profile $g(\eta)$ decrease with increase of N_1 but it increases when N_2 increases.





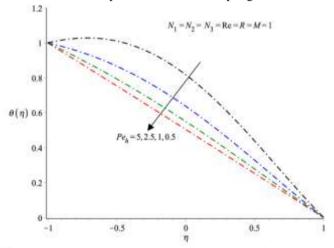
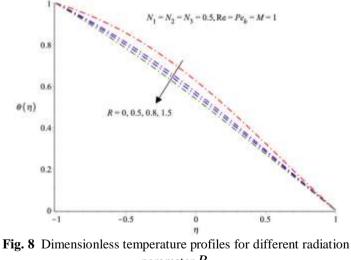


Fig. 7 Dimensionless temperature profiles for different Peclet number Pe_h



parameter R

Temperature profile for various values of R Peclet number Pe_h and radiation parameter R are presented graphically through Figs. 7 and 8.

It is noticed that the effect of Peclet number and the effect of radiation parameter have the similar effect over temperature profile so as to increase it. Fig. 9 displays the effect of radiation parameter and Reynolds number on Nusselt number. Increase in radiation parameter and Reynolds number leads to increase in Nusselt number

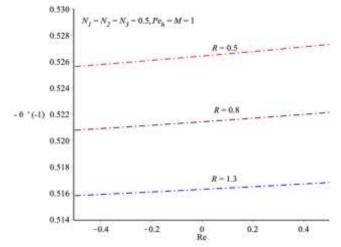


Fig. 9 Effects of Reynolds number and Peclet number on Nusselt number

6. CONCLUSION

The DTM was employed successfully to find the semianalytical solution of the MHD micropolar fluid and heat transfer in a permeable channel with viscous dissipation. Based on this analysis we made the following conclusion:

- Differential transformation method does not require small parameters in the mathematical formulation, so one limitation of the traditional perturbation method can be eliminated.
- Effects of radiation parameter with magnetic field is to decelerate the temperature.
- The velocity increases by increasing in N_1 , but it decreases with increasing N_2 .
- In the presence of magnetic field, dimensional temperature increases with increasing of Reynolds number and Peclet number.

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