

NORMAL CONTACT FORCE ON TWO IDENTICAL SPHERE BY PARTICLE – PARTICLE CONTACT USING DISCRETE ELEMENT METHOD

OLEENA S H

University of Kerala

ABSTRACT: *The objective of this contribution is to exist a numerical simulation technique to model the collision of particles in a plane using object-oriented procedures. This approach is based on the second law of Newton for the translational motion of each particle in the granular material. This comprises a possession track of all forces acting on each particle at every time-step. The back-ground version of DEM and time integration algorithm are established and executed into C++ code. A simple test is regarding the application of time- integration algorithm by particle-particle interaction for which analytical expression exist. In this paper elastic force due to particle – particle contact and particle – wall contact on identical spherical particles are investigated.*

Keywords: *DEM simulation, Granular materials, Elastic effect, damping effect;*

1. Introduction

A granular material is a conglomeration of many discrete solid particles which is characterized by a loss of energy due to dissipative collisions whenever the particles interact. They can be considered as the fourth state of matter which is very different from solids, liquids or gas. Granular materials are very simple. If the particles are non-cohesive, then the forces between them are basically only repulsive so that the shape of the material is determined by external boundaries and gravity. Practically many solid particles which we make use of in the kitchen are granular particles like sugar, rice, coffee, cereals etc. Walking outside we step on the soil which is again a particulate and hence falls under the category of granular matter. The unusual and unique character displayed by granular material systems have led to a resurgence of interest within several scientific and engineering disciplines ranging from physics, soil mechanics and chemical engineering (Jaeger and Nagel [1992]; Behringer [1995]; Bideau and Hansen [1993]; Jaeger et al. [1994, 1996a]). Much of the engineering literature has been dedicated to understanding how to deal with these materials. Prominent contributions in the literature include Coulomb [1773], who proposed the ideas of static friction; Faraday [1831], who discovered the convective instability in a vibrated container filled with powder, and Reynolds [1885], who introduced the motion of dilatancy, which implies that a compacted granular material must expand in order for it to undergo shear. Processes involving particulate or granular flows are prevalent throughout the pharmaceutical, chemical, energy, food handling, mineral processing, powder metallurgy, and mining industries. In addition, numerous phenomena found in nature involve such material flows.

The discrete element method (DEM), originally developed by Cundall and Strack [1971, 1979], has been used successfully to simulate chute flow (Dippel et.al 1996), heap formation (Luding, 1997), hopper discharge (Thompson and Grest, 1991; Ristow and Herrmann, 1994), blender segregation (Wightman et al., 1998; Shinbrot et al., 1999; Moakher et al., 2000) and flows in rotating drums (Ristow, 1996; Wightman et al., 1998). The DEM allows for the simulation of particle motion and interaction between the particles. DEM considering not only the obvious geometric and material effects such as particle shape, material non-linearity, viscosity, friction, etc., but also the effect of various physical fields of surrounding media, level of chemical reactions (Kantor et al. 2000). One of the most auspicious area of future applications of discrete element method seems to be geotechnical engineering. The discrete approach assumes the soil is an assembly of granular or discrete particle.

2. Discrete State formulation.

The dynamic behavior of granular media is considered as the dynamics of each particles. The overall response of media is foretold by the behaviour of individual particles and the dynamics of particles is evaluated by applying the second Newton's law. Detection of interaction force between contacting particles is considered by discrete approach is one of the most important issues. The interaction forces of each contacting particle are locally resolved based on actual geometry of kinematic contact between two spherical particles, inter-particle contact forces and boundary conditions.

Granular material is considered as a collection of frictional elastic spherical particles and it is termed as discrete elements. The particles are assumed to be composed of spherical particles with same radii R_i . The granular particles are assumed to be deformable bodies, deforming each other by normal and shear force. The composition of media is time-

dependent because distinct particle changes their position by free rigid body motion or by contacting with neighbor particles or walls. Each particle may be in contact with other particles. The boundary conditions of media are determined by planes and treated as particles with an infinite radius and mass

3. Geometry of Kinematic contact of spherical particles.

Consider two particles i and j be in contact with position vectors x_i and x_j with center of gravity lying at O_i and O_j having linear velocities v_i and v_j .

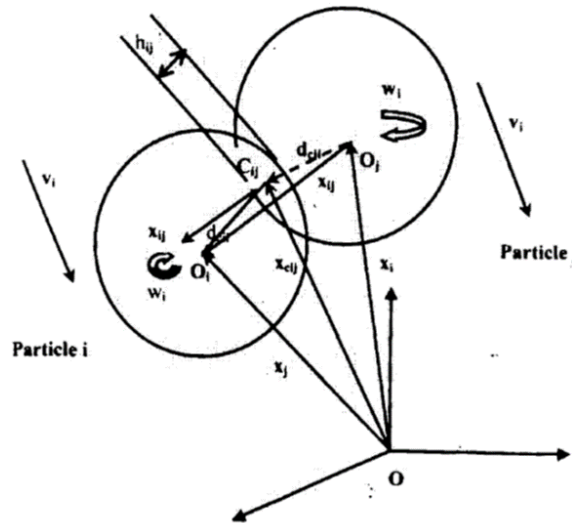


Figure 1. Inter particle Contact between two identical spheres i and j

The contact point C_{ij} is defined to be at the center of the overlap area position vector x_{cij} . The vector x_{ij} of the relative position point from the center to gravity of particle i to that of particle j is defined as $x_{ij} = x_i - x_j$. The depth of overlap is h_{ij} . Unit vector in the normal direction of the contact surface through the center of the overlap area is denoted by n_{ij} . It extends from the contact point to the inside of the particle i as $n_{ij} = -n_{ji}$

Since the particle shape is assumed to be spherical, for sphere of any dimension the contact parameters can be written as follows:

$$h_{ij} = \begin{cases} R_i + R_j - |x_{ij}|, & |x_{ij}| < R_i + R_j \\ 0, & |x_{ij}| \geq R_i + R_j \end{cases}$$

Where R_i is the radius of the particle. The relative velocity of the contact point is defined as

$$v_{ij} = v_{cij} - v_{cji}$$

4. Inter particle contact force.

The contact force between particles can be expressed as the sum of normal and tangential components;

$$F_{ij} = F_{n,ij} + F_{t,ij}$$

The contact forces between particles depend on the overlap geometry, the properties of the material and the relative velocity between the particles in the contact area and it is modeled as spring, dash-pots and a friction slider. If the particles are in perfect contact model, it is required to describe the effects of elasticity, energy loss through internal friction and attraction on the contact surface for the contact force calculations. The normal component of contact force between particles can be expressed as the sum of elastic force and viscous force.

$$F_{n,ij} = F_{n,ij,elastic} + F_{n,ij,viscous}$$

Normal elastic force is based on the linear Hooke's law of a spring with a spring stiffness constant $k_{n,ij}$ and is given by the expression,

$$F_{n,ij,elastic} = K_{n,ij} h_{ij} n_{ij}$$

Normal viscous force is dissipated during real collisions between particles. The linear dependency of force on the relative velocity of the particles at the contact point with a constant normal dissipation coefficient γ_n and is expressed as

$$F_{n,ij,viscous} = -\gamma_n m_{ij} v_{n,ij}$$

Equation for the motion of granular material in a plane,

$$\begin{aligned} m_i \frac{d^2 x_i}{dt^2} &= m_i a_i \\ &= F_i \\ v_i &= \frac{dx_i}{dt} \end{aligned}$$

Force acting on i^{th} particle F_i is, $F_i = m_i g + \sum_{\substack{j=i \\ j \neq i}}^N F_{n,ij} + \sum_{\substack{j=i \\ j \neq i}}^N F_{t,ij}$ i.e., sum of gravitational force and contact force

5. Computer Implementation

The key computational tasks of DEM at each time step of contact particle can be summarized as follows:

- Finding of contacts between a particle i and j .
- Calculation of contact forces from relative displacement between particles
- Summary of contact forces to determine the total unbalanced force
- Computation of acceleration from force
- Velocity and displacement by integrating the acceleration
- Updating the position of particles

6. Result and Discussion

6.1. Elastic impact of two identical particles by Particle – Particle collision using Discrete Element Method

This test simulates first particle moving and collide with the other particle, and thus particle-particle contact force are formed. This test also confirms the particle-particle interaction. The diameter of the particle is 0.005 m and the value of the normal stiffness parameter is 3×10^5 units. The particle is moving and collide the other particle and it result in change in velocity and position of the particle.

6.1.1. Elastic Effect of Force with respect to displacement by particle – particle contact

Considers the simplest case of an elastic normal impact of two identical spheres with the incoming velocity magnitude of first particle was set at 1 m/s and the initial position is 5m.

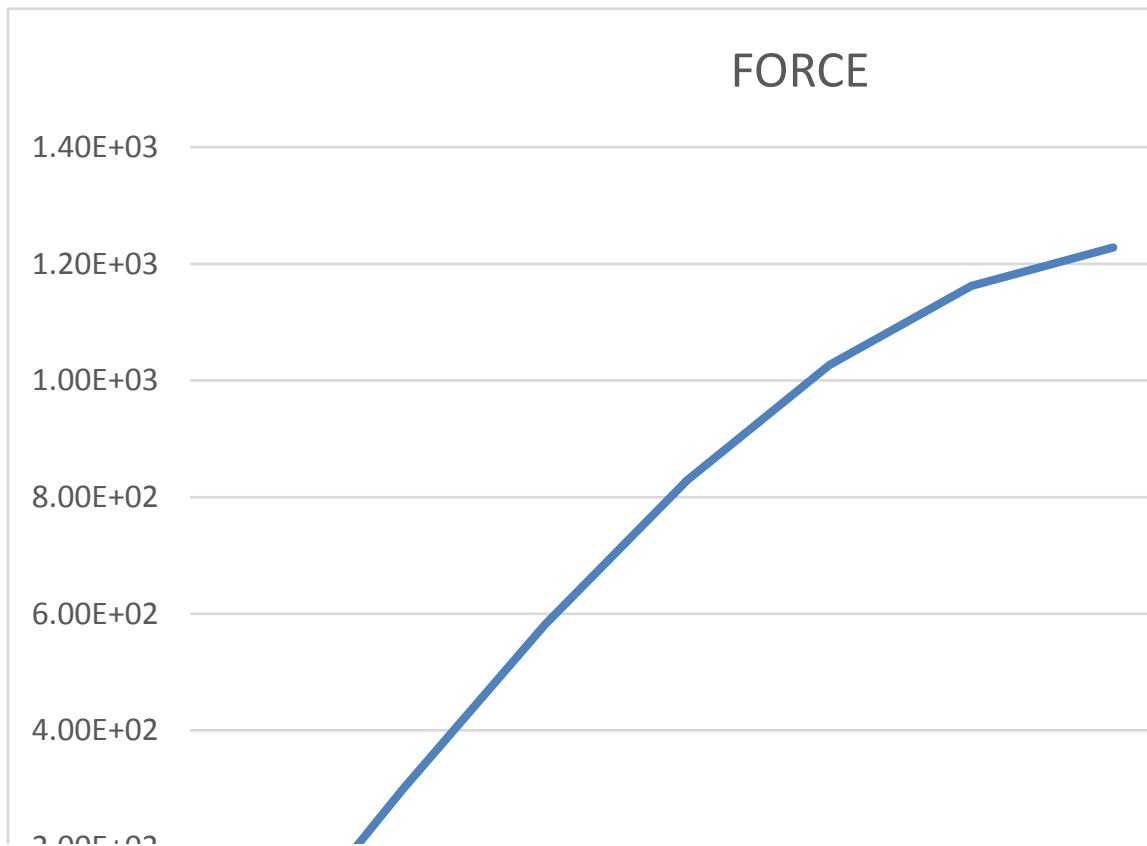


Figure 2. Elastic normal impact of first particle with change in Displacement by particle – particle collision

The first particle moving with the same velocity and collide the second particle, then particle moves in the direction of the second particle and the normal elastic force reaches a peak during contact as shown in Figure 2. Figure shows the results of a particle with displacement in the x-direction and elastic force in the y-directions.

From the result we can conclude that the two particle attain same elastic force with respect to displacement. When two particles are collide then two different displacement are obtained but the elastic forces are same.

6.1.2. Elastic Effect of Force with respect to Time by particle – particle contact

Considers two identical spherical particles collide with an initial velocity 1 m/s and the initial position is 5 m. The first particle moves and collides the second particle. When collision increases, then elastic force reaches a peak if collision decreases then elastic force decreases as shown in Figures 3. Figure shows the results for a particle with change in time in the x-direction and elastic force in the y-directions.

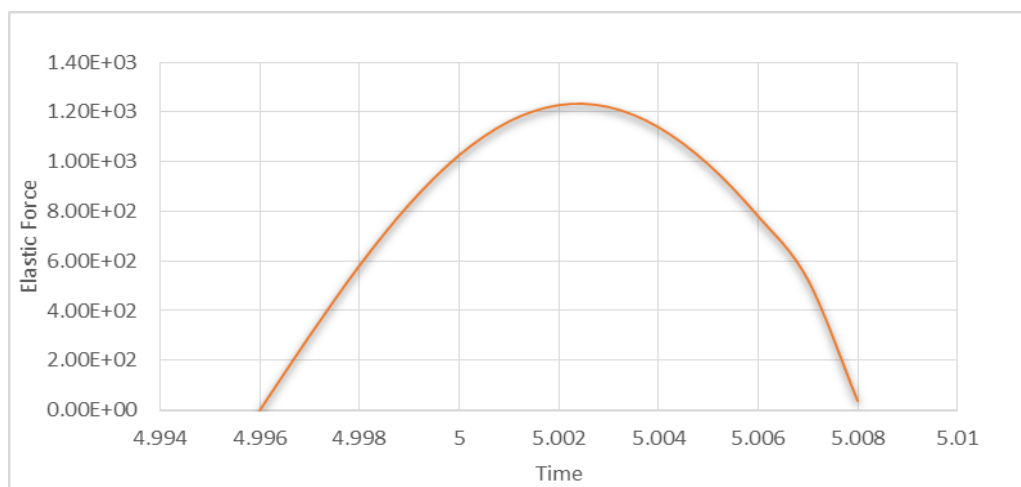


Figure 3. Elastic normal impact of particle with change in time by particle – particle collision

Collision is depends on time step if time step increases then collision time also increases, if collision time increases then elastic force also increases. Similarly there is a less collision time then there is a less collision force is obtained. From the result we can say that two particle attain a same elastic force with respect to time. When time step increases collision also increases then elastic force reaches a peak when collision decreases the elastic force decreases and reaches a constant.

6.1.3. Elastic Effect of Kinetic energy with respect to Time by particle – particle contact

Considers two identical spheres with initial velocity 1 m/s and the initial position is 5m. And the second particle with constant velocity and position 10m. The first particle is moving with the same velocity and collide with the second particle, then the velocity of two particles changes with respect to time.

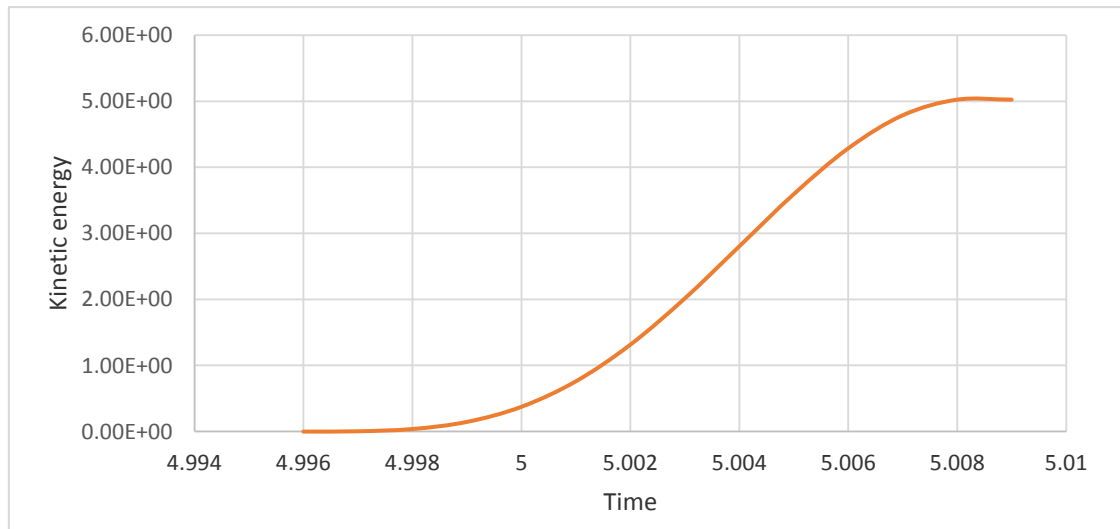


Figure 4. Elastic normal effect of particle with change in kinetic energy with respect to time by particle – particle collision

The first particle moving with the same velocity and collide the second particle, the kinetic energy varies and reduces to a constant level. Kinetic energy of a particle is proportional to its velocity. If the velocity is constant then Kinetic energy will also be constant. If first particle continuously increases in velocity, the Kinetic energy too continuously varies. The amount of kinetic energy of a particle depends on two things they are mass of the object and the speed at which it is moving. Faster-moving particles have more kinetic energy. Faster-moving particles hit other particle harder, which pushes them further apart. When there is no collision the kinetic energy remains constant. Figure 4 shows the results for a particle with time in the x-direction and kinetic energy in the y-directions. If there is a continuous increase in velocity, then Kinetic energy of the particle also varies.

If the velocity of the particle increases then kinetic energy also increases. If the velocity reaches a constant then kinetic energy will also be constant. By comparing the two particles the second particle acquire more kinetic energy.

6.1.4. Elastic Effect of change in displacement with respect to time by particle – particle contact

The two identical spheres with initial velocity 1 m/s and the initial position is 5m and the second particle with constant velocity and position. The first particle is moving with an initial position and collide the second particle, then the position of two particles changes with respect to time. The damping and gravitational effect are set to be zero.

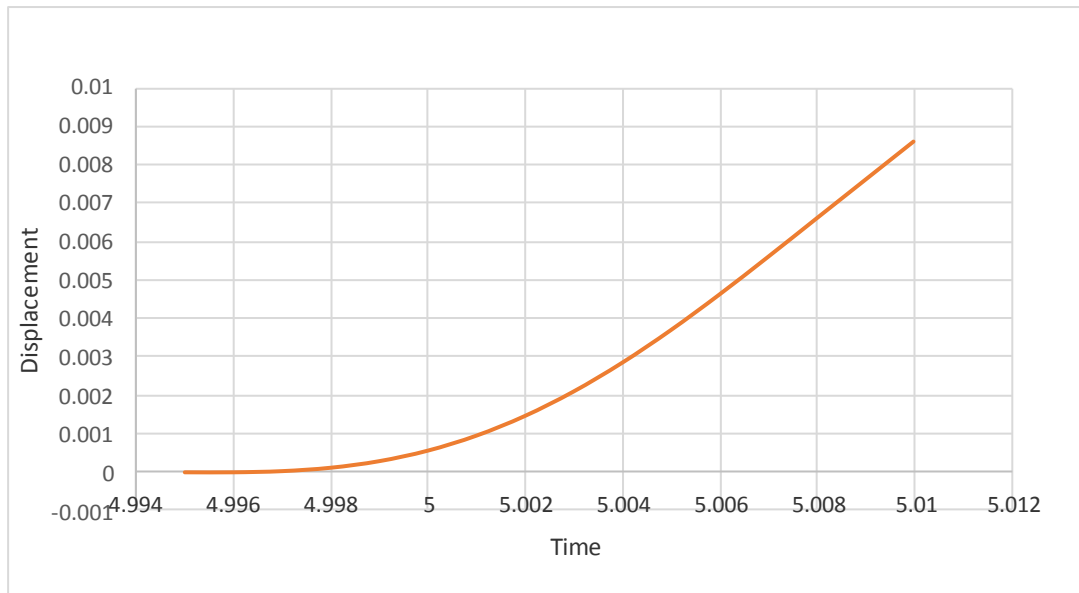


Figure 5. Elastic normal effect of particle with change in displacement with respect to time by particle – particle collision

The particle is moving with the same velocity and collide with the second particle. When first particle collide with the other, then the position of the particle change with change in time. If time step increases then collision increases, and the displacement reaches a peak as shown in Figures 5. Figure shows the results for a first particle with change in time in the x-direction and displacement in the y-directions.

When two spherical particles are collide then first particle attain more displacement compared to the second particle. Particles with same time step but they acquire different displacement.

6.2. Elastic impact of Particle – Wall collision using Discrete Element Method

This test simulates particle moving and collide with the wall, and their particle-wall contact force are formed. Here the wall is considered to be a particle with infinite mass. This test also confirms the particle-wall collision, the particle moving at the initial velocity and hitting the wall base were simulated. The diameter of the particle and the wall is 0.005 m and the value of the normal stiffness parameter is 3×10^5 units. This simulates the collision of particles with initial velocity and tests the particle-wall contact force for the elastic impact of sphere and the wall. For this test, parameters corresponding to the tangential and damping forces are set zero and the effect of normal elastic force with respect to particle – wall collision is studied.

6.2.1. Elastic impact of Force with respect to Displacement by particle – wall contact

Considers the elastic normal impact of a particle with particle – wall contact. Wall is considered as a particle with diameter 0.005m and has an infinite mass. The incoming velocity magnitude of first particle was set at 1 m/s and the initial position is 5m.

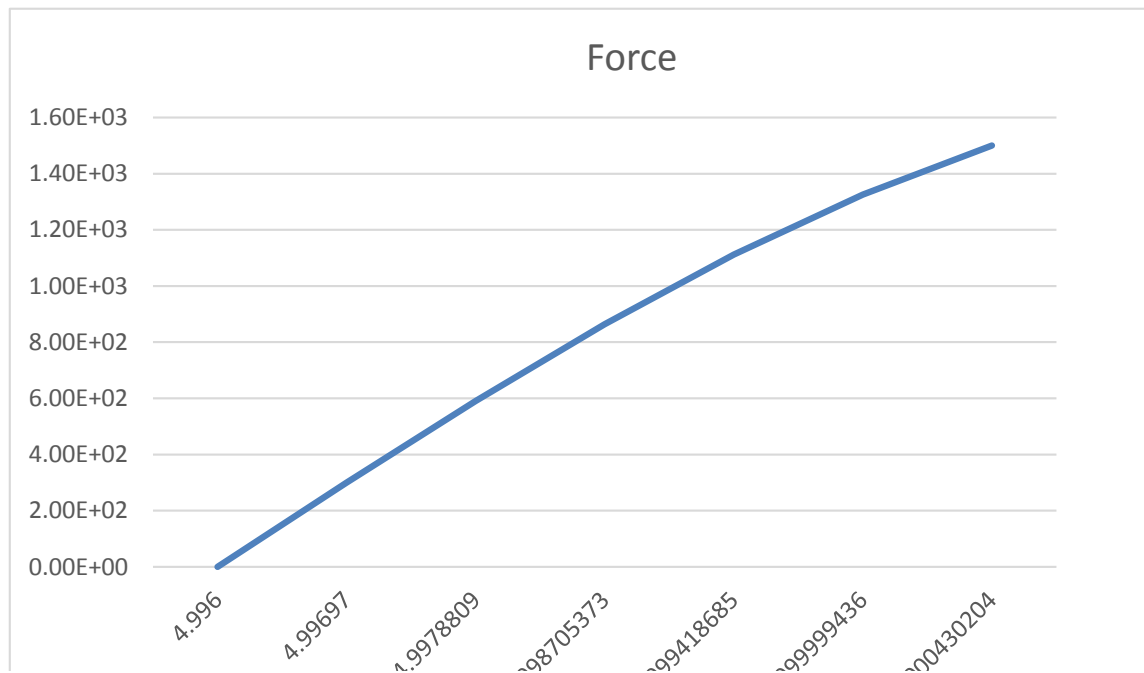


Figure 6. Elastic normal impact of force with change in Displacement by particle – wall collision

The particle moving with the same velocity and collide with the wall, the particle rebounds to the original position and the normal elastic force reaches a peak during contact as shown in Figures 6. Figure shows the results for a particle with displacement in the x-direction and elastic force in the y-directions. The first particle is moving with the same velocity and collide the wall, it attain more force compared with particle – particle collision.

When particle collide with the wall the particle change their position with respect to time. Thus displacement changes, by change in displacement the elastic force reaches a maximum.

6.2.2. Elastic impact of Force with respect to Time by particle – wall contact

Considers the simplest case of an elastic normal impact of a spherical particle with incoming velocity was set at 1 m/s and the initial position is 5 m. The particle is moving with the same velocity and collide with the wall. When time step increases then the collision also increases, and the elastic force reaches a peak then collision decreases then elastic force decreases as shown in Figures 7. Figure shows the results for a particle with change in time in the x-direction and elastic force in the y-directions.

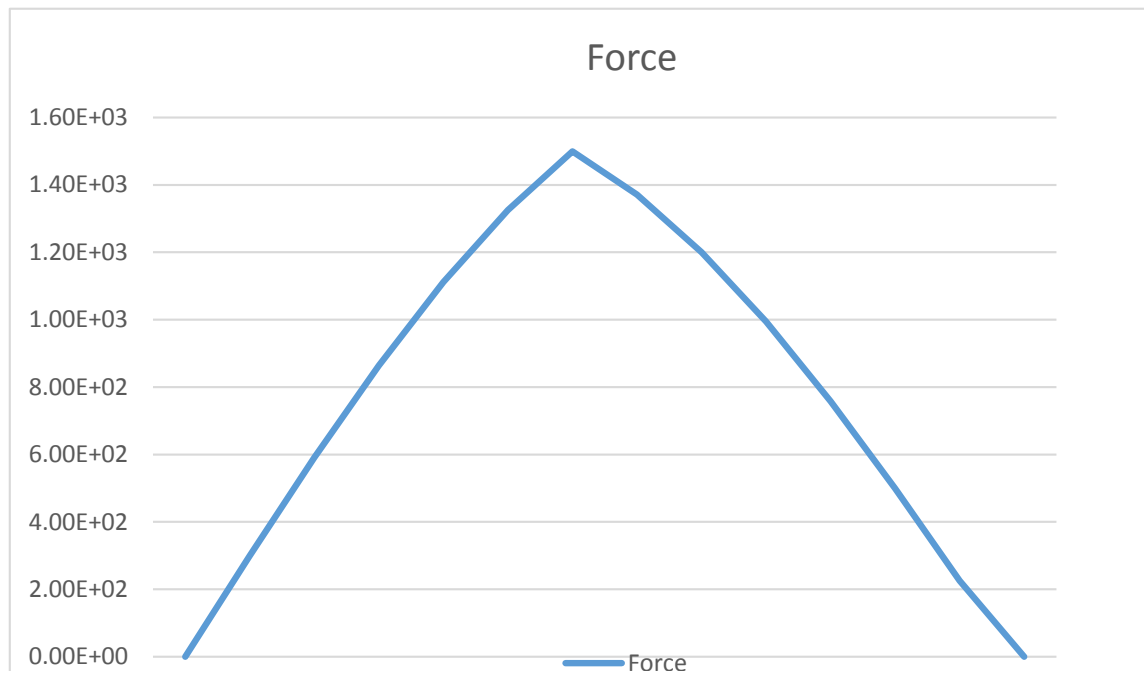


Figure 7. Elastic normal impact of the particle with change in time

When particle collide with the wall then particle – wall attain more elastic force compared with particle – particle collision. In particle – wall collision particle attain maximum elastic force and then slight reduction in elastic force with respect to time and then elastic force reduces to zero.

6.2.3. Elastic impact of Kinetic energy with respect to Time by particle – wall contact

Elastic normal impact of spheres with incoming velocity magnitude of the particle was set at 1 m/s and the initial position is 5m. The first particle is moving with the same velocity and collide the second particle, then there is a change in kinetic energy with respect to time.

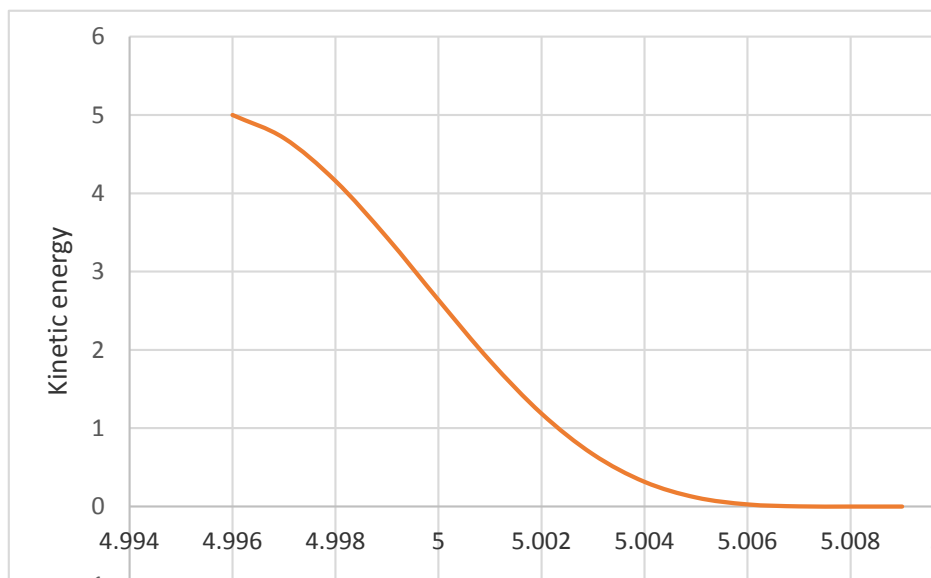


Figure 8. Elastic normal effect of particle with change in kinetic energy with respect to time

The particle moving with the same velocity and collide the wall, the kinetic energy varies and reduces to a constant. Kinetic energy depends on mass of the object and the speed at which it is moving. Faster-moving particles have more kinetic energy. When there is no collision the kinetic energy remains constant. Figure 8 shows the results for a particle with displacement in the x-direction and kinetic energy in the y-directions.

When particle collide with the wall then velocity decreases and then reaches a constant. Therefore kinetic energy of a particle also decreases with respect to time and then reaches a constant if there is no collision.

6.2.4. Elastic impact of Displacement with respect to Time by particle – wall contact

Elastic effect of particle with the initial velocity 1 m/s and the initial position is 5 m. in this case damping and gravitational effect are set to be zero. The particle is moving with the same velocity and collide the wall. When particle collide with wall then displacement increase. , time step increases then the collision also increases, and the displacement reaches a peak as shown in Figures 9. Figure shows the results for a particle with change in time in the x-direction and displacement in the y-directions.

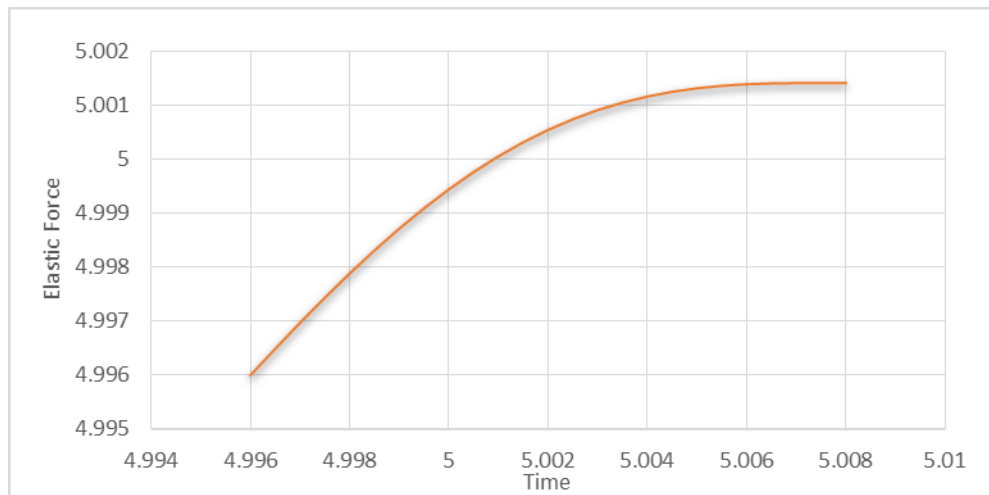


Figure 9. Elastic normal effect of particle with change in displacement with respect to time

When particles collide with the wall then there is an increase in displacement with respect to time. If time changes then displacement also changes. The time step for particle – particle and particle – wall collision are same but the displacement in two cases are different. Displacement occurred by particle – particle collision is more than particle wall collision.

7. Conclusion

The result obtained in the present investigation may be generally described as follows:

- The discrete element method calm into the visco – elastic spherical particles is executed into the advanced C++ code.
- The elastic impact of two identical spheres have been examined and derived using DEM.
- In particle-particle and particle wall collision the particle change their position with respect to time and the elastic normal force reaches a peak during contact.
- Elastic force on identical spherical particles by particle – particle contact and particle – wall contact with respect to time and displacement are analyzed.
- Elastic effect on kinetic energy and displacement on identical spherical particles by particle – particle contact and particle – wall contact with different time step are analyzed.
- Elastic effect of displacement on identical spherical particles by particle – particle contact and particle – wall contact with different time step are analyzed.

Reference

1. Cundall P. A. and O. D. L. Strack, (1979) "A discrete numerical model for granular assemblies", Geotechnique 29, 47.
2. Cundall, P.A., (1971). A computer model for simulating progressive large-scale movements in blocky rock systems. Proceedings of Symposium International Society of Rock Mechanics 2, 129.
3. Dippel, S., Batrouni, G.G., Wolf, D.E., (1996). Collision-induced friction in the motion of a single particle on a bumpy inclined line. Physical Review E 54, 6845.
4. Kantor, A. L.; Long, L. N.; Micci, M. M. (2000) Molecular dynamics simulation of dissociation kinetics. In: AIAA Aersospace Science Meeting, AIAA Paper 2000-0213.

5. Luding, S., (1997). Stress distribution in static two-dimensional granular model media in the absence of friction. *Physical Review E* 55, 4720.
6. Moakher, M., Shinbrot, T., Muzzio, F.J., (2000). Experimentally validated computations of flow, mixing and segregation of non-cohesive grains in 3D tumbling blenders. *Powder Technology* 109, 58.
7. Ristow, G.H., (1996). Dynamics of granular material in a rotating drum. *Euro physics Letters* 34, 263.
8. Ristow, G.H., Herrmann, H.J., (1994). Density patterns in two-dimensional hoppers. *Physical Review E* 50, R5.
9. Shinbrot, T., Alexander, A., Moakher, M., Muzzio, F.J., 1999. Chaotic granular mixing. *Chaos* 9, 611.
10. Thompson, P.S., Grest, G.S., (1991). Granular flow: friction and the dilatancy transition. *Physical Review Letters* 67, 1751.
11. Wightman, C., Moakher, M., Muzzio, F.J., Walton, O.R., (1998). Simulation of flow and mixing of particles in a rotating and rocking cylinder. *A.I.Ch.E. Journal* 44, 1226.
12. Jaeger H. M. and S. R. Nagel, (1995) "The physics of the granular state", *Science* 255, 1523.
13. Behringer R. P., (1995) "Mixed Predictions", *Nature* 374, 15.
14. Behringer R. P., (1993) "The Dynamics of flowing sand", *Nonlinear Science Today*.
15. Bideau D. and A. Hansen, Eds., (1993) "Disorder and Granular Media", *Random Materials and Processes Series*, Elsevier Science Publishers B. V., Amsterdam, North-Holland.
16. Jaeger H. M., S. R. Nagel and R. P. Behringer., (1996b). "Granular solids, liquids and glasses", *Rev. Mod. Phys.* 68, 1259.
17. Jaeger H. M., S. R. Nagel, and R. P. Behringer, (1996a) "The Physics of Granular Materials", *Physics Today* 4, 32.
18. Jaeger, H. M., J. B. Knight, C. H. Liu, and S. R. Nagel, (1994) "What is shaking in the sand box", *Mater. Res. Bull.* 19, 25.
19. Mehta, A., Ed., "Granular Matter: An Interdisciplinary Approach", Springer, New York, 1994.
20. Hayakawa H., H. Nishimori, S. Sasa, and Y. H. Taguchi, (1995) "Dynamics of granular matter", *Jpn. J. Appl. Phys.* 34, 397.
21. Coulomb C. (1773), in *Memoir de Mathematique et de Phy-sique* 7, Academie des.
22. Faraday M. (1831), "On a peculiar class of acoustical figures; and on certain forms assumed by groups of particles upon vibrating elastic surfaces", *Phil. Trans. R. Soc. London* 52, 299
23. Reynolds (1885)., on the dilatancy of media composed of rigid particles in contact with-experimental illustrations", *Philos. Mag.* 20, 469
24. Dippel, S., Batrouni, G.G., Wolf, D.E., (1996). Collision-induced friction in the motion of a single particle on a bumpy inclined line. *Physical Review E* 54, 6845.