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Experimental Validation of the Proposed SHM Technique form Limited Extracted Dynamic Responses of a Steel Beam Adopting Inverse Dynamic Approach

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I. Abstract

Abstract— Condition monitoring of structures is important from safety consideration. Damage detection techniques, using inverse dynamic approaches, are important tools to improve the mathematical models for structural health monitoring. However, uncertainties in the measured data might lead to unreliable identification of damage in structural system. The measurement of dynamic responses at all degrees of freedom of a structure is also not feasible in practice. An analysis methodology is proposed for condition monitoring of beam structures in the framework of finite element model with limited randomly measured dynamic responses. The structural properties viz. axial rigidity and bending rigidity are identified at the element level in the updated models of the system. Damage detection technique is employed considering the uncertainties associated with the measured dynamic responses data. The proposed model is suitable for practical problem, as it is able to identify the structural parameters with limited modal data of first few modes, measured at selected degrees of freedom. Different numerical examples with various damage scenarios are explored to demonstrate the applicability of the proposed model. The model is able to identify the structural damage with greater accuracy from the noisy dynamic responses even if the extent of damage is small. Experimental studies, on simple cantilever beams, establish the potential of the proposed methods for its practical implementation.

Keywords: Condition Monitoring, Damage, Inverse Dynamic Approach, Modal data,

II. Introduction

Condition monitoring and damage detection in civil, mechanical and aerospace engineering communities has become one of the most important keys in maintaining the integrity and safety of a structure. The objective of structural condition monitoring is to provide a continuous diagnosis of the state of the structure during its life span. More and more researchers have made efforts in developing efficient, reliable and low-cost damage diagnosis approaches using static and vibration parameters measured from a condition monitoring system in aerospace, mechanical, and structural engineering disciplines. A detailed review on damage detection is provided by *Doebling et al* (1998) that includes various approaches in system identification algorithm covering static as well as dynamic data. Maia et al. (2002) expressed a series of numerical simulation of a simple beam to compare several damage detection techniques based on mode shape changes. Mode shapes reflect the information of geometry, materials, mass, and stiffness. It is, therefore, another popular parameter incorporated in forward damage detection methods (*Li et al., 2005; Ndambi et al.,* 2002). *West* (1984) and *Wolff & Richardson* (1989) presented early work making use of mode shapes in damage detection. The authors proposed modal assurance criterion (MAC) to detect the existence and the location of structural damage *R.B. Randall, G. Zurita and T.Wardrop (2004)* compares a number of common curve fitting methods for the detection of damping and natural frequencies were obtained by using the Ibrahim time domain method, with the rational fraction polynomial method very similar. *Brincker et al (2000)* showed that by considering the Singular Value Decomposition (SVD) of the spectral matrix, the spectral matrix is decomposed into a set of auto spectral density functions, each corresponding to a single degree of freedom (SDOF) system.

The objective of the present study is to develop a of the present investigation is to condition monitoring of beam structures using limited randomly measured dynamic responses adopting frequency domain technique. For fulfilling the objective numerical model of a cantilever beam is prepared in the finite element framework and subsequent damage identification is done based on the inverse dynamic approach using measured modal data like mode shapes at few selected degree of freedom and also natural frequencies. Experimental validation is also done for the same cantilever beam.

III. THEORITICAL FORMULATION

The dynamic equilibrium equation of motion of finite element model can be expresses as,

$$[m]\{\ddot{u}\}+[k]\{u\}=\{f\} \tag{1}$$

Where [k] and [m] are the stiffness and consistent mass matrices of the element respectively. {u} and {f} are the nodal displacements and force vectors. Assembling the element equations and applying boundary conditions, one can obtain the following equilibrium equation for damped forced vibration

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=\{F(t)\} \tag{2}$$

The same equation for the undamped forced vibration shall be

$$[M]\{\ddot{U}\}+[K]\{U\}=\{F(t)\} \tag{3}$$

Where [M], [K], [C], {U} and {F(t)} indicates the global mass matrix, stiffness matrix, Global damping Matrix, nodal displacement and force vector respectively. For free vibration, substitution of F(t) = 0 and $\{U\}=\{\varphi \exp(i\omega t)\}$ in equation (3) gives,

$$([K]-\omega 2\,[M])\{\varphi\}=\{0\} \text{ i.e. } [K]\{\varphi\}=\lambda[M]\{\varphi\}, \text{ where } \lambda=\omega 2\,. \tag{4}$$

Equation (4) is a generalized eigenvalue problem that can be solved for λ and $\{\varphi\}$. The eigenvector $\{\varphi\}$ is the mode shape and eigenvalue λ is the square of the associated natural frequency in radians per seconds. The identification of the structural parameters can be formulated as minimization of a nonnegative error function ε defined as,

$$\text{Minimize, } \varepsilon=\sum_{k=1}^{p}\left([K]\{\varphi_k\}-\lambda_k[M]\{\varphi_k\}\right)^2 \tag{5}$$

Where, λk and $\{\varphi k\}$ are the k-th measured modal data; p is the total number of measured modes. The Characteristics equation for the motion can be partitioned in terms of mode shapes at measured and unmeasured DOF for each measured frequency as shown below.

$$\left[\frac{K_{aa}}{K_{ba}} \Big| \frac{K_{ab}}{K_{bb}}\right]\left\{\frac{\phi_a}{\phi_b}\right\}_i=\lambda_i\left[\frac{M_{aa}}{M_{ba}} \Big| \frac{M_{ab}}{M_{bb}}\right]\left\{\frac{\phi_a}{\phi_b}\right\}_i \tag{6}$$

By condensing out the unmeasured mode shapes the following equation can be obtained.

$$\begin{aligned}&[K_{aa}]\{\phi_a\}_i-[K_{ab}]\left([K_{bb}]-\lambda_i[M_{bb}]\right)^{-1}\left([K_{ba}]-\lambda_i[M_{ba}]\right)\{\phi_a\}_i\\&=\lambda_i[M_{aa}]\{\phi_a\}_i-\lambda_i[M_{ab}]\left([K_{bb}]-\lambda_i[M_{bb}]\right)^{-1}\left([K_{ba}]-\lambda_i[M_{ba}]\right)\{\phi_a\}_i\end{aligned} \tag{7}$$

[=-p $\bar{S}(p_j)$ associated with the "modal stiffness –based error function" are derivative of equation No (7) for each unknown parameter p_j.

$$\left[\bar{S}(p_j)\right]=\begin{pmatrix} \left(\dfrac{\partial[K_{aa}]}{\partial p_j}-\lambda\dfrac{\partial[M_{aa}]}{\partial p_j}\right)-\left(\dfrac{\partial[K_{ab}]}{\partial p_j}-\lambda\dfrac{\partial[M_{ab}]}{\partial p_j}\right) \\ *\left([K_{bb}]-\lambda[M_{bb}]\right)^{-1}\left([K_{ba}]-\lambda[M_{ba}]\right) \\ +\left([K_{ab}]-\lambda[M_{ab}]\right)\left([K_{bb}]-\lambda[M_{bb}]\right)^{-1}*\left(\dfrac{\partial[K_{bb}]}{\partial p_j}-\lambda\dfrac{\partial[M_{bb}]}{\partial p_j}\right).\left([K_{bb}]-\lambda[M_{bb}]\right)^{-1}.\left([K_{ba}]-\lambda[M_{ba}]\right) \\ -\left([K_{bb}]-\lambda[M_{bb}]\right)^{-1}.\left([K_{ab}]-\lambda[M_{ab}]\right)\left(\dfrac{\partial[K_{ba}]}{\partial p_j}-\lambda\dfrac{\partial[M_{ba}]}{\partial p_j}\right) \end{pmatrix}\{\phi_a\}$$

A Gauss -Newton method is used to find the change the parameters $\{\Delta p\}$ for each iteration k. $\{p_{k+1}\}=\{p_k\}+\{\Delta p\}$ Iteration continues till estimation of the parameters are found. The convergence is specified by the desired tolerance level with the relative change in $\{\Delta p\}$ on the order of $10^{-3}, 10^{-6}$ etc.

IV. EXPERIMENTAL STUDY AND RESULTS

Experimental studies are essential and helpful to understand the effect of various uncertainties, inherently present in real situations. It is also important for validation of the proposed numerical model. In the present study experimentation on simple structure has been performed. Most importantly, the feasibility of the developed numerical model for the structural health monitoring is studied with the use of these experimentally obtained data. For this experiment, a cantilever beam of steel of dimension 500mm × 50mm × 15mm is selected as show in Fig. 1. Three different damage cases are developed such as Undamaged, Single element damage (1D3) (10%) and Double element damage (2D34) (10%).

Fig 1: Plan and Elevation of Undamaged, Single damaged and double damaged cantilever beam

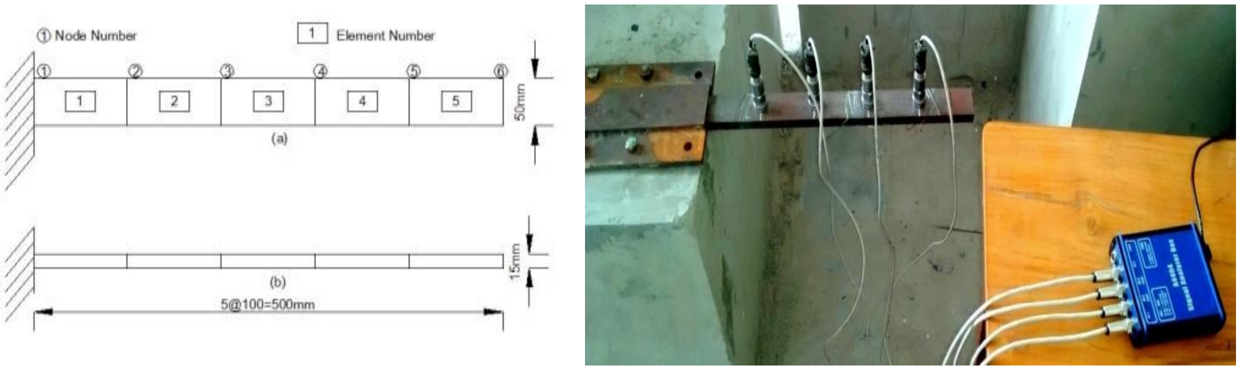

Fig 1: Schematic and Actual Experimental Setup of Undamaged Cantilever Steel Beam

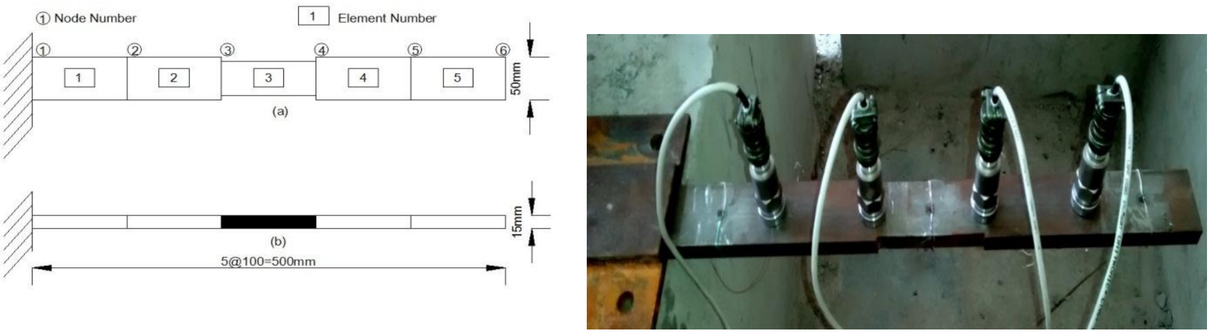

Fig 2: Schematic and Actual Experimental Setup of Single Element Damaged (1D3) Cantilever Steel Beam

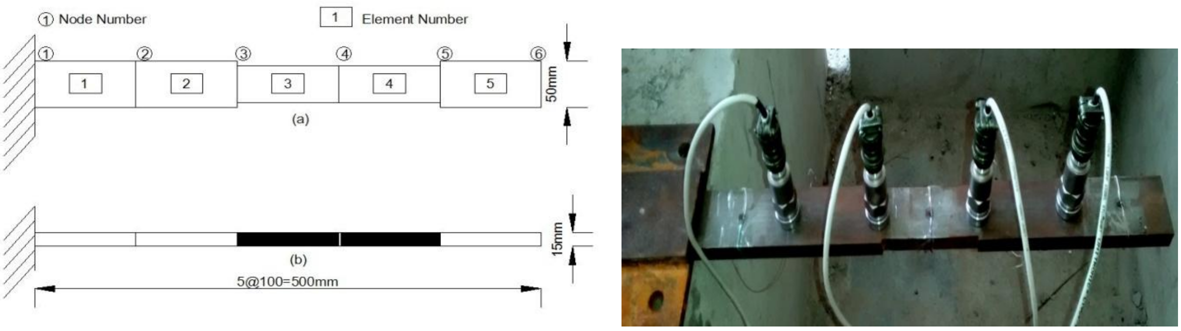

Fig 3: Schematic and Actual Experimental Setup of Multiple Element Damaged (2D34) Cantilever Steel Beam

Table 1: Estimated Structural Parameters of Cantilever Beams

Element No.	**Cross Sectional Area (mm^2)**			**Moment of Inertia (10^4 X mm^4)**		
Damaged Condition	**UD**	**1D3**	**2D3,4**	**UD**	**1D3**	**2D3,4**
1	750	750	750	1.4062	1.4062	1.4062
2	750	750	750	1.4062	1.4062	1.4062
3	750	**675**	**675**	1.4062	**1.265**	**1.265**
4	750	750	**675**	1.4062	1.4062	**1.265**
5	750	750	750	1.4062	1.4062	1.4062

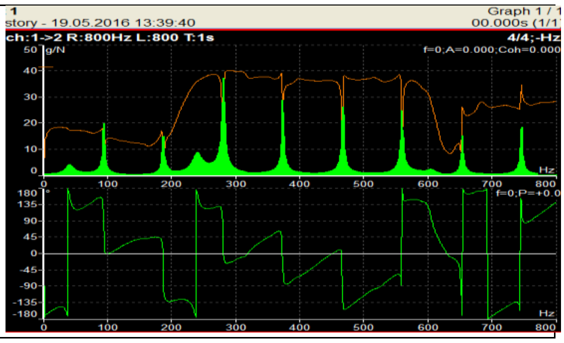

Fig. 4: Auto spectrum of undamaged beam experiment

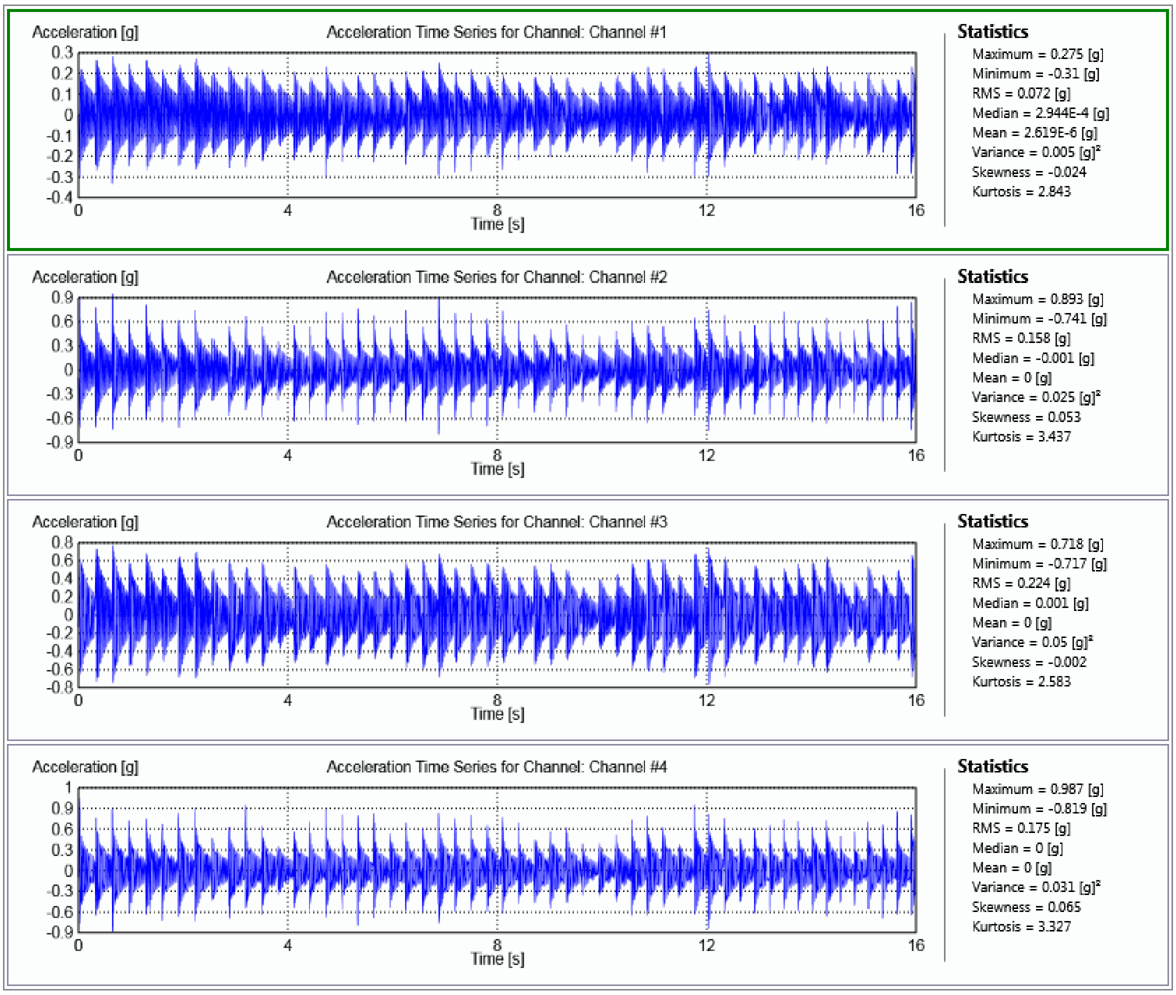

Fig. 5: Experimental setup of undamaged and single element damaged cantilever beam

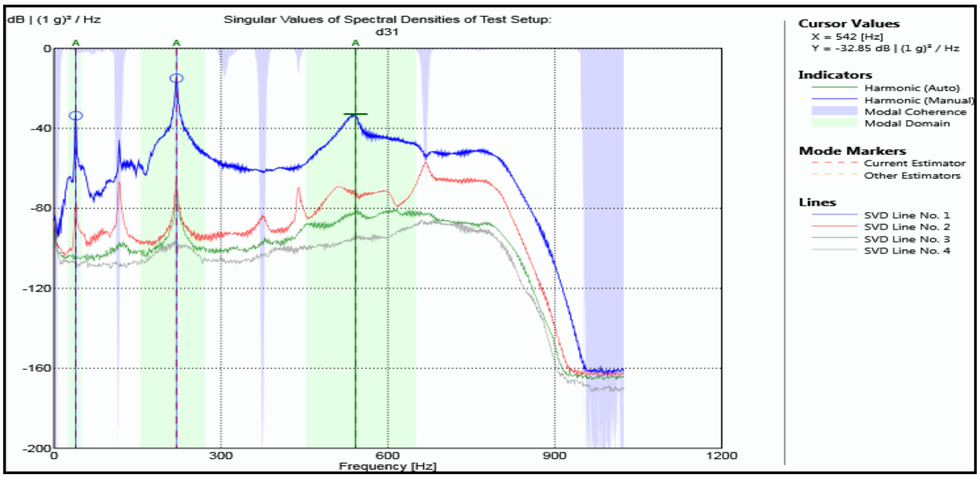

Fig. 6: Estimation from frequency domain decomposition (FDD) for double element damage (2D34)

The experimental modal data as extracted after post processing using ARTeMIS software are shown below.

TABLE 2 Extracted Natural Frequencies

Mode No	1st Mode	2nd	3rd
Analytical Nat Freq (HZ)	37	230	552
Experimental Nat Freq (HZ)	38	235	556

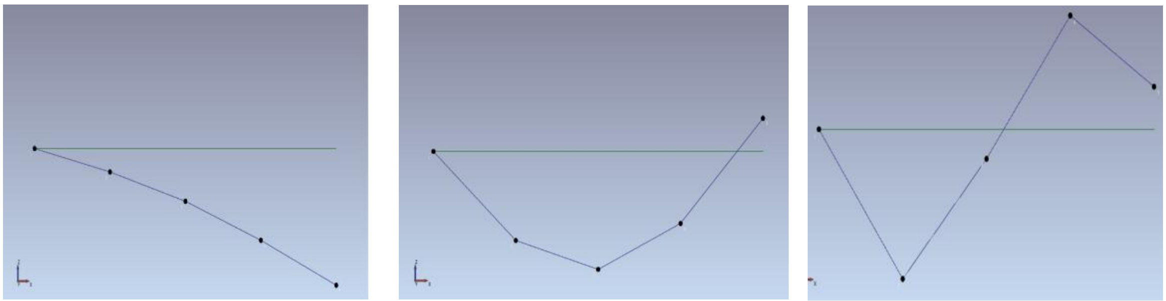

Fig. 7: Extracted Modal Parameters of First, Second and Third operational Mode

The validation of the proposed models for the detection of damage in structural systems is carried out first with several examples. The effect of the noise in measured modal data on the identified structural properties is studied. The applicability of the proposed methods for various multiple damage scenarios with various degree of damage is also examined. The effect of number of measurements of modal data on the identified properties is also studied.

Measured modal data are mixed with the random noises is used for this model for UD and different damage stated to check the accuracy of this model. The predicted error and estimated value for axial rigidity and bending rigidity of each element with various noise level are shown form Fig.8 UD and different damage level. It is noted that the structural parameters like axial rigidity, bending rigidity of both undamaged and damaged each element are accurately predicted for the various damage state of the frame, provided accurate measured modal data are available. However, the presence of noise in the measurement of modal data is inevitable. Thus the effect of the presence of noise in the measured data on the accuracy of the predicted structural properties is important to be studied. Similarly, the applicability of the proposed method in case of large structure with multiple damages is also important to be studied. It is seen that accuracy in the estimated axial rigidity and bending rigidity decreases for increased noise level. Also it is observed that the accuracy in prediction parameter for different element are varying due to the presence of random type noise.

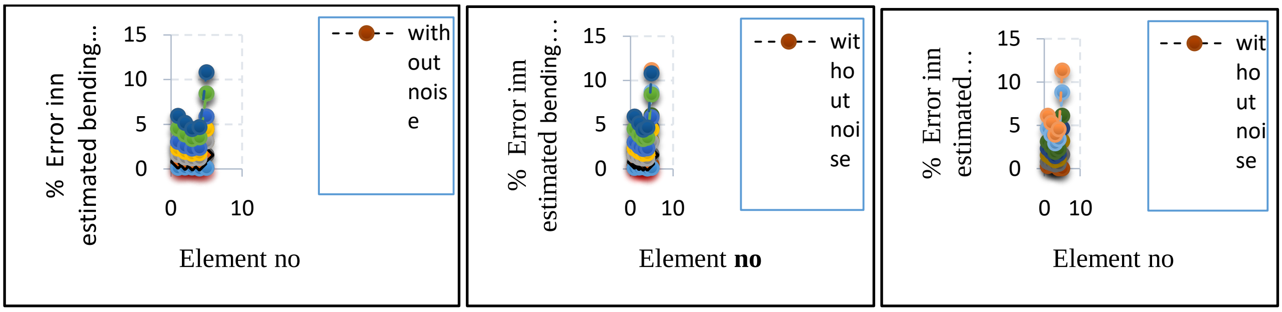

Fig 8: Plot of error (%) for identified bending rigidity with element for different noise in UD, 1D3 and 2D34 case

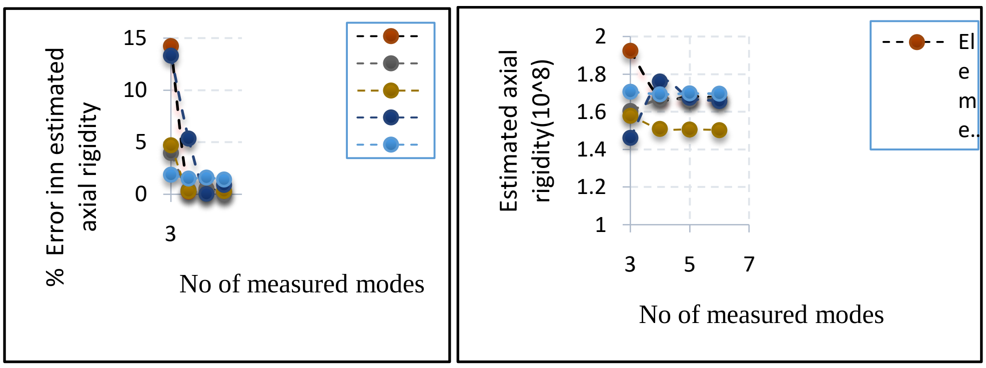

Fig 9: Plot of error (%) and estimated axial rigidity with measured mode for different element for 1D3 case

It is clearly observed from Fig.9 that the accuracy of prediction of the structural parameters depend on the number of measurement of modal data. The model is able to accurately predict the structural parameters provided noise-free data of large number of measured modes are available as observed in the example. But in practice, the availability of large number of modal data is difficult. The presence of noise, which is generally random in the measured data, is also inevitable in practice.

V. CONCLUSIONS

Damage Identification with dynamic data based approaches may address the condition monitoring aspect in much better way. The proposed model is feasible for practical problem as it is able to identify the structural parameters with first few modal data measured at selected degrees of freedom. The proposed model are able to identify the damage from limited noisy data with great accuracy even at a very low damage level. The accuracy of the proposed model increases with the increase of the number of measured modes and decreases with the increase of noise level. The condition of all the elements may be monitored continuously if the measured data are available. The experimental study even with simple structures demonstrates the potential of the proposed numerical model for its practical implementation.

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