International Journal of Technical Innovation in Modern Engineering & Science (IJTIMES)
Impact Factor: 5.22 (SJIF-2017), e-ISSN: 2455-2585
Volume 4, Issue 12, December-2018

Lexicographic method of Fully Fuzzy Transportation problem

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Abstract: The Transportation Problem in operations Researches deals with the shipment of units of a commodity from a given number of sources to a given number of destinations. Much research was done on this topic but the same subject is dealt with a new approach within the limits of linear programming to solve the real life problems of transportation. The real life situations are affected by uncertainty and sometimes fictional evidences. The fully fuzzy transportation problem with Lexicographic method is used to achieve the computational preciseness.

Key Words: Fully Fuzzy Transportation Problem, Lexicographic method

1. Introduction: The objective of the transportation problem is to supply the required number of units of a certain commodity (product) to a certain number of locations (destinations) from certain origins (sources) at a total minimum cost. The objective function and the constraints being linear, the transportation problem can be conveniently formulated as an integer linear programming problem. Quantities needed at the destinations, quantities available at the origins and unit transportation costs are called parameters of the transportation problem. The classical transportation problem involves all precise parameters. This, being a linear programming problem, can be solved by various methods available in the literature. But in most practical situations, for example, when a transportation problem is made in advance, these parameters could not be estimated precisely. This impreciseness of parameters can be dealt with by the use of fuzzy numbers.

The transportation problem was first proposed by Hitchcock in 1941[1]. Methods for determining optimal solution of the classical transportation problem were developed by Dantzig in1951 [2].and then by Charnels, Cooper and Henderson in 1953[3].

A linear programming problem with all fuzzy parameters may be called a Fully Fuzzy Linear Programming problem (FFLP).

In Adel Hatami-Marbini,Madjid Tavana 2011[6], a new method to solve FFLP has been developed by Adel Hatami-Marbini, Madjid Tavana. In this paper, the L-R fuzzy numbers are considered for the imprecise parameters and the lexicography method is applied. In the present paper we consider a Fully Fuzzy Transportation Problem (FFTP) and develop a lexicographic method for its solution. Application to a numerical example proved the efficiency of the proposed method.

2. Preliminaries:

Definition 1: (see [1]). Usually the left shape function denoted by L and the right shape function denoted by R, is reference function of a fuzzy number if and only if L(x)=L(-x),L(0)=1, and L is non increasing on [0,+∞]. Naturally, a right shape function R (·) is similarly defined as L (·).

Definition 2: (see [1]). A fuzzy number $\widetilde{M}$ is said to be a LR fuzzy number, if there exists left reference function L, right reference function R, and scalars $\alpha > 0, \beta > 0$ with

$$\mu_{\widetilde{M}} = \begin{cases} L\left(\frac{m-x}{\alpha}\right), x \leq m, \\ R\left(\frac{x-m}{\beta}\right), x \geq m, \end{cases} \quad (1)$$

Where m is the mean value of $\widetilde{M}$ and α and β are called the left and right spreads,respectively.Using its mean value and left and right spreads, and shape functions, such a LR fuzzy number $\widetilde{M}$ is symbolically written as $\widetilde{M}$=$(m, \alpha, \beta)_{LR}$.

Definition3: (see [1]). Two L-R type fuzzy numbers $\widetilde{M}$=$(m, \alpha, \beta)_{LR}$, and$\widetilde{N}$= (n, γ, δ)are said to be equal if and only if m= n and $\alpha = \beta$ and $\gamma = \delta$.

Theorem4: (see [1]). Let $\widetilde{M}$ = $(m, \alpha, \beta)_{LR}$ and $\widetilde{N}$ = (n,$\gamma, \delta)_{LR}$ be two fuzzy numbers of LR-type. Then one has

(1) $(m, \alpha, \beta)_{LR} + (n, \gamma, \delta)_{LR} = (m + n, \alpha + \beta, \gamma + \delta)_{LR}$,
(2) - $(m, \alpha, \beta)_{LR} = (-m, \gamma, \delta)_{LR}$,
(3) $(m, \alpha, \beta)_{LR} - (n, \gamma, \delta)_{LR} = (m - n, \alpha + \beta, \gamma + \delta)_{LR}$,

Remark5.The LR-Type fuzzy number $\widetilde{M}$ = $(m, \alpha, \beta)_{LR}$ is said to be non negative fuzzy number if and only if m$\geq 0, m - \alpha \geq 0, m + \beta \geq 0$.

Theorem 6 :(see [1]). From Theorem 4 it is assumed that,

(1) for $\widetilde{M}, \widetilde{N}$ positive,

$$(m, \alpha, \beta)_{LR} * (n, \gamma, \delta)_{LR} \approx (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR}, \quad (2)$$

(2) for $\widetilde{N}$ *positive and* $\widetilde{M}$ *negative*,

$$(m, \alpha, \beta)_{RL} * (n, \gamma, \delta)_{LR} \approx (mn, n\alpha - m\delta, n\beta - m\gamma)_{RL}, \quad (3)$$

(3) for $\widetilde{M}$, $\widetilde{N}$ negative

$$(m, \alpha, \beta)_{LR} * (n, \gamma, \delta)_{LR} \approx (mn, -n\beta - m\delta, -n\alpha - m\gamma)_{LR}, \quad (4)$$

Following [10], to compare two LR-type fuzzy numbers we propose a new definition.

Definition 7: Let the two arbitrary LR-type fuzzy numbers are $\widetilde{M} = (m_1, \alpha_1, \beta_1)_{LR}$ and $\widetilde{N} = (m_2, \alpha_2, \beta_2)_{LR}$ be represents that if and only if $\widetilde{M}$ is relatively less than $\widetilde{N}$, which is denoted by $\widetilde{M} < \widetilde{N}$.

(i) $m_1 < m_2$,
(ii) $m_1 = m_2\, and (\alpha_1 + \beta_1) > (\alpha_2 + \beta_2)$or,
(iii) $m_1 = m_2, (\alpha_1 + \beta_1) = (\alpha_2 + \beta_2), and\ (2m_2 - \alpha_1 + \beta_1) < (2m_2 - \alpha_2 + \beta_2)$.

Remark 8: It is clear from the above definition that $m_1 = m_2, (\alpha_1 + \beta_1) = (\alpha_2 + \beta_2)$,

$and\ (2m_2 - \alpha_1 + \beta_1) < (2m_2 - \alpha_2 + \beta_2)$. if and only if $\widetilde{M} = \widetilde{N}$.

Definition 9: R: F(R) →R is a ranking function, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real numbers and real line, where a natural order exists. Let $\widetilde{M} = (m, \alpha, \beta)_{LR}$ be a LR-type fuzzy number; then R ($\widetilde{M}$) = m+ $(\beta - \alpha)$/4.

Remark10: If $\widetilde{M} = (a, b, c)$is a triangle fuzzy number.

3. FFTP Formulation and Proposed Method:

Fully Fuzzy Transportation Problems with m fuzzy equality constraints and n fuzzy variables may be formulated as follows:

Objective function:

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{C_{ij}} \widetilde{x_{ij}}$

s.t. $\sum_{j=i}^{n} \widetilde{x_{ij}} = \widetilde{a_i}$,i=1,2,3- - - - - - - n.

and $\sum_{i=1}^{m} \widetilde{x_{ij}} = \widetilde{b_j}$,j=1,2,3- - - - - - - m. (5)

$\widetilde{x_{ij}} \geq (0,0,0)$,

$\widetilde{C_{ij}} = (C_{ij1}, C_{ij2}, C_{ij3})$

Step1: with respect to the definition fuzzy numbers (1), we have

Let $\widetilde{x_{ij}} = (x_{ij}', y_{ij}', z_{ij}')$

Then the constrains become

$\sum_{j=1}^{n}(x_{ij}', y_{ij}', z_{ij}') = (a_i', \alpha_i', \beta_i')$

$\sum_{i=1}^{m}(x_{ij}', y_{ij}', z_{ij}') = (b_j', \gamma_j', \delta_j')$

i.e $\sum_{j=1}^{n} x_{ij}' = a_i'$

$\sum_{j=1}^{n} y_{ij}' = \alpha_i'$

$\sum_{j=1}^{n} z_{ij}' = \beta_i'$

$\sum_{i=1}^{m} x_{ij}' = b_j'$ (6)

$\sum_{i=1}^{m} y_{ij}' = \gamma_j'$

$\sum_{i=1}^{m} z_{ij}' = \delta_j'$

$x_{ij}' - y_{ij}' \geq 0$

$x_{ij}' + y_{ij}' \geq 0$

Step2: Regarding Definition 7, we convert into the following multi objective TP problem.

Max $(y_{ij}' + z_{ij}')$

Min $(2x_{ij}' - y_{ij}' - z_{ij}')$ (7)

i.e. $Min(\sum \sum C_{ij} x_{ij}')$

$Max(\sum\sum(C_{ij}y_{ij}{}' + C_{ij}z_{ij}{}'))$

$Min(\sum\sum(2C_{ij}x_{ij}{}' - C_{ij}y_{ij}{}' - C_{ij}z_{ij}{}'))$

$\sum_{j=1}^{n} x_{ij}{}' = a_i{}'$

$\sum_{j=1}^{n} y_{ij}{}' = \alpha_i{}'$

$\sum_{j=1}^{n} z_{ij}{}' = \beta_i{}'$

$\sum_{i=1}^{m} x_{ij}{}' = b_j{}'$

$\sum_{i=1}^{m} y_{ij}{}' = \gamma_j{}'$

$\sum_{i=1}^{m} z_{ij}{}' = \delta_j{}'$

$\sum_{i=1}^{m} z_{ij}{}' = \delta_j{}'$

$x_{ij}{}' - y_{ij}{}' \geq 0$

$x_{ij}{}' + y_{ij}{}' \geq 0$

Step3: In priority of objective functions, the lexicographic method for any TP will be used to obtain a lexicographically optimal solution.

s.t $\sum\sum C_{ij}x_{ij}{}'$

$\sum_{j=1}^{n} x_{ij}{}' = a_i{}'$

$\sum_{j=1}^{n} y_{ij}{}' = \alpha_i{}'$

$\sum_{j=1}^{n} z_{ij}{}' = \beta_i{}'$ (8)

$\sum_{i=1}^{m} x_{ij}{}' = b_j{}'$

$\sum_{i=1}^{m} y_{ij}{}' = \gamma_j{}'$

$\sum_{i=1}^{m} z_{ij}{}' = \delta_j{}'$

$x_{ij}{}' - y_{ij}{}' \geq 0$

$x_{ij}{}' + y_{ij}{}' \geq 0$

Since the constraints are transportation constraints, existence of feasible solution i.e., guaranteed optimal solution of the above LPP gives the optimal solution of the FFTP.

Solution of Example by proposed method:

Min Z= (2,1,1) $\widetilde{x_{11}}'$+ (3,1,1) $\widetilde{x_{12}}'$ +(4,1,1) $\widetilde{x_{21}}'$ + (5,1,1) $\widetilde{x_{22}}'$

$\widetilde{x_{11}} + \widetilde{x_{12}}$ = (3, 1, 1)

$\widetilde{x_{21}} + \widetilde{x_{22}}$ = (4, 1, 1)

$\widetilde{x_{11}} + \widetilde{x_{21}} = (4,1,1)$

$\widetilde{x_{12}} + \widetilde{x_{22}} = (3, 1, 1)$

$\widetilde{x_{ij}} \geq (0,0,0)$

Step1: Let $\widetilde{x_{ij}} = (x_{ij}', y_{ij}', z_{ij}')$

∴ The constraints are

$(x_{11} + x_{12}, y_{11} + y_{12}, z_{11} + z_{12}) = (3,1,1)$

$(x_{21} + x_{22}, y_{21} + y_{22}, z_{21} + z_{22}) = (4,1,1)$

$(x_{11} + x_{21}, y_{11} + y_{21}, z_{11} + z_{21}) = (4,1,1)$

$(x_{12} + x_{22}, y_{12} + y_{22}, z_{21} + z_{22}) = (3,1,1)$

Multiplication of two fuzzy numbers i.e., is given by

$(m, 1,1) * (x_{ij}', y_{ij}', z_{ij}') = (mx_{ij}', my_{ij}' + x_{ij}', mz_{ij}' + x_{ij}')$

∴ $(2, 1, 1)\, \widetilde{x_{11}} = (2x_{11}', 2y_{11}' + x_{11}', 2z_{11}' + x_{11}')$

$(3, 1, 1)\, \widetilde{x_{12}} = (3x_{12}', 3y_{12}' + x_{12}', 2z_{12}' + x_{12}')$

$(4, 1, 1)\, \widetilde{x_{21}} = (4x_{21}', 4y_{21}' + x_{21}', 4z_{21}' + x_{21}')$

$(5, 1, 1)\widetilde{x_{22}} = (5x_{22}', 5y_{22}' + x_{22}', 5z_{22}' + x_{22}')$

Objective function: Using (7), we formulate the FFTP as

Min $(2x_{11}' + 3x_{12}' + 4x_{21}' + 5x_{22}'$, $2y_{11}'+x_{11}'+3y_{12}'+x_{12}'+4y_{21}'+x_{21}'+5y_{22}'+x_{22}'$, $2z_{11}'+x_{11}'+3z_{12}'+x_{12}'+4z_{21}'+x_{21}'+5z_{22}'+x_{22}')$

Max $(2x_{11}'+3x_{12}'+4x_{21}'+5x_{22}'$, $2x'_{11}+y_{11}'+3x_{12}'+y_{12}'+4x_{21}'+y_{21}'+5x_{22}'+y_{22}'$,

$2x_{11}'+z_{11}'+3x_{12}'+z_{12}'+4x_{21}'+z_{21}'+5x_{22}' + z_{22}')$

Min $((4x_{11}'+6x_{12}'+8x_{21}'+5x_{22}') - (2y_{11}'+x_{11}'+3y_{12}'+x_{12}'+4y_{21}'+x_{21}'+5y_{22}'+x_{22}') + (2z_{11}'+x_{11}'+3z_{12}'+x_{12}'+4z_{21}'+x_{21}'+5z_{22}'+x_{22}'))$

S.t $x_{11}' + x_{12}' = 3$

$y_{11}' + y_{12}' = 1$

$z_{11}' + z_{12}' = 1$

$x_{21}' + x_{22}' = 4$

$y_{21}' + y_{22}' = 1$

$z_{21}' + z_{22}' = 1$

$x_{11}' + x_{21}' = 4$

$y_{11}{}' + y_{21}{}' = 1$

$z_{11}{}' + z_{21}{}' = 1$

$x_{12}{}' + x_{22}{}' = 3$

$y_{12}{}' + y_{22}{}' = 1$

$z_{12}{}' + z_{22}{}' = 1$

Using (8), we obtain

Min $(2\,x_{ij}{}' + 3\,x_{ij}{}' + 4\,x_{ij}{}' + 5\,x_{ij}{}'\,,\; 2\,y_{ij}{}' + x_{ij}{}' + 3\,y_{ij}{}' + x_{ij}{}' + 4\,y_{ij}{}' + x_{ij}{}' + 5\,y_{ij}{}' + x_{ij}{}'\,,\; 2z_{ij}{}' + x_{ij}{}' + 3z_{ij}{}' + x_{ij}{}' + 4z_{ij}{}' + x_{ij}{}' + 5z_{ij}{}' + x_{ij}{}')$

S.t. $x_{11}{}' + x_{12}{}' = 3$

$y_{11}{}' + y_{12}{}' = 1$

$z_{11}{}' + z_{12}{}' = 1$

$x_{21}{}' + x_{22}{}' = 4$

$y_{21}{}' + y_{22}{}' = 1$

$z_{21}{}' + z_{22}{}' = 1$

$x_{11}{}' + x_{21}{}' = 4$

$y_{11}{}' + y_{21}{}' = 1$

$z_{11}{}' + z_{21}{}' = 1$

$x_{12}{}' + x_{22}{}' = 3$

$y_{12}{}' + y_{22}{}' = 1$

$z_{12}{}' + z_{22}{}' = 1$

The optimal solution of the above TP is obtained by LINGO14.0 as

$x_{12}{}' \to 3, x_{21}{}' \to 4, y_{12}{}' \to 1, y_{21}{}' \to 1, z_{11}{}' \to 1, z_{22}{}' \to 1$

$x_{11} = (0,0,0)$

$x_{12} = (3,1,0)$

$x_{21} = (4,1,1)$

$x_{22} = (0,0,1)$

$\therefore$ MinZ = (2,1,1)(0,0,0)+(3,1,1)(3,1,0)+(4,1,1)(4,1,1)+(5,1,1)(0,0,1)

= (9,6,3)+(16,8,8)+(0,5,10)

= (24, 19, 21)

4. Conclusions

The present study focused on solution of fully fuzzy transportation problem using the L-R numbers. To solve this problems the lexicographic methods and transportation methods are used .This algorithm is applied to single –objective transportation problems with triangular fuzzy numbers for normal and abnormal data to obtain a fully fuzzy optimal solution. The proposed method foresees the efficiency in computing and also a notable result in the performance levels of the solution..

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