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$\emptyset -$ graceful labeling of merge graph , bistar graph, friendship graph, C_n^d relation graph .

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ABSTRACT: Let G be a graph. The $\emptyset -$ graceful labeling of a graph $G(V,E)$ with 'm' vertices and 'n' edges is a injective function $f^*: V(G) \rightarrow \{0,1,2,\dots,p-1\}$ such that the induced function $f^*: E(G) \rightarrow N$ is given by $f^*(g,h) = 2\{f(g)+f(h)\}$, then the resulting edge labels are distinct. In this paper we we investigate some result $\emptyset -$ graceful labeling of merge graph, bistar graph, friendship graph, C_n^+(graph obtained by adding a pendent edge for each vertex of the cycleC_n) , C_n^d(A cycle C_n with non-intersecting chords).

Mathematics Subject Classification (2010): 05C78

Keywords: $\emptyset -$ Graceful graph, Graceful labeling and C_n^d relation graph.

I. Introduction

Graph labeling is the task of labels, where vertices or edges or both are allocating real values field to certain conditions. We have frequently motivated by their uses in different applied fields and their intrinsic mathematical interest.

The concept of graceful labelling is a useful area of research in graph theory which has lots of application in communication networks, optimal circuits layer, coding theory and graph domination problems with a dynamic survey of various graph labeling problems with an immense bibliography, here we refer a Gallian[1].

The following 3- types of problems at observe in the area zone of graph labeling.

(1) How certain labeling is affected in the different operations.
(2) Study of new families of graph which reveal a certain graph labeling.
(3) Consisting a graph theoretic property and represent the classes of graphs with property that reveal a particular graph labeling.

It is useful to recall following definition from graph theory.

II. BASIC TERMINOLOGY

Definition 1.1([1]): *A ϕ −graceful graph* is a graph $G(V,E)$ with *'m'* vertices and *'n'* edges is a injective function $f^*: V(G) \rightarrow \{0,1,2,\dots,p-1\}$ such that the induced function $f^*: E(G) \rightarrow N$ is given by $f^*(g,h) = 2\{f(g)+f(h)\}$, then the resulting edge labels are distinct.

Definition 1.2([4]): *A Merge graph* $G_1 * G_2$can be formed from two graphs G_1 and G_2 by merging a vertex of G_1 with a vertex of G_2.

Definition 1.3([4]): *A Bistar graph* $B(m,n)$ is a graph obtained by attaching m pendent edges to one end point and n pendent edges to the order point of K_2.

Definition 1.4([5]): *A Friendship graph* T_n is a planner undirected graph with 2n+1 vertices and 3n edges constructed by joining n copies of the cycle graph C_3 with a common vertex.

Definition1.5.([6]): A Let $G = (V,E,R_1,R_2)$. The vertex set $V = \{1,2,\dots\dots,n\}$ and the edge set is defined by the relations R_1 and R_2 such that $R_1: b = a+1$ & $R_2 : a+b = n+1$, $\forall a,b \in V$. If $n \equiv 0\ (mod 2)$ We get cycle C_n with

$d = (n-2)/2$ non-intersecting chords and when $n \equiv 1(mod 2)$ we get cycle C_n with $d = (n-3)/2$ non interesting chords. The graph is obtained by this *relation is* $C_n^d,\ n \geq 5$

III. CONCEPTS OF ϕ −GRACEFUL LABELING

Theorem-2.1: *The Merge graph* $(C_3 * K_{1,n})$ *is* $\emptyset -$ *graceful graph.*

Proof: Let G be the Merge graph $(C_3 * K_{1,n})$. Note that the graph G has an order $p = 3 + n$ and size $q = 3 + n$. Let vertex set of graph $(C_3 * K_{1,n})$ is $\{u_0, u_1, u_{2,} v_1, v_2, \dots, v_n\}$, where u_2 is an apex vertex and v_j's are adjacent vertices of u_2. Here u_i are the vertices of the cycle as shown in Fig 1. Now we define a function $f: V(G) \rightarrow \{0,1,2, \dots, p-1\}$ as follows:

$f(u_i) = i\ ;\ 0 \leq i \leq 2$

$f(v_j) = j + 2\ ;\ 1 \leq j \leq n$

Then we get the induced edge function $f^*: E(G) \rightarrow N$ which is given by $f^*(gh) = 2\{f(g) + f(h)\}$ as

$f^*(u_0 u_i) = 2i\ ; i = 1,2$

$f^*(u_i v_j) = 2(u_i + v_j)\ ; i = 2, 1 \leq j \leq n,$

$f^*(u_1 u_2) = 6$

From above mentioned edge function, it is clear that all edges are getting distinct edge label. Hence, the Merge

$(C_3 * K_{1,n})$ is $\emptyset -$ graceful graph.

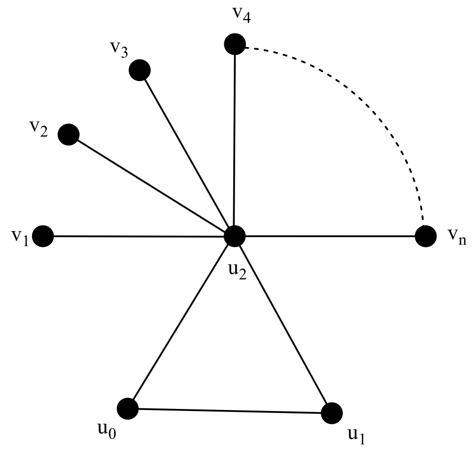
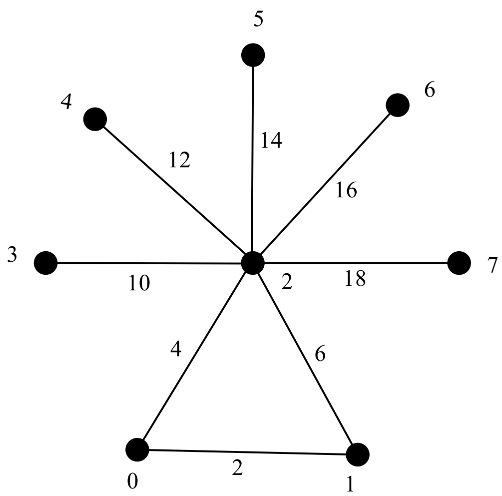


Illustration-2.2: *The* ϕ *−graceful labelling of* $(C_3 * K_{1,5})$ *in fig 2.*



Theorem-2.3: *The Bistar graph $B(m,n)$ is $\emptyset$ −graceful graph.*

Proof: Let G be the Bistar graph $B(m,n)$ as shown in Fig 3. The graph G has an order $p = m + n + 2$ and size $q = m + n + 1$.Note that the vertex set $V(G) = \{u_0, u_1, u_2, \dots, u_m, v_0, v_1, v_2, \dots, v_n\}$, where $u_1, u_2, \dots, u_m$ are pendent vertices adjacent to u_0 and $v_1, v_2, \dots, v_n$ are pendent vertices adjacent to v_0.We define a function $f: V(G) \to \{0,1,2, \dots, p-1\}$ as follows:

$f(u_0) = 0, f(v_0) = 1$

$f(u_j) = 1 + j\,; 1 \le j \le m$

$f(v_j) = m + 1 + j\,; 1 \le j \le n$

Then we get the induced edge function $f^*: E(G) \to N$ which is given by $f^*(gh) = 2\{f(g) + f(h)\}$ as

$f^*(u_0u_j) = 2j + 2\,; 1 \le j \le n$

$f^*(v_0v_j) = 2(j + m + 2)\,;\ 1 \le j \le n.$

As, $m \ge 1$, it's clear that $2(j + m + 2) > 2j + 2$, for $1 \le j \le n$. From above mentioned edge function, it is clear that all edges are getting distinct edge label. Hence, the Bistar graph $B(m,n)$ is $\emptyset$ −graceful graph.

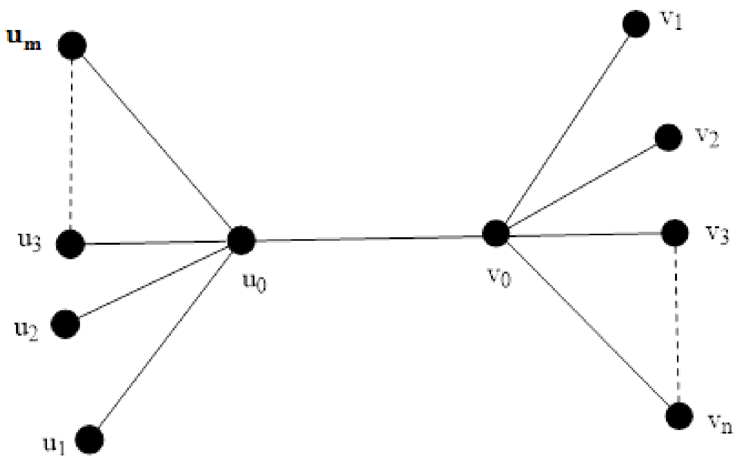
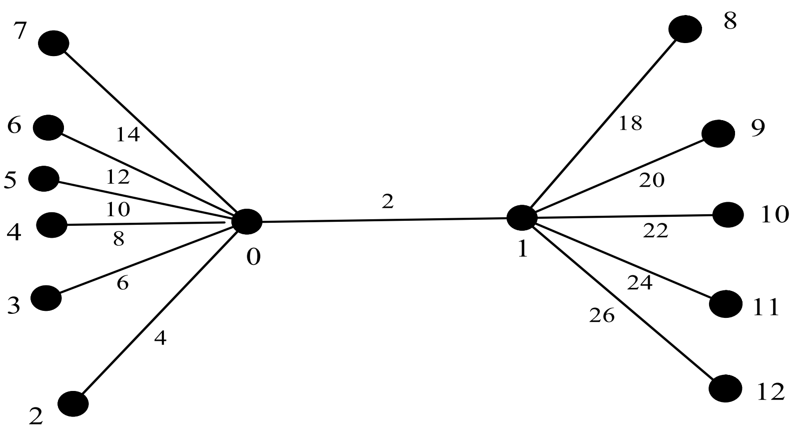


Fig 3: Bistar graph $B(m,n)$

Illustration-2.4: The $\emptyset$ −graceful labeling of Bistar graph $B(6,5)$ is shown in Fig 4.



Theorem-2.5: *The Friendship graph T_n is $\emptyset -$ graceful graph.*

Proof: Let $G = T_n$ be a friendship graph as shown in the Fig 5. Let the vertex set $V(G) = \{v_0, v_1, v_2, \dots, v_{2n}\}$, where v_0 be the centre of the vertex.Note that $|V(G)| = n + 1$ and $|E(G)| = 3n$. We define a function $f: V(G) \to \{0,1,2, \dots ,2n\}$

$f(v_0) = 0$

$f(v_{2j-1}) = 5i - 2 \ ; \ 1 \le i, j \le n$

$f(v_{2j}) = 5i + 1 \ ; 1 \le i, j \le n$

Then we get the induced edge function $f^*: E(G) \to N$ which is given by $f^*(gh) = 2\{f(g) + f(h)\}$ as

$f^*(v_0 v_i) = 2\{f(v_0) + f(v_i)\}; 1 \le i, j \le 2n$

$f^*(v_i v_{i+1}) = 2\{f(v_i) + f(v_{i+1})\}; 1 \le i \le n$

from above mentioned edge function, it is clear that all edges are getting distinct edge label. Hence, the Friendship graph T_n is $\emptyset -$ graceful graph.

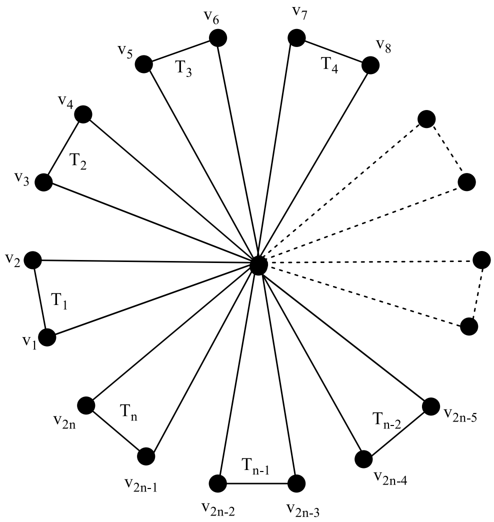
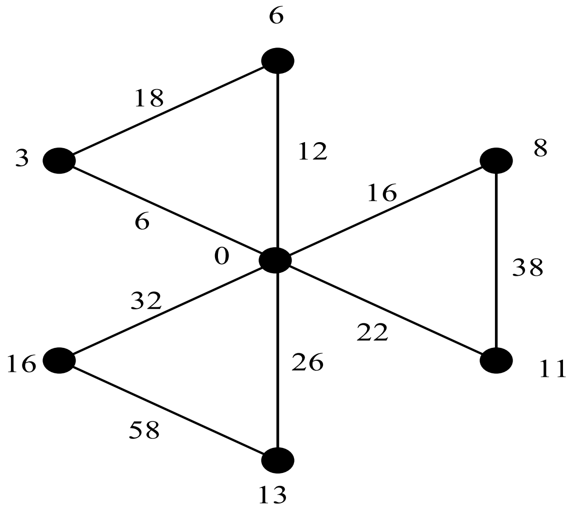


Fig 5: - Friendship graph T_n

Illustration-2.6: The $\emptyset -$graceful labeling of Friendship graph T_3 is shown in Fig 6.



Theorem-2.7: *The graph C_n^+ is $\emptyset-$ graceful graph.*

Proof: Let $V(G) = \{v_1, v_2, \dots, v_n, v_{n+1}, \dots, v_{2n}\}$ be the vertex set and $E(G) = E_1 \cup E_2 \cup E_3$ be the edge set where $E_1 = \{v_j v_{j+1} | \ 1 < j < n\}, E_2 = \{v_j v_{j+n} | \ 1 < j < n\}$ and $E_3 = \{v_n v_1\}$ of the graph C_n^+. Define a bijection $f: V(G) \to \{1,2,3, \dots ,2n\}$ such that

Case(i): When $n \equiv 0 \ (mod\ 2)$

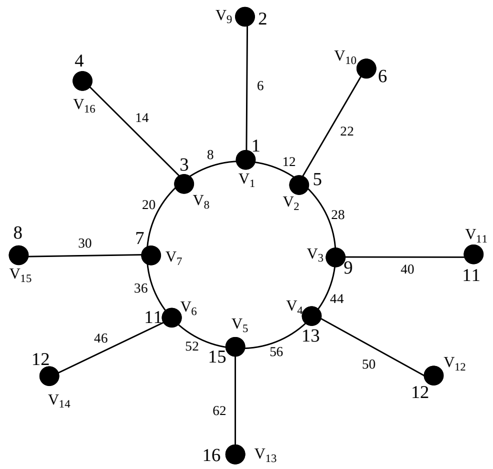
$$f(v_j) = \begin{cases} 4j-3, & 1 \le j \le \frac{n}{2} \\ 4(n-j)+3, & \frac{n}{2}+1 \le j \le n \end{cases}$$

And

$$f(v_{n+j}) = \begin{cases} 4j-2, & 1 \le j \le \frac{n}{2} \\ 4(n-j)+4, & \frac{n}{2}+1 \le j \le n \end{cases}$$

Then we get the induced edge function $f^*: E(G) \to N$ which is given by $f^*(gh) = 2\{f(g) + f(h)\}$ as $f^*(v_j v_{j+1}) = 2\{f(v_j) + f(v_{j+1})\}$ where $1 < j < n$, $f^*(v_j v_{n+j}) = 2\{f(v_j) + f(v_{n+j})\}$ where $1 < j < n$ and $f^*(v_n v_1) = 2\{f(v_n) + f(v_1)\}$. From above mentioned edge function, it is clear that all edges are getting distinct edge label. Hence, the graph C_n^+ is a $\emptyset-$ graceful graph ($n \equiv 0 \ (mod\ 2)$).

Illustration-2.8(a): The $\emptyset$ $-$graceful labeling of C_8^+ is shown in Fig 7.



Case(ii): When $n \equiv 1 \ (mod\ 2)$

The bijection $f: V(G) \to \{1,2,3, \dots ,2n\}$ is defined as the following

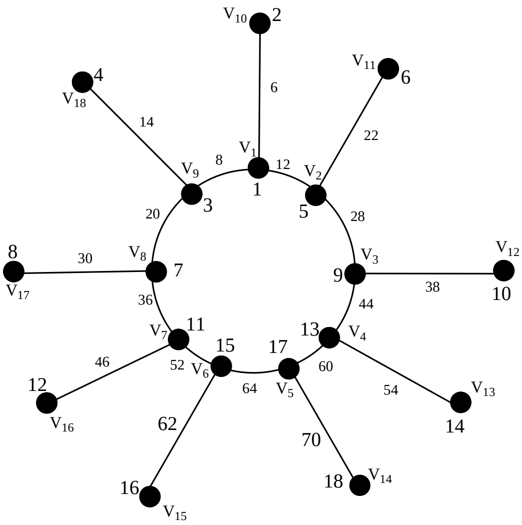
$$f(v_j) = \begin{cases} 4j-3, & 1 \le j \le \frac{n+1}{2} \\ 4(n-j)+3, & \frac{n+1}{2}+1 \le j \le n \end{cases}$$

And

$$f(v_{n+j}) = \begin{cases} 4j-2, & 1 \le j \le \frac{n+1}{2} \\ 4(n-j)+4, & \frac{n+1}{2}+1 \le j \le n \end{cases}$$

Then we get the induced edge function $f^*: E(G) \to N$ which is given by $f^*(gh) = 2\{f(g)+f(h)\}$ as $f^*(v_j v_{j+1}) = 2\{f(v_j)+f(v_{j+1})\}$ where $1 < j < n$, $f^*(v_j v_{n+j}) = 2\{f(v_j)+f(v_{n+j})\}$ where $1 < j < n$ and $f^*(v_n v_1) = 2\{f(v_n)+f(v_1)\}$. From above mentioned edge function, it is clear that all edges are getting distinct edge label. Hence, the graph C_n^+ is a $\emptyset-$ graceful graph ($n \equiv 1\ (mod\ 2)$).

Illustration-2.8(b): The $\emptyset$ −graceful labeling of C_9^+ is shown in Fig 8.



Theorem-2.9: *The graph $C_n^d, n \ge 5$ with non-intersecting chords is $\emptyset - graceful$ graph.*

Proof :-Let $G = \{v_1, v_2, \dots \dots \dots v_n\}$ be the vertex set and $E = E_1 \cup E_2 \cup E_3$ be the edge set of the graph C_n^d with non-intersecting chords. Where $E_1 = \{v_j v_{j+1}, 1 \le j < n\}$, $E_2 = \left\{v_{j+1} v_{n-j+1}, 1 \le j < \frac{n}{2} - 1\right\}$ and $E_3 = \{v_n v_1\}$

Define a bijection $f: V \to \{1,2,3, \dots \dots \dots, n\}$ such that

Case-1:- When $n \equiv 0\ (mod\ 2)$

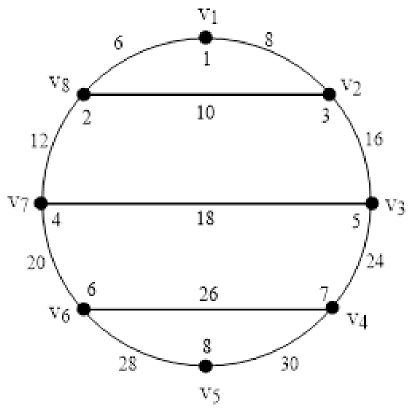
$$f(v_j) = \begin{cases} 2j-1, & 1 \le j \le \frac{n}{2} \\ 2(n-j)+2, & \frac{n}{2}+1 \le j \le n \end{cases}$$

Then we get the induced edge function $f^*: E(G) \to N$ which is given by $f^*(gh) = 2\{f(g)+f(h)\}$ as
$f^*(v_j v_{j+1}) = 2\{f(v_j)+f(v_{j+1})\}$ Where $1 \le j < n$

$f^*(v_{j+1} v_{n-j+1}) = 2\{f(v_{j+1})+f(v_{n-j+1})\}$ Where $1 \le j < \frac{n}{2} - 1$ and

$f^*(v_n v_1) = 2\{f(v_n)+f(v_1)\}$. From above mentioned edge function, it is clear that all edges are getting distinct edge label. Hence, the graph C_n^d is a $\emptyset-$ graceful graph ($n \equiv 0\ (mod\ 2)$).

Illustration-2.10(a): The $\emptyset$ −graceful labeling of C_8^3 is shown in Fig 9.



Case-2:- When $n \equiv 1\ (mod\ 2)$

Define a bijection $f: V \rightarrow \{1,2,3, \dots \dots \dots, n\}$ such that

$$f(v_j) = \begin{cases} 2j-1, & 1 \le j \le \frac{n+1}{2} \\ 2(n-j)+2, & \frac{n+1}{2}+1 \le j \le n \end{cases}$$

With the edge set $E = E_1 \cup E_2 \cup E_3$

Where $E_1 = \{v_j v_{j+1}, 1 \le j < n\}, E_2 = \left\{v_{j+1} v_{n-j+1}, 1 \le j < \frac{n-1}{2} - 1\right\}$ and $E_3 = \{v_n v_1\}$

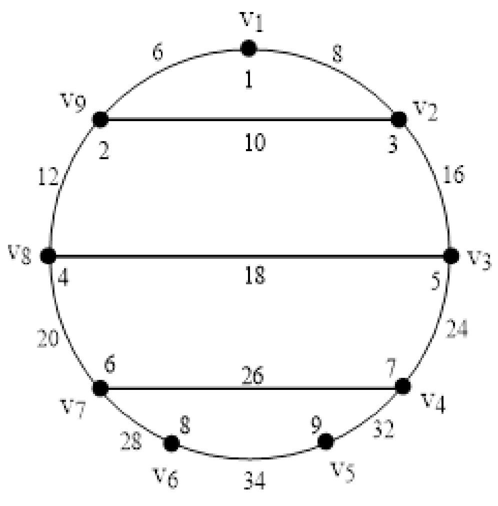
Then we get the induced edge function $f^*: E(G) \rightarrow N$ which is given by $f^*(gh) = 2\{f(g) + f(h)\}$ as

$f^*(v_j v_{j+1}) = 2\{f(v_j) + f(v_{j+1})\}$ Where $1 \le j < n$

$f^*(v_{j-1} v_{n-j+1}) = 2\{f(v_{j-1}) + f(v_{n-j+1})\}$ Where $1 \le j < \frac{n-1}{2} - 1$ and $f^*(v_n v_1) = 2\{f(v_n) + f(v_1)\}$,.

From above mentioned edge function, it is clear that all edges are getting distinct edge label. Hence, the graph C_n^d is a $\emptyset$ − graceful graph ($n \equiv 1\ (mod\ 2)$).

Illustration-2.10(b): The $\emptyset$ −graceful labeling of C_9^3 is shown in Fig 10.



IV. Conclusion.

In this paper we have to show that merge graph, Bistar graph, friendship graph, $C_n^+ \, graph$, C_n^d relation graph are $\emptyset -$ graceful graph.

REFERENCE

[1]S.N. Daoud, "Edge odd graceful labelling of some path and cycle related graphs" **International Journal of Graphs and Combinatorics,** 14,(2017),178-203

[2] J.A. Gallian, *"A Dynamic Survey of graph labeling"*, **The Electronics Journal of Combinatorics**, Dec 20, (2013).

[3]Chidanand.A.Masarguppi and Shigehalli V. S, *"$\emptyset -$gracefullabeling of some graphs",*
International Journal of Mathematical Archive, 9(7), (2018), 23-25.

[4] G.Subashini," Quad difference labelling For mechanical process operating at repetitive cycles", **International Journal of Innovative Technology and Exploring Engineering** ,9(254), 2019,413-416

[5]Himayat ullah, Gohar Ali, Murtaza Ali, and Andrea Semanicova- Fenovcikova,," On super(a,d)-Edge-Antimagic Total Labeling of Special Typesof Crown Graphs", **Journal Of Applied Mathematics, Hindawi Publishing Corporation,** Article ID 896815, 2013, 6 Pages.

[6] M.Muthusamy, K.C.Raajasekar, J.Baskar Babujee, "On Strongly Multiplicative Graphs", International journal of mathematicstrends and technology, Vol 3, Issue 2012,PP 13-18

[7] R.Uma, S.Divya , "Cube difference labelling of star related graphs", **International Journal of Scientific Research In Computer Science, Engineering and Information Technology,**2(4),2017,298-301.

[8]Shigehalli V. S and Chidanand. A. Masarguppi, *"$\emptyset -$gracefullabeling of some graphs",*
Journal of Computer and Mathematical Science, 6(2), (2015), 127-133.

[9] Dr. K.V. Vaithilingam," Difference labelling of some graph families", **International Journal of Mathematics and Statistics Invention ,**2(6),2014,34-43