International Journal of Technical Innovation in Modern Engineering & Science (IJTIMES)
Impact Factor: 5.22 (SJIF-2017), e-ISSN: 2455-2585
Volume 4, Issue 10, October-2018

ON CERTAIN TRANSFORMATION FORMULAE FOR BASIC HYPERGEOMETRIC FUNCTIONS AND BIBASIC HYPERGEOMETRIC SERIES

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Abstract: In this paper, making use of Bailey's transformation, an attempt has been made to establish certain transformation formulae for Basic Hypergeometric Functions and Bibasic series.

1. Introduction

In 1944, Bailey established a powerful series identity which was later known as Bailey's Transform

The Bailey's Transform states that, if

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r} \tag{1.1}$$

and

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n}\, v_{n+r} \tag{1.2}$$

where α_r, δ_r, u_r and v_r are any function of r only such that γ_n the series exists then

$$\sum_{n=0}^{\infty} \alpha_n \Upsilon_n \;=\; \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.3}$$

Making use of Bailey's Transform, Slater [5] gave a long list of Rogers-Ramanujan type identities. Later on a number of mathematicians, notably, Denis [1], Singh [4], Slater [6], Andrews [2], Gasper and Rahman [3], Verma [7] and others made use of Bailey's identity (1.3) and established a number of transformation formulae. In this paper, making use of certain formulae due to Gasper and Rahman [3] and identity (1.3), an attempt has been made to establish certain transformation formulae for Basic Hypergeometric Functions.

1. Notation

Suppose that $|q| < 1$ where q is non zero complex number, the condition ensures that all the infinite product that we use will converge. The following identities will be used:

$$(a;q)_n = \begin{cases} (1-a)(1-aq)\ldots\ldots\ldots(1-aq^{n-1}) & ;n>0 \\ 1; & n=0, \end{cases} \tag{2.1}$$

$$(a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k) \tag{2.2}$$

$$(a;q)_{-n} = \frac{(-)^n q^{n(n+1)/2}}{a^n [q/a\,;q]_n}\,, \tag{2.3}$$

$$(a;q)_{2n} = (a;q)_n (aq^n;q)_n \tag{2.4}$$

$$(a;q)_{n-k} = \frac{(a;q)_n \left(-qa^{-1}\right)^k q^{\binom{k}{2}-nk}}{(a^{-1}q^{1-n};q)_k} \tag{2.5}$$

$$(a;q)_{n+k} = (a;q)_n (aq^n;q)_k \tag{2.6}$$

$$(aq^{-n};q)_k = \frac{(a;q)_k \left(qa^{-1};q\right)_n}{\left(a^{-1}q^{1-k};q\right)_n} q^{-nk} \tag{2.7}$$

where k and n are integers.

The Basic Hypergeometric Function is defined as

$$ {}_r\Phi_s\left[\begin{matrix} a_1,a_2,a_3,\ldots\ldots a_r;q;z \\ b_1,b_2,b_3,\ldots\ldots b_s \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(a_1,a_2,a_3,\ldots\ldots a_r;q)_n z^n}{(q,b_1,b_2,b_3,\ldots\ldots b_s;q)_n} \quad (2.8)$$

Max ($|q|, |z| < 1$), where

$$(a_1, a_2, a_3, \ldots \ldots a_r; q)_n = (a_1; q)_n (a_2; q)_n \ldots \ldots \ldots (a_r; q)_n$$

We shall make use of the following known results

$$ {}_2\Phi_1\left[\begin{matrix} a,\ b\ ;q\ ;c/ab \\ c \end{matrix}\right] = \frac{(c/a,c/b;q)_\infty}{(c,c/ab;q)_\infty} \quad (2.9)$$

(Slater [6;App.IV(IV.2)])

$$ {}_2\Phi_1\left[\begin{matrix} a,\ b\ ;q\ ;c/ab \\ cq \end{matrix}\right] = \frac{(cq/a,cq/b;q)_\infty}{(cq,cq/ab;q)_\infty} \left\{\frac{ab(1+c)-c(a+b)}{ab-c}\right\} \quad (2.10)$$

(Verma [7;(1.4)])

$$ {}_3\Phi_2\left[\begin{matrix} a\ ,\ \ b\ ,q^{-n}\ ;q;q \\ c\ ,\ \ d \end{matrix}\right] = \frac{(c/a,c/b;q)_n}{(c,c/ab;q)_n} \quad (2.11)$$

(Slater [6;App.IV(IV.4)])

$$ {}_6\Phi_5\left[\begin{matrix} a,q\sqrt{a}\ ,-q\sqrt{a}\ ,b\ ,kq^n,q^{-n};q;aq/bk \\ \sqrt{a}\ ,-\sqrt{a},aq/b,aq^{1-n}/k,aq^{1+n} \end{matrix}\right] = \frac{(aq,kb/a;q)_n}{(k/a,aq/b;q)_n b^n} \quad (2.12)$$

(Gasper & Rahman [3;App.II(II.21)])

$$ {}_6\Phi_5\left[\begin{matrix} a,\ q\sqrt{a}\ ,\ -q\sqrt{a}\ ,b,\ \ c\ ,d; \\ \sqrt{a}\ ,-\sqrt{a},aq/b,aq/c,aq/d; \end{matrix} q, aq/bcd\right] = \frac{(aq,aq/cd,aq/bd,aq/bc;q)_\infty}{(aq/b,aq/c,aq/d,aq/bcd;q)_\infty} \quad (2.13)$$

(Slater [6;App.IV(IV.7)])

$$ {}_8\Phi_7\left[\begin{matrix} a,\ q\sqrt{a}\ ,\ -q\sqrt{a}\ ,b,\ \ c\ ,d,a^2q/bck,kq^n,q^{-n}; \\ \sqrt{a}\ ,-\sqrt{a},aq/b,aq/c,bck/a,aq^{n+1}/k,aq^{1+n}; \end{matrix} q, q\right] = \frac{(aq,aq/bc,kb/a,kc/a;q)_n}{(aq/b,aq/c,k/a,kbc/a;q)_n} \quad (2.14)$$

(Gasper & Rahman [3;App.II(II.22)])

2. Main results

$$ {}_3\Phi_2\left[\begin{matrix} k,kq,q^2;q^2;aq/k \\ aq\ ,\ \ aq^2 \end{matrix}\right] = \frac{(aq;q)_\infty (a^2q/k^2;q)_\infty}{(aq/k;q)_\infty (a^2q/k;q)_\infty} \sum_{n=0}^{\infty} \frac{(k,k/a,aq/k;q)_n (a;q^2)_n}{(q,a,a/k;q)_n (aq^2;q^2)_n} \left(\frac{a^2q}{k^2}\right)^n \quad (3.1)$$

$$ {}_3\Phi_2\left[\begin{matrix} k,kq,q^2;q^2;a/k \\ a^2q/k\ ,a^2q^2/k \end{matrix}\right] + a\,{}_3\Phi_2\left[\begin{matrix} k,kq,q^2;q^2;aq^2/k \\ a^2q/k\ ,a^2q^2/k \end{matrix}\right]$$

$$ = \frac{(aq;q)_\infty (a^2q/k^2;q)_\infty (k+a)}{k(aq/k;q)_\infty (a^2q/k;q)_\infty} \sum_{n=0}^{\infty} \frac{(k,k/a,aq/k;q)_n (a;q^2)_n}{(q,a,a/k;q)_n (aq^2;q^2)_n} \left(\frac{a^2}{k^2}\right)^n \quad (3.2)$$

$$ {}_6\Phi_5\left[\begin{matrix} k,q\sqrt{k},-q\sqrt{k},,c,d,kb/a;q;aq/bcd \\ \sqrt{a},-\sqrt{a},aq/b,kq/c,kq/d \end{matrix}\right] = \frac{(kq,kq/cd,aq/c,aq/d,;q)_\infty}{(aq,aq/cd,kq/c,kq/d,;q)_\infty} {}_6\Phi_5\left[\begin{matrix} a,q\sqrt{a},-q\sqrt{a},b,c,d;q;aq/bcd \\ \sqrt{a},-\sqrt{a},aq/b,aq/c,aq/d \end{matrix}\right] \quad (3.3)$$

$$ {}_8\Phi_7\left[\begin{matrix} k,q\sqrt{k},-q\sqrt{k},kb/a,kc/a,c,d,aq/bc;q;aq/cd \\ \sqrt{k},-\sqrt{k},aq/b,aq/c,kq/c,kq/d,kbc/a \end{matrix}\right]$$

$$= \frac{(kq,kq/cd,aq/c,aq/d;q)_\infty}{(aq,aq/cd,kq/c,kq/d;q)_\infty} {}_8\Phi_7\left[\begin{matrix} a,q\sqrt{a},-q\sqrt{a},b,c,c,d,a^2q/bck;q;kq/cd \\ \sqrt{a},-\sqrt{a},aq/b,aq/c,aq/c,aq/d,bck/a \end{matrix}\right] \tag{3.4}$$

3.1.3 Proof of (3.1.2.1) to (3.1.2.4) :

Considering

$u_r = \frac{(k/a;q)_r}{(q;q)_r}$ & $v_r = \frac{(k;q)_r}{(aq;q)_r}$ in (1.1) & (1.2), of Bailey's transformation,

$$\beta_n = \sum_{r=0}^{n} \frac{(k/a;q)_{n-r}(k;q)_{n+r}}{(q;q)_{n-r}(aq;q)_{n+r}} \alpha_r \tag{4.1}$$

$$\gamma_n = \sum_{r=n}^{\infty} \frac{(k/a;q)_{r-n}(k;q)_{n+r}}{(q;q)_{r-n}(aq;q)_{n+r}} \delta_r \tag{4.2}$$

By applying (2.5) & (2.6) in (4.1)

$$\beta_n = \frac{(k/a;q)_n(k;q)_n}{(q;q)_n(aq;q)_n} \sum_{r=0}^{n} \frac{(kq^n;q)_r(q^{-n};q)_r}{(aq^{1-n}/k;q)_r(aq^{n+1};q)_r} \left(\frac{aq}{k}\right)^r \alpha_r \tag{4.3}$$

Now, replacing r by $r+n$ in (4.2)

$$\gamma_n = \sum_{r=0}^{\infty} \frac{(k/a;q)_r(k;q)_{r+2n}}{(q;q)_r(aq;q)_{r+2n}} \delta_{r+n}$$

By applying (2.5),

$$\gamma_n = \frac{(k;q)_{2n}}{(aq;q)_{2n}} \sum_{r=0}^{\infty} \frac{(k/a;q)_r(kq^{2n};q)_r}{(q;q)_r(aq^{1+2n};q)_r} \delta_{r+n} \tag{4.4}$$

Taking $\alpha_r = \left(\frac{k}{a}\right)^r$ in (4.3) of Bailey's transformation,

$$\beta_n = \frac{(k/a;q)_n(k;q)_n}{(q;q)_n(aq;q)_n} \sum_{r=0}^{n} \frac{(kq^n;q)_r(q^{-n};q)_r}{(aq^{1-n}/k;q)_r(aq^{n+1};q)_r} q^r$$

$$\beta_n = \frac{(k/a;q)_n(k;q)_n}{(q;q)_n(aq;q)_n} {}_3\Phi_2\left[\begin{matrix} kq^n,q,q^{-n};q;q \\ aq^{n+1},aq^{1-n}/k \end{matrix}\right]_n \tag{4.5}$$

By using known result (2.11),

$$\beta_n = \frac{(k/a;q)_n(k;q)_n(aq/k;q)_n(aq^n;q)_n}{(q;q)_n(aq;q)_n(aq^{n+1};q)_n(a/k;q)_n} \tag{4.6}$$

Now, taking $\delta_r = \left(\frac{a^2q}{k^2}\right)^r$ in (4.4),

$$\gamma_n = \frac{(k;q)_{2n}}{(aq;q)_{2n}} \left(\frac{a^2q}{k^2}\right)^n \sum_{r=0}^{\infty} \frac{(k/a;q)_r(kq^{2n};q)_r}{(q;q)_r(aq^{1+n};q)_r} \left(\frac{a^2q}{k^2}\right)^r$$

$$\gamma_n = \frac{(k;q)_{2n}}{(aq;q)_{2n}} \left(\frac{a^2q}{k^2}\right)^n {}_2\Phi_1\left[\begin{matrix} k/a,kq^{2n};q;a^2q/k^2 \\ aq^{1+2n} \end{matrix}\right] \tag{4.7}$$

By using known result (2.9),

$$\gamma_n = \frac{(k;q)_{2n}(aq/k;q)_\infty(a^2q/k;q)_\infty}{(aq;q)_{2n}(aq;q)_\infty(a^2q/k^2;q)_\infty} \left(\frac{a^2q}{k^2}\right)^n \tag{4.8}$$

By putting the value of $\alpha_n, \beta_n, \gamma_n and \delta_n$ in equation (1.3),

$$\sum_{n=0}^{\infty} \left(\frac{k}{a}\right)^n \frac{(k;q)_{2n}(aq/k;q)_\infty(a^2q/k;q)_\infty}{(aq;q)_{2n}(aq;q)_\infty(a^2q/k^2;q)_\infty} \left(\frac{a^2q}{k^2}\right)^n = \sum_{n=0}^{\infty} \frac{(k/a;q)_n(k;q)_n(aq/k;q)_n(aq^n;q)_n}{(q;q)_n(aq;q)_n(aq^{n+1};q)_n(a/k;q)_n} \left(\frac{a^2q}{k^2}\right)^n$$

$$\frac{(aq/k;q)_\infty(a^2q/k;q)_\infty}{(aq;q)_\infty(a^2q/k^2;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,kq;q^2)_n}{(aq,aq^2;q^2)_n}\left(\frac{aq}{k}\right)^n=\sum_{n=0}^{\infty}\frac{(k,k/a,aq/k,aq^n;q)_n}{(q,aq,aq^{n+1},a/k;q)_n}\left(\frac{a^2q}{k^2}\right)^n$$

$$\sum_{n=0}^{\infty}\frac{(k,kq;q^2)_n}{(aq,aq^2;q^2)_n}\left(\frac{aq}{k}\right)^n=\frac{(aq;q)_\infty(a^2q/k^2;q)_\infty}{(aq/k;q)_\infty(a^2q/k;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,k/a,aq/k,aq^n;q)_n}{(q,aq,aq^{n+1},a/k;q)_n}\left(\frac{a^2q}{k^2}\right)^n$$

$${}_3\Phi_2\left[\begin{matrix}k,kq,q^2;q^2;aq/k\\ aq\ ,\ aq^2\end{matrix}\right]=\frac{(aq;q)_\infty(a^2q/k^2;q)_\infty}{(aq/k;q)_\infty(a^2q/k;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,k/a,aq/k;q)_n(a;q^2)_n}{(q,a,a/k;q)_n(aq^2;q^2)_n}\left(\frac{a^2q}{k^2}\right)^n \tag{4.9}$$

This is main result (3.1)

Now, taking $\delta_r=\left(\frac{a^2}{k^2}\right)^r$ in (4.4),

$$\gamma_n=\frac{(k;q)_{2n}}{(aq;q)_{2n}}\left(\frac{a^2}{k^2}\right)^n\sum_{r=0}^{\infty}\frac{(k/a;q)_r(kq^{2n};q)_r}{(q;q)_r(aq^{1+n};q)_r}\left(\frac{a^2}{k^2}\right)^r$$

$$\gamma_n=\frac{(k;q)_{2n}}{(aq;q)_{2n}}\left(\frac{a^2}{k^2}\right)^n{}_2\Phi_1\left[\begin{matrix}k/a,kq^{2n};q;a^2/k^2\\ aq^{1+2n}\end{matrix}\right] \tag{4.10}$$

By using known result (2.10),

$$\gamma_n=\frac{(k;q)_{2n}(1+aq^{2n})}{(a^2q/k;q)_{2n}}\left(\frac{k}{k+a}\right)\frac{(aq/k,a^2q/k;q)_\infty}{(aq,a^2q/k^2;q)_\infty}\left(\frac{a^2}{k^2}\right)^n \tag{4.11}$$

By putting the values

$$\alpha_n=\left(\frac{k}{a}\right)^n,\qquad \beta_n=\frac{(k/a;q)_n(k;q)_n(aq/k;q)_n(aq^n;q)_n}{(q;q)_n(aq;q)_n(aq^{n+1};q)_n(a/k;q)_n},$$

$$\gamma_n=\frac{(k;q)_{2n}(1+aq^{2n})}{(a^2q/k;q)_{2n}}\left(\frac{k}{k+a}\right)\frac{(aq/k,a^2q/k;q)_\infty}{(aq,a^2q/k^2;q)_\infty}\left(\frac{a^2}{k^2}\right)^n\ \&\ \delta_n=\left(\frac{a^2}{k^2}\right)^n\ \text{in (1.3)},$$

$$\sum_{n=0}^{\infty}\left(\frac{k}{a}\right)^n\frac{(k;q)_{2n}(aq/k;q)_\infty(a^2q/k;q)_\infty(1+aq^{2n})k}{(aq;q)_{2n}(aq;q)_\infty(a^2q/k^2;q)_\infty(k+a)}\left(\frac{a^2}{k^2}\right)^n=\sum_{n=0}^{\infty}\frac{(k/a;q)_n(k;q)_n(aq/k;q)_n(aq^n;q)_n}{(q;q)_n(aq;q)_n(aq^{n+1};q)_n(a/k;q)_n}\left(\frac{a^2}{k^2}\right)^n$$

$$\frac{k(aq/k;q)_\infty(a^2q/k;q)_\infty}{(aq;q)_\infty(a^2q/k^2;q)_\infty(k+a)}\sum_{n=0}^{\infty}\frac{(1+aq^{2n})(k,kq;q^2)_n}{(aq,aq^2;q^2)_n}\left(\frac{a}{k}\right)^n=\sum_{n=0}^{\infty}\frac{(k,k/a,aq/k,aq^n;q)_n}{(q,aq,aq^{n+1},a/k;q)_n}\left(\frac{a^2}{k^2}\right)^n$$

$$\sum_{n=0}^{\infty}\frac{(1+aq^{2n})(k,kq;q^2)_n}{(a^2q/k,a^2q^2/k;q^2)_n}\left(\frac{a}{k}\right)^n=\frac{(aq;q)_\infty(a^2q/k^2;q)_\infty(k+a)}{k(aq/k;q)_\infty(a^2q/k;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,k/a,aq/k,aq^n;q)_n}{(q,aq,aq^{n+1},a/k;q)_n}\left(\frac{a^2}{k^2}\right)^n$$

$${}_3\Phi_2\left[\begin{matrix}k,kq,q^2;q^2;a/k\\ a^2q/k\ ,a^2q^2/k\end{matrix}\right]+a\,{}_3\Phi_2\left[\begin{matrix}k,kq,q^2;q^2;aq^2/k\\ a^2q/k\ ,a^2q^2/k\end{matrix}\right]$$

$$=\frac{(aq;q)_\infty(a^2q/k^2;q)_\infty(k+a)}{k(aq/k;q)_\infty(a^2q/k;q)_\infty}\sum_{n=0}^{\infty}\frac{(k,k/a,aq/k;q)_n(a;q^2)_n}{(q,a,a/k;q)_n(aq^2;q^2)_n}\left(\frac{a^2}{k^2}\right)^n \tag{4.12}$$

This is main result (3.2)

Now, by taking $\alpha_r=\frac{(a,q\sqrt{a},-q\sqrt{a},b;q)_r}{(q,\sqrt{a},-\sqrt{a},aq/b;q)_r b^r}$ in (4.1) ,

$$\beta_n=\frac{(k,k/a;q)_n}{(q,aq;q)_n}\sum_{r=0}^{n}\frac{(a,q\sqrt{a},-q\sqrt{a},b,kq^n,q^{-n};q)_r}{(q,\sqrt{a},-\sqrt{a},aq/b,aq^{1-n}/k,aq^{n+1};q)_r}\left(\frac{aq}{bk}\right)^r$$

$$\beta_n=\frac{(k,k/a;q)_n}{(q,aq;q)_n}{}_6\Phi_5\left[\begin{matrix}a,q\sqrt{a},-q\sqrt{a},b,kq^n,q^{-n};q;aq/bk\\ \sqrt{a},-\sqrt{a},aq/b,aq^{1-n}/k,aq^{n+1}\end{matrix}\right]_n \tag{4.13}$$

By using known result (2.12),

$$\beta_n = \frac{(k,kb/a;q)_n}{(q,aq/b;q)_n b^n} \tag{4.14}$$

By taking $\delta_r = \frac{(q\sqrt{k},-q\sqrt{k},c,d;q)_r}{(\sqrt{k},-\sqrt{k},kq/c,kq/d;q)_r}\left(\frac{aq}{cd}\right)^r$ in (4.4) ,

$$\gamma_n = \frac{(k;q)_{2n}}{(aq;q)_{2n}}\frac{(c,d;q)_n}{(kq/c,kq/d;q)_n}\left(\frac{1-kq^{2n}}{1-k}\right)\left(\frac{aq}{cd}\right)^n \sum_{r=0}^{\infty}\frac{(kq^{2n},q^{n+1}\sqrt{k},-q^{n+1}\sqrt{k},cq^n,dq^n,k/a;q)_r}{(q,q^n\sqrt{k},-q^n\sqrt{k},kq^{n+1}/c,kq^{n+1}/d,aq^{1+2n};q)_r}\left(\frac{aq}{cd}\right)^r$$

$$\gamma_n = \frac{(k;q)_{2n}}{(aq;q)_{2n}}\frac{(c,d;q)_n}{(kq/c,kq/d;q)_n}\left(\frac{aq}{cd}\right)^n\left(\frac{1-kq^{2n}}{1-k}\right){}_6\Phi_5\left[\begin{matrix}kq^{2n},q^{n+1}\sqrt{k},-q^{n+1}\sqrt{k},cq^n,dq^n,k/a;q;aq/cd\\ q^n\sqrt{k},-q^n\sqrt{k},kq^{n+1}/c,kq^{n+1}/d,aq^{1+2n}\end{matrix}\right] \tag{4.15}$$

By using known result (2.13),

$$\gamma_n = \frac{(kq,kq/cd,aq/c,aq/d;q)_\infty}{(aq,aq/cd,kq/c,kq/d;q)_\infty}\frac{(c,d;q)_n}{(aq/c,aq/d;q)_n}\left(\frac{aq}{cd}\right)^n \tag{4.16}$$

By putting the values $\alpha_n = \frac{(a,q\sqrt{a},-q\sqrt{a},b;q)_n}{(q,\sqrt{a},-\sqrt{a},aq/b;q)_n b^n}$, $\beta_n = \frac{(k,kb/a;q)_n}{(q,aq/b;q)_n b^n}$,

$\gamma_n = \frac{(kq,kq/cd,aq/c,aq/d;q)_\infty}{(aq,aq/cd,kq/c,kq/d;q)_\infty}\frac{(c,d;q)_n}{(aq/c,aq/d;q)_n}\left(\frac{aq}{cd}\right)^n$ & $\delta_n = \frac{(q\sqrt{k},-q\sqrt{k},c,d;q)_n}{(\sqrt{k},-\sqrt{k},kq/c,kq/d;q)_n}\left(\frac{aq}{cd}\right)^n$ in (1.3),

$$\frac{(kq,kq/cd,aq/c,aq/d,;q)_\infty}{(aq,aq/cd,kq/c,kq/d,;q)_\infty}\sum_{n=0}^{\infty}\frac{(a,q\sqrt{a},-q\sqrt{a},b,c,d;q)_n}{(q,\sqrt{a},-\sqrt{a},aq/b,aq/c,aq/d;q)_n}\left(\frac{aq}{bcd}\right)^n$$

$$= \sum_{n=0}^{\infty}\frac{(k,kb/a,q\sqrt{k},-q\sqrt{k},b,c,d;q)_n}{(q,aq/k,\sqrt{k},-\sqrt{k},kq/b,kq/c;q)_n}\left(\frac{aq}{bcd}\right)^n$$

$${}_6\Phi_5\left[\begin{matrix}k,q\sqrt{k},-q\sqrt{k},,c,d,kb/a;q;aq/bcd\\ \sqrt{a},-\sqrt{a},aq/b,kq/c,kq/d\end{matrix}\right] = \frac{(kq,kq/cd,aq/c,aq/d,;q)_\infty}{(aq,aq/cd,kq/c,kq/d,;q)_\infty}{}_6\Phi_5\left[\begin{matrix}a,q\sqrt{a},-q\sqrt{a},b,c,d;q;aq/bcd\\ \sqrt{a},-\sqrt{a},aq/b,aq/c,aq/d\end{matrix}\right] \tag{4.17}$$

This is main result (3.3).

Now, by taking $\alpha_r = \frac{(a,q\sqrt{a},-q\sqrt{a},b,c,a^2q/bck;q)_r}{(q,\sqrt{a},-\sqrt{a},aq/b,aq/c,bck/a;q)_r}\left(\frac{k}{a}\right)^r$ in (4.1), we get

$$\beta_n = \frac{(k,k/a;q)_n}{(q,aq;q)_n}\sum_{r=0}^{n}\frac{(a,q\sqrt{a},-q\sqrt{a},b,c,kq^n,q^{-n},a^2q/bck;q)_r}{(q,\sqrt{a},-\sqrt{a},aq/b,aq/c,aq^{1-n}/k,aq^{n+1},bck/a;q)_r}q^r$$

$$\beta_n = \frac{(k,k/a;q)_n}{(q,aq;q)_n}{}_8\phi_7\left[\begin{matrix}a,q\sqrt{a},-q\sqrt{a},b,c,kq^n,q^{-n},a^2q/bck;q;q\\ \sqrt{a},-\sqrt{a},aq/b,aq/c,aq^{1-n}/k,aq^{n+1},bck/a\end{matrix}\right]_n \tag{4.18}$$

By using known result (2.14), we get

$$\beta_n = \frac{(k,kb/a,kc/a,aq/bc;q)_n}{(q,aq/b,aq/c,kbc/a;q)_n} \tag{4.19}$$

By putting the values $\alpha_r = \frac{(a,q\sqrt{a},-q\sqrt{a},b,c,a^2q/bck;q)_r}{(q,\sqrt{a},-\sqrt{a},aq/b,aq/c,bck/a;q)_r}\left(\frac{k}{a}\right)^r$, $\beta_n = \frac{(k,kb/a,kc/a,aq/bc;q)_n}{(q,aq/b,aq/c,kbc/a;q)_n}$,

$\gamma_n = \frac{(kq,kq/cd,aq/c,aq/d;q)_\infty}{(aq,aq/cd,kq/c,kq/d;q)_\infty}\frac{(c,d;q)_n}{(aq/c,aq/d;q)_n}\left(\frac{aq}{cd}\right)^n$ & $\delta_n = \frac{(q\sqrt{k},-q\sqrt{k},c,d;q)_n}{(\sqrt{k},-\sqrt{k},kq/c,kq/d;q)_n}\left(\frac{aq}{cd}\right)^n$ in (1.3),

$$\frac{(kq,kq/cd,aq/c,aq/d;q)_\infty}{(aq,aq/cd,kq/c,kq/d;q)_\infty}\sum_{n=0}^{\infty}\frac{(a,q\sqrt{a},-q\sqrt{a},b,c,c,d,a^2q/bck;q)_n}{(q,\sqrt{a},-\sqrt{a},aq/b,aq/c,aq/c,aq/d,bck/a;q)_n}\left(\frac{kq}{cd}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{(k, q\sqrt{k}, -q\sqrt{k}, kb/a, kc/a, c, d, aq/bc; q)_n}{(q, \sqrt{k}, -\sqrt{k}, aq/b, aq/c, kq/c, kq/d, kbc/a; q)_n} \left(\frac{aq}{cd}\right)^n$$

$${}_8\Phi_7\left[\begin{matrix} k, q\sqrt{k}, -q\sqrt{k}, kb/a, kc/a, c, d, aq/bc; q; aq/cd \\ \sqrt{k}, -\sqrt{k}, aq/b, aq/c, kq/c, kq/d, kbc/a \end{matrix}\right]$$

$$= \frac{(kq, kq/cd, aq/c, aq/d; q)_\infty}{(aq, aq/cd, kq/c, kq/d; q)_\infty} {}_8\Phi_7\left[\begin{matrix} a, q\sqrt{a}, -q\sqrt{a}, b, c, c, d, a^2q/bck; q; kq/cd \\ \sqrt{a}, -\sqrt{a}, aq/b, aq/c, aq/c, aq/d, bck/a \end{matrix}\right] \quad (4.20)$$

This is main result (3.4).

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