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MODULAR MULTIPLICATIVE DIVISOR LABELING OF VARIOUS GRAPHS

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Abstract :-In this paper we investigate modular multiplicative divisor labeling of various graphs. We prove that triangular book with n-pages, triangular snake TS_n and the graph obtained by duplication of every edge by a vertex in cycle C_n admit modular multiplicative divisor labeling. We also prove that the graph obtained by switching of end vertices in path P_n and switching of vertex in cycle C_n admits modular multiplicative divisor labeling except $n \equiv 0 (mod\ 3)$

Keywords: *Modular multiplicative divisor labeling, triangular book, triangular snake, duplication of edge by vertex, switching of a vertex.*

Introduction

In this research article, by a graph, we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)| = p$ and size $|E(G)| = q$. For any undefined term in graph theory, we refer to Gross and Yellen[3].

Many branches of mathematics such as statistics, algebra, geometry and topology have close correlation with graph theory. There are many potential fields of research in graph theory. Algebraic graph theory, domination in graphs, algorithmic graph theory, energy of graphs and labeling of graphs are among them.

Definition 1.1. A *graph labeling* is an assignment of numbers to the vertices or edges or both subject to certain condition(s). Graph labeling can be useful in social network, electrical circuit theory and energy crisis management, coding theory, mobile location tracking, the spread of disease, data processing and many more[6]. A latest survey of all the graph labeling techniques can be found in Gallian[2].

Definition 1.2. A graph with p vertices is said to be *strongly multiplicative* if the vertices of graph can be labelled with distinct integer $1,2,\dots,p$ such that the label induced on the edges by the product of end vertices are distinct.
In 2001 Beineke and Hegde[1] introduced strongly multiplicative labeling

Definition 1.3. A graph with p vertices is said to be *modular multiplicative* if the vertices of the graph can be labelled with distinct integers $0,1,2,\dots,p-1$ such that the label induced on the edges by the product of end vertices modulo p are distinct.
In 2013 Krawec[5] introduced modular multiplicative labeling.

Definition 1.4. The *modular multiplicative divisor labeling* of a graph $G = (V(G), E(G))$ with p vertices is a bijection f from the vertices of G to the set of positive integers $\{1,2,3,\dots,p\}$
and the labels induced on the edges by the product of labels of end vertices modulo p such that
p divides sum of all edge labels of G. The graph admits modular multiplicative divisor labeling is called *modular multiplicative divisor (MMD) graph.*
Revathi and Rajeswari[7] introduced modular multiplicative divisor labeling.

R. Revathi et.al.[7, 8] proved the following results:

- Path $P_n (n > 1)$ admits modular multiplicative divisor labeling.
- The shadow graph of a path $D_2(P_n)$ admits modular multiplicative divisor labeling.
- The cycle graph C_n (n is not multiple of 3) admits modular multiplicative divisor labeling.
- The split graph of cycle C_n admits modular multiplicative divisor labeling.
- The helm graph H_n admits modular multiplicative divisor labeling.
- The star graph S_n admits modular multiplicative divisor labeling.

- The split graph of star graph $spl\ (S_n)$ admits modular multiplicative divisor labeling.
- The shadow graph of star graph $D_2(S_n)$ admits modular multiplicative divisor labeling.
- The shadow graph of bistar $B_{m,n}$ admits modular multiplicative divisor labeling.
- The square graph of bistar $B_{n,n}$ admits modular multiplicative divisor labeling.

Definition 1.5. The *triangular book with n-pages* is defined as *n* copies of cycle C_3 sharing a common edge. The common edge is called spine or base of the book. This graph is denoted by $B(3, n)$.
Definition 1.6. The *triangular snake* TS_n is obtained from the path$P_n$byreplacing every edge of the path by the triangle C_3.
Definition 1.7. The *duplication of an edge* $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.
Definition 1.8. The *switching of a vertex v* in a graph G means removing all the edges incident to *v* and adding edges joining *v* to every other vertex which are not adjacent to *v* in G. The graph obtained by switching of a vertex *v* in a graph G is denoted by G_v.

Main Theorems

Theorem 2.1 *The triangular book graph* $B(3, n)$ *with n-pages admits modular multiplicative divisor labeling for even n.*

Proof: Let $G = B(3, n)$ be the triangular book graph with *n*-pages. Let u_0 and v_0 be spine vertices. Let $v_1, v_{2,\dots,}v_n$ be the *n*-vertices and $E = E_1 \cup E_2 \cup E_3$ as the edge set where;
$E_1 = \left\{u_0 v_i : 1 \le i \le n+1; i \ne \dfrac{n+2}{2}\right\}$
$E_2 = \left\{v_0 v_i : 1 \le i \le n+1; i \ne \dfrac{n+2}{2}\right\}$
$E_3 = \{u_0 v_0\}$
It is noted that $|V\ (G)| = n + 2$ and $|E\ (G)| = 2n + 1$.

The vertex labeling $f: V(G) \to \{1,2, \dots, n + 2\}$ is defined as follows:
$f(v_0) = \frac{n+2}{2}$;
$f(u_0) = n + 2$;
$f(v_i) = i;\ 1 \le i \le n + 1; i \ne \frac{n+2}{2}$

Then induced function $f^*: E(G) \to \{0,1,2, \dots, n + 1\}$ define as
$f^*(uv) = f(u)f(v)(mod\ (n + 2))$ for each edge $uv \in E(G)$

Let the sum of all the edge labels be S then,

$$S = \sum_{i=1, i\ne\frac{n+2}{2}}^{n+1} [f^*(u_0 v_i) + f^*(v_0 v_i)] + f^*(u_0 v_0)$$

$$= \sum_{i=1, i\ne\frac{n+2}{2}}^{n+1} [f(u_0)f(v_i) + f(v_0)f(v_i)] + f(u_0)f(v_0)\ \ (mod\ (n + 2))$$

$$= \sum_{i=1, i\ne\frac{n+2}{2}}^{n+1} \left[\frac{n+2}{2}\ i + (n + 2)i\right] + \left(\frac{n+2}{2}\right)(n + 2)\ \ (mod\ (n + 2))$$

$$= \left(\frac{3(n+2)}{2}\right) \sum_{i=1, i\ne\frac{n+2}{2}}^{n+1} (i) + \left(\frac{n+2}{2}\right)(n + 2)\ \ (mod(n + 2))$$

$$= (n+2)\left[\frac{3}{2}\sum_{i=1, i\neq\frac{n+2}{2}}^{n+1}(i) + \left(\frac{n+2}{2}\right)\right] \ (mod\ (n+2))$$

$$= (n+2)\left[\frac{3(n+1)(n+2)}{2} - \left(\frac{n+2}{2}\right) + \left(\frac{n+2}{2}\right)\right] \ (mod\ (n+2))$$

$$= (n+2)\left[\frac{3(n+1)(n+2)}{2}\right] \ (mod\ (n+2))$$
$$\equiv 0 \ (mod\ (n+2))$$

That is, the triangular book graph $B(3, n)$ with n-pages admits modular multiplicative divisor labeling.

Illustration 2.2 The triangular book graph$B(3,6)$ with 6-pages and its modular multiplicative divisor labeling is shown in Figure 1.

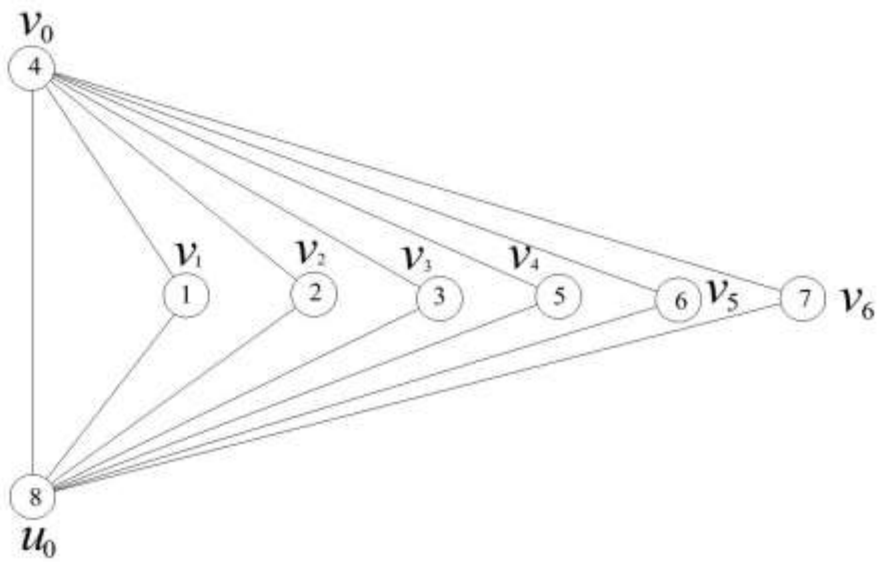

Figure 1

Theorem 2.3 The triangular snake TS_n admits modular multiplicative divisor labeling except $2n - 1 \equiv 0(mod\ 3)$.
Proof: Let $G = TS_n$ be the triangular snake obtained from the path P_n. Let $\{v_1, v_{2,\dots,}v_n\}$ be the vertices of path P_n. Let $\{u_1, u_{2,\dots,}u_{n-1}\}$ be the newly added vertices to replace every edge in the path P_n by cycle C_3. The edge set $E = E_1 \cup E_2 \cup E_3$ where;
$E_1 = \{v_i v_{i+1}: 1 \leq i \leq n-1\}$
$E_2 = \{u_i v_i: 1 \leq i \leq n-1\}$
$E_3 = \{u_i v_{i+1}: 1 \leq i \leq n-1\}$
It is noted that $|V\ (G)| = 2n - 1$ and $|E\ (G)| = 3n - 3$.

The vertex labeling $f: V(G) \to \{1,2, \dots, 2n-1\}$ is defined as follows:
$f(u_i) = i;\quad 1 \leq i \leq n-1.$
$f(v_i) = 2n - i;\quad 1\leq i \leq n.$

Then induced function $f^*: E(G) \to \{0,1,2, \dots, 2n-2\}$ define as; $f^*(uv) = f(u)f(v)\ \ (mod\ (2n-1))$ for each edge $uv \in E(G)$

Let the sum of all edge labels be S then,

$$S = \sum_{i=1}^{n-1}[f^*(v_i v_{i+1})] + \sum_{i=1}^{n-1}[f^*(u_i v_i) + f^*(u_i v_{i+1})$$

$$= \sum_{i=1}^{n-1}[f(v_i)f(v_{i+1})] + \sum_{i=1}^{n-1}[f(u_i)f(v_i) + f(u_i)f(v_{i+1}) \ (mod \ (2n-1))$$

$$= \sum_{i=1}^{n-1}[(2n-i)(2n-i-1)] + \sum_{i=1}^{n-1}[(i)(2n-i) + (i)(2n-i-1)] \ (mod \ (2n-1))$$

$$= \sum_{i=1}^{n-1}[(2n-i+i)(2n-i-1)] + \sum_{i=1}^{n-1}[(i)(2n-i)] \ (mod \ (2n-1))$$

$$= (2n)\sum_{i=1}^{n-1} \left((2n-1)-i\right) + \sum_{i=1}^{n-1}(2ni-i^2) \ \left(mod \ (2n-1)\right)$$

$$= (2n)(2n-1)(n-1) - (2n)\sum_{i=1}^{n-1} i + \ (2n)\sum_{i=1}^{n-1} i - \frac{(n-1)(n)(2n-1)}{6} \ (mod \ (2n-1))$$

$$= \frac{n(n-1)}{6}[(12n)(2n-1) - 2n + 1] \ (mod \ (2n-1))$$

$$= \frac{11n(n-1)(2n-1)}{6} \ (mod \ (2n-1))$$

$$\equiv 0 \ (mod \ (2n-1))$$

That is, the triangular snake TS_n admits modular multiplicative divisor labeling except $2n-1 \equiv 0 (mod \ 3)$.

Illustration 2.4(a) The triangular snake TS_6 and its modular multiplicative divisor labeling is shown in Figure 2.

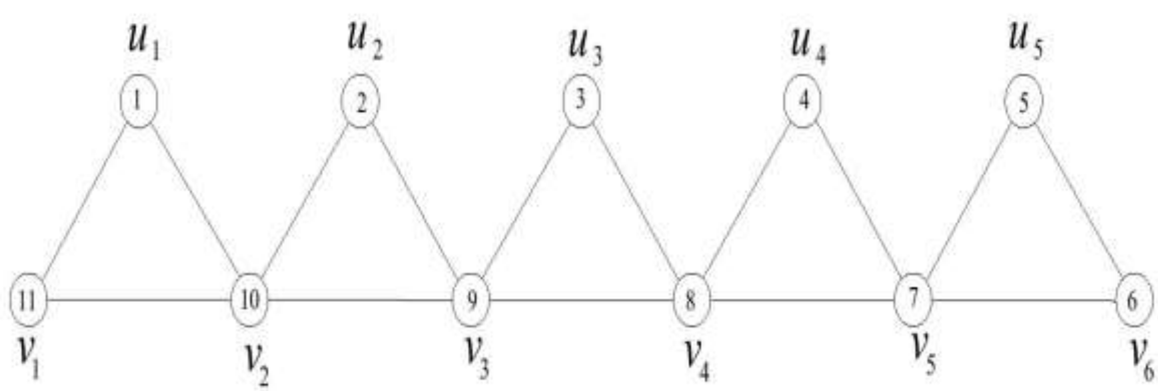

Figure 2

Illustration 2.4(b) The triangular snake TS_7 and its modular multiplicative divisor labeling is shown in Figure 3.

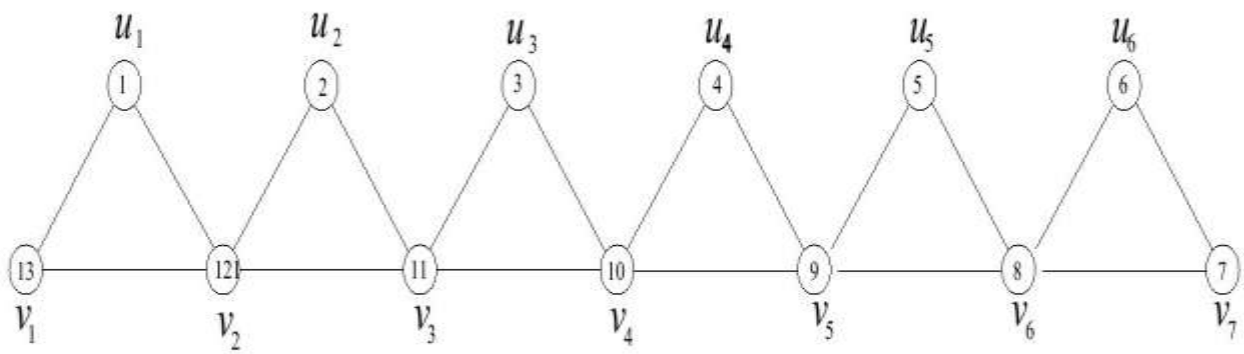

Figure 3

Theorem 2.5 The graph obtained by duplication of every edge by a vertex in cycle C_n admits modular multiplicative divisor labeling except $n \equiv 0(mod\ 3)$.

Proof: Let $\{v_1, v_{2,\dots,} v_n\}$ be the vertices of cycle C_n. Let $\{u_1, u_{2,\dots,} u_n\}$ be the added vertices for duplication of each edge in cycle C_n. Let G be the graph obtained by duplication of every edge by a vertex in cycle C_n and the edge set $E = E_1 \cup E_2 \cup E_3$ where;

$E_1 = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_n v_1\}$
$E_2 = \{u_i v_{i+1} : 2 \le i \le n\}$
$E_3 = \{u_i v_i : 1 \le i \le n\} \cup \{u_1 v_n\}$
It is noted that $|V\ (G)| = 2n$ and $|E\ (G)| = 3n$.

The vertex labeling $f: V(G) \rightarrow \{1,2,\dots,2n\}$ is defined as follows:
$f(u_i) = 2i - 1;\ \ 1 \le i \le n.$
$f(v_i) = 2i;\ \ \ \ \ 1 \le i \le n.$

Then induced function $f^*: E(G) \rightarrow \{0,1,2,\dots,2n-1\}$ define as;
$f^*(uv) = f(u)f(v)(mod\ 2n)$ for each edge $uv \in E(G)$

Let the sum of all edge labels be S then,

$$S = \sum_{i=1}^{n-1}[f^*(v_i v_{i+1})] + f^*(v_n v_1) + \sum_{i=1}^{n}[f^*(u_i v_i) + \sum_{i=2}^{n} f^*(u_i v_{i-1}) + f^*(u_1 v_n)$$

$$= \sum_{i=1}^{n-1}[f(v_i)f(v_{i+1})] + f(v_n)f(v_1) + \sum_{i=1}^{n}[f(u_i)f(v_i) + \sum_{i=2}^{n} f(u_i)f(v_{i-1}) + f(v_n)f(u_1) \quad (mod\ 2n)$$

$$= \sum_{i=1}^{n-1}(2i)(2i+2) + (2n)(2) + \sum_{i=1}^{n}(2i-1)(2i) + \sum_{i=2}^{n}(2i-1)(2i-2) + (1)(2n) \quad (mod\ 2n)$$

$$= 4\sum_{i=1}^{n-1}(i^2 + i) + (4n) + 2\sum_{i=1}^{n}(2i^2 - i1) + 2\sum_{i=2}^{n}(2i-1)(i-1) + (2n) \quad (mod\ 2n)$$

$$= 4\left(\frac{(n-1)(n)(2(n-1)+1)}{6} + \frac{(n-1)(n)}{2}\right) + 2\ \left(\frac{2n(n+1)(2n+1)}{6} - \frac{(n)(n+1)}{2}\right)$$

$$+ 2\left(\frac{2n(n+1)(2n+1)}{6} - \frac{3n(n+1)}{2} + n\right) + 6n \quad (mod\ 2n).$$

$$= 4n\left(\frac{(n-1)(2n-1)}{6} + \frac{(n-1)}{2}\right) + 2\text{n}\ \left(\frac{2(n+1)(2n+1)}{6} - \frac{(n+1)}{2}\right)$$

$$+2n\left(\frac{2(n+1)(2n+1)}{6} - \frac{3(n+1)}{2} + 1\right) + 6n \quad (mod\ 2n).$$

$$= 4n\left(\frac{(n-1)}{6}(2n-1+3)\right) + 2\text{n} \ \left(\frac{(n+1)}{6}(4n+2-3)\right)$$

$$+2n\left(\frac{(n+1)}{6}(4n+2-9+6)\right) + 6n \ \ (mod\ 2n).$$

$$= 4n\left(\frac{(n-1)(2n+2)}{6}\right) + 2\text{n} \ \left(\frac{(n+1)(4n-1)}{6}\right) + 6n \ \ (mod\ 2n).$$

$$= 4n\left(\frac{(n^2-1)}{3}\right) + 2\text{n} \ \left(\frac{(n+1)(4n-1)}{6}\right) + 6n \ \ (mod\ 2n).$$

$$\equiv 0\ (mod\ 2n)$$

That is, the graph obtained by duplication of every edge by a vertex in cycle C_n admits modular multiplicative divisor labeling except $n \equiv 0(mod\ 3)$.

Illustration 2.6 The graph obtained by duplication of every edge by a vertex in cycle C_5 and its modular multiplicative divisor labeling is shown in Figure 4.

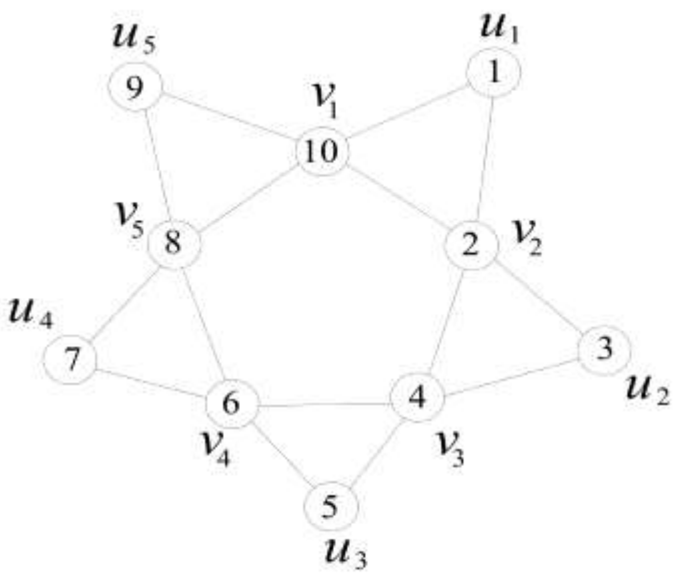

Figure 4

Theorem 2.7 The graph obtained by switching of pendant vertex in path P_n admits modular multiplicative divisor labeling except $n \equiv 0(mod\ 3)$.

Proof: Let $\{v_0, v_{1,\dots,}v_{n-1}\}$ be the vertices of path P_n. Without loss of generality let the switched vertex be v_0. Let G be the graph obtained by switching of pendant vertex in path P_n.Then the vertex set of G is $\{v_0, v_{1,\dots,}v_{n-1}\}$ and the edge set $E = E_1 \cup E_2$ where;

$E_1 = \{v_0v_i: 2 \le i \le n-1\}$

$E_2 = \{v_iv_{i+1}: 1 \le i \le n-1\}$

It is noted that $|V\ (G)| = n$ and $|E\ (G)| = 2n-3$.

The vertex labeling $f: V(G) \to \{1,2,\dots,n\}$ is defined as follows:

$f(v_0) = n;$

$f(v_i) = i; \qquad 1 \le i \le n-1.$

Then induced function $f^*: E(G) \to \{0,1,2,\dots,n-1\}$ define as;

$f^*(uv) = f(u)f(v)(mod\ n)$ for each edge $uv \in E(G)$.

Let the sum of all edge labels be S then,

$$S = \sum_{i=2}^{n-1}[f^*(v_0v_i)] + \sum_{i=1}^{n-2}[f^*(v_iv_{i+1})]$$

$$= \sum_{i=2}^{n-1}[f(v_0)f(v_i)] + \sum_{i=1}^{n-2}[f(v_i)f(v_{i+1})]\ (mod\ n)$$

$$= \sum_{i=2}^{n-1}(ni) + \sum_{i=1}^{n-2}(i)(i+1)\ \ (mod\ n)$$

$$= n\sum_{i=2}^{n-1} i + \sum_{i=1}^{n-2}(i^2) + \sum_{i=1}^{n-2}(i)\ \ (mod\ n)$$

$$= n\left[\frac{n(n-1)}{2} - 1\right] + \frac{(n-2)(n-1)(2n-3)}{6} + \frac{(n-1)(n-2)}{2}\quad (mod\ n)$$

$$= \frac{n}{2}[n^2 - n - 2] + \frac{(n-2)(n-1)}{6}[2n - 3 + 3]\quad (mod\ n)$$

$$= \frac{n}{2}(n-2)(n-1) + \frac{n(n-2)(n-1)}{3}\quad (mod\ n)$$

$$= \frac{n(n-2)}{6}[3(n+1) + 2(n-1)]\quad (mod\ n)$$

$$= \frac{n(n-2)(5n+1)}{6}\quad (mod\ n)$$

$$\equiv 0\quad (mod\ n)$$

That is, the graph obtained by switching of pendant vertex in path P_n admits modular multiplicative divisor labeling except $n \equiv 0(mod\ 3)$.

Illustration 2.8 The graph obtained by switching of pendant vertex in path P_7 and its modular multiplicative divisor labeling is shown in Figure 5.

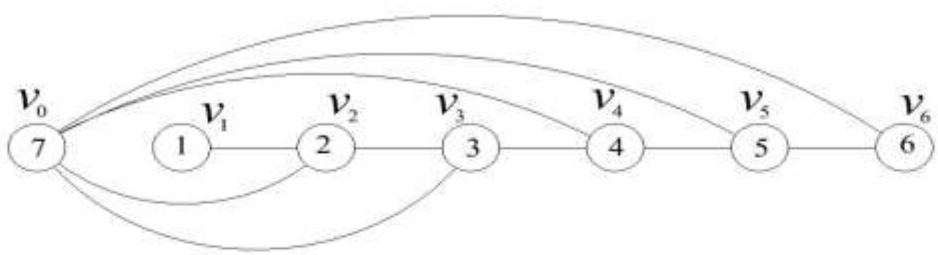

Figure 5

Theorem 2.9 The graph obtained by switching of a vertex in cycle C_n admits modular multiplicative divisor labeling except $n \equiv 0(mod\ 3)$.

Proof: Let $\{v_0, v_{1,\dots,}v_{n-1}\}$ be the vertices of cycle C_n. Without loss of generality let the switched vertex be v_0. Let G be the graph obtained by switching a vertex in cycle C_n.Then the vertex set of G is $\{v_0, v_{1,\dots,}v_{n-1}\}$ and the edge set $E = E_1 \cup E_2$ where;

$E_1 = \{v_0v_i\colon\ 1 \le i \le n-2\}$

$E_2 = \{v_iv_{i+1}\colon 2 \le i \le n-1\}$

It is noted that $|V\ (G)| = n$ and $|E\ (G)| = 2n - 4$.

The vertex labeling $f\colon V(G) \to \{1,2,\dots,n\}$ is defined as follows:

$f(v_0) = n;$

$f(v_i) = i;\qquad 1 \le i \le n-1.$

Then induced function $f^*\colon E(G) \to \{0,1,2,\dots,n-1\}$ define as;

$f^*(uv) = f(u)f(v)(mod\ n)$ for each edge $uv \in E(G)$.

Let the sum of all edge labels be S then,

$$S = \sum_{i=2}^{n-2} [f^*(v_0 v_i)] + \sum_{i=1}^{n-2} [f^*(v_i v_{i+1})]$$

$$= \sum_{i=2}^{n-2} [f(v_0) f(v_i)] + \sum_{i=1}^{n-2} [f(v_i) f(v_{i+1})] \ (mod\ n)$$

$$= \sum_{i=2}^{n-2} (ni) + \sum_{i=1}^{n-2} (i)(i+1) \ (mod\ n)$$

$$= n\left[\sum_{i=2}^{n-2} i - 1\right] + \sum_{i=1}^{n-2} (i^2) + \sum_{i=1}^{n-2} (i) \ (mod\ n)$$

$$= n\left[\frac{(n-2)(n-1)}{2} - 1\right] + \frac{(n-2)(n-1)[2(n-2)+1]}{6} + \frac{(n-1)(n-2)}{2} \ (mod\ n)$$

$$= \frac{(n-2)(n-1)}{6}[3n + 2n - 3 + 3] - n \ (mod\ n)$$

$$= \frac{5n(n-2)(n-1)}{6} - n \ (mod\ n)$$

$$= \frac{5n(n^2 - 3n + 2) - 6n}{6} \ (mod\ n)$$

$$= \frac{n}{6} \ (5n^2 - 15n + 10 - 6) \ (mod\ n)$$

$$= \frac{n}{6} \ (5n^2 - 15n + 4) \ (mod\ n)$$

$$\equiv 0 \ (mod\ n).$$

That is, the graph obtained by switching of a vertex in cycle C_n admits modular multiplicative divisor labeling except $n \equiv 0 (mod\ 3)$.

Illustration 2.10 The graph by switching of a vertex in cycle C_7 and its modular multiplicative divisor labeling is shown in

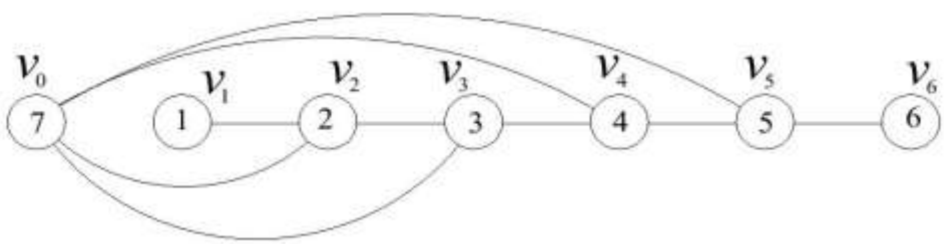

Figure 6.

Figure 6

Conclusion:

Here, we have derived some new results related to modular multiplicative divisor labeling.

To derive similar results for other graph families is an open problem.

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