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VARIOUS GRAPH LABELING TECHNIQUES FOR THE LINE GRAPH OF BISTAR

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Abstract— The line graph $L(G)$ of a graph G is the graph whose vertex set is edge set of G and two vertices are adjacent in $L(G)$ whenever they are incident in G. In this paper an investigation on the line graph of bistar $B_{n,n}$ for cordial labeling, product cordial labeling, total product cordial labeling, edge product cordial labeling and total edge product cordial labeling has been done.

Keywords— Line Graph, Bistar, Cordial Labeling.
2010 Mathematics Subject Classification: 05C78.

1 INTRODUCTION

In the present work, all graphs under consideration are finite, connected, undirected and simple $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$. For any undefined notation and terminology related to graph theory Gross and Yellen[4] is referred while for number theory Burton[1] is referred.

Many graph labeling techniques have been introduced so far and explored as well by many researchers. These techniques have massive applications not only in mathematics but in several areas of computer science and communication networks. A dynamic survey on various graph labeling problems with a wide-ranging bibliography can be found in Gallian[3].

Definition 1.1 If the vertices or edges or both are assigned numbers subject to certain condition(s) then it is known as *graph labeling.*

Definition 1.2 A mapping $f: V(G) \to \{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the label of the vertex v of G under f.

Notations 1.3 If for an edge $e = uv$, the induced edge labeling $f^*: E(G) \to \{0,1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$. Then
$v_f(i) =$ number of vertices of G having label i under f,
$e_f(i) =$ number of edges of G having label i under f^*.

Definition 1.4 A binary vertex labeling f of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits cordial labeling is called a *cordial graph*.

The concept of cordial labeling was introduced by Cahit[2] as a weaker version of graceful and harmonious labeling in *1987*. In the same paper Cahit has proved that all trees, complete graph K_n (only for $n \leq 3$) and complete bipartite graph $K_{m,n}$ are cordial. Vaidya and Shah[10] have proved that shadow graph $D_2(B_{n,n})$, splitting graph $S'(B_{n,n})$ and degree splitting graph $DS(B_{n,n})$ of bistar $B_{n,n}$ are cordial graphs.

Definition 1.5 *Bistar* $B_{n,n}$ is the graph obtained by joining the center(apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition 1.6 The line graph $L(G)$ of a graph G is the graph whose vertices are the edges of G, with $ef \in E(L(G))$ when $e = uv$ and $f = vw$ in G.

Illustration: Bistar $B_{7,7}$ and its line graph $L(B_{7,7})$ are shown in Figure 1 and Figure 2 respectively.

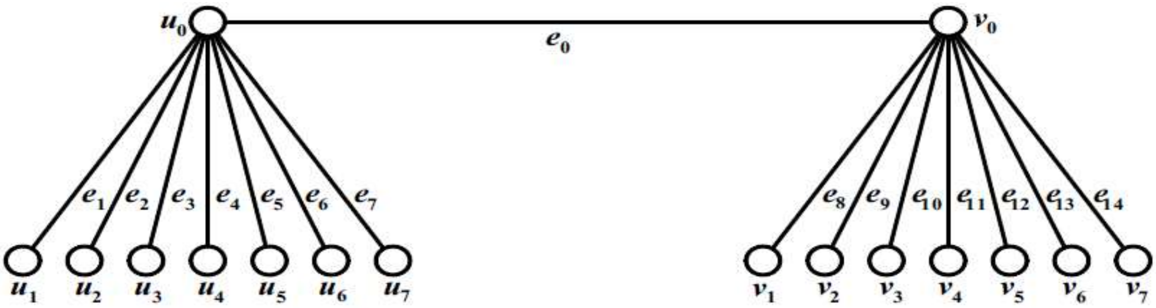

Figure 1: Bistar $B_{7,7}$.

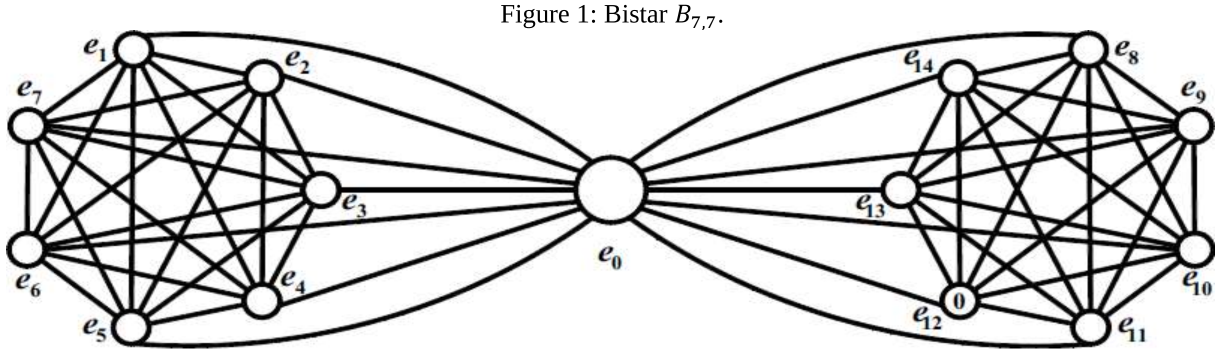

Figure 2: $L(B_{7,7})$ Line Graph of Bistar $B_{7,7}$.

Some characteristic features of line graph $L(B_{n,n})$ of bistar $B_{n,n}$ are:

- $L(B_{n,n})$ is isomorphic to $2K_n + K_1$.
- The vertex e_0 in $L(B_{n,n})$ is the apex vertex with degree $d(e_0) = 2n$.
- If the apex vertex e_0 is removed in $L(B_{n,n})$ then two complete graphs of order n as two components of a vertex deleted subgraph are obtained.

2 CORDIAL LABELING OF LINE GRAPH OF BISTAR

Theorem 2.1 *$L(B_{n,n})$ is cordial if and only if $n = t^2$ or $n = (t+1)^2 - 1$ for $t \in \mathbb{N}$.*

Proof: Let $B_{n,n}$ be the bistar with vertex set $\{u_0, v_0, u_i, v_i : 1 \le i \le n\}$ where u_0, v_0 are apex vertices and u_i, v_i are pendant vertices for all $1 \le i \le n$. Let $\{e_0 = u_0v_0,\ e_i = u_0u_i,\ e_{n+i} = v_0v_i : 1 \le i \le n\}$ be the edge set of $B_{n,n}$. Then $V\left(L(B_{n,n})\right) = \{e_0, e_1, e_2, \dots, e_n, e_{n+1}, e_{n+2}, \dots, e_{2n}\}$. Hence, $|V\left(L(B_{n,n})\right)| = 2n + 1$ and $|E\left(L(B_{n,n})\right)| = n(n+1)$.

To define vertex labeling $f : V\left(L(B_{n,n})\right) \to \{0,1\}$ following two cases are considered:

<u>Case-I</u>: when $f(e_0) = 1$.

In order to satisfy vertex condition $|v_f(0) - v_f(1)| \le 1$ it must be either $v_f(0) = n$ and $v_f(1) = n + 1$ or $v_f(0) = n + 1$ and $v_f(1) = n$.

<u>Subcase-(i):</u> when $v_f(0) = n$ and $v_f(1) = n + 1$.

Define $f: V\left(L(B_{n,n})\right) \to \{0,1\}$ as follows:

$f(e_0) = 1$.
Now, n vertices must be labeled with 0 and remaining n with 1. To consider all different possibilities of labeling the variable j is defined as j Number of vertices having label 0 from the vertices $e_1, e_2, \dots, e_n$ = Number of vertices having label 1 from the vertices $e_{n+1}, e_{n+2}, \dots, e_{2n}$. It is noted that $1 \le j \le n$.

Without loss of generality $f(e_i)$ is defined as:

$$f(e_i)=\begin{cases}0\,; & 1\le i\le j\\ 1\,; & j+1\le i\le n\\ 1\,; & n+1\le i\le n+j\\ 0\,; & n+j+1\le i\le 2n\end{cases}$$

In view of above defined labeling pattern $e_f(1)=2j(n-j)+n=2nj-2j^2+n$.

$L(B_{n,n})$ is cordial if and only if : $|e_f(0)-e_f(1)|\le 1$
: $e_f(0)=e_f(1)=\frac{n(n+1)}{2}$ because $|E\left(L(B_{n,n})\right)|=n(n+1)$
: $2nj-2j^2+n=\frac{n^2+n}{2}$
: $n^2+(-4j-1)n+4j^2=0$.

Now, Discriminant of the equation $n^2+(-4j-1)n+4j^2=0$ is $8j+1$.
$L(B_{n,n})$ is cordial if and only if $8j+1$ is a perfect square number.
It is known that the integer j is a triangular number if and only if $8j+1$ is a perfect square and a number is triangular if and only if it is of the form $\frac{k(k+1)}{2}$ for some $k\in\mathbb{N}$.

$L(B_{n,n})$ is cordial if and only if : $j=\frac{k(k+1)}{2}$ for some $k\in\mathbb{N}$
: $n=k^2$ or $(k+1)^2$ for $k\in\mathbb{N}$
: $n=t^2$ for $t\in\mathbb{N}$.

<u>Subcase-(ii):</u> when $v_f(0)=n+1$ and $v_f(1)=n$.

Define $f:V\left(L(B_{n,n})\right)\to\{0,1\}$ as follows:
$f(e_0)=1$.
Now, $n+1$ vertices must be labeled with 0 and remaining $n-1$ with 1. To consider all different possibilities of labeling the variable j is defined as j Number of vertices having label 0 from the vertices $e_1, e_2,\dots, e_n = 1+$ Number of vertices having label 1 from the vertices $e_{n+1}, e_{n+2},\dots,e_{2n}$. It is noted that $1\le j\le n$.

Without loss of generality $f(e_i)$ is defined as:

$$f(e_i)=\begin{cases}0\,; & 1\le i\le j\\ 1\,; & j+1\le i\le n\\ 1\,; & n+1\le i\le n+j-1\\ 0\,; & n+j\le i\le 2n\end{cases}$$

In view of above defined labeling pattern $e_f(1)=j(n-j)+(j-1)\big(n-(j-1)\big)+n+1=2nj+2j-2j^2$.

$L(B_{n,n})$ is cordial if and only if : $|e_f(0)-e_f(1)|\le 1$
: $e_f(0)=e_f(1)=\frac{n(n+1)}{2}$ because $|E\left(L(B_{n,n})\right)|=n(n+1)$
: $2nj+2j-2j^2=\frac{n^2+n}{2}$
: $n^2+(1-4j)n+4j^2-4j=0$.

Now, Discriminant of the equation $n^2+(1-4j)n+4j^2-4j=0$ is $8j+1$.
$L(B_{n,n})$ is cordial if and only if $8j+1$ is a perfect square number.
It is known that the integer j is a triangular number if and only if $8j+1$ is a perfect square and a number is triangular if and only if it is of the form $\frac{k(k+1)}{2}$ for some $k\in\mathbb{N}$.

$L(B_{n,n})$ is cordial if and only if : $j=\frac{k(k+1)}{2}$ for some $k\in\mathbb{N}$
: $n=k^2-1$ or $(k+1)^2-1$ for $k\in\mathbb{N}$
: $n=(t+1)^2-1$ for $t\in\mathbb{N}$.

<u>Case-II</u>: when $f(e_0)=0$.

In order to satisfy vertex condition $|v_f(0)-v_f(1)|\le 1$ it must be either $v_f(0)=n$ and $v_f(1)=n+1$ or $v_f(0)=n+1$ and $v_f(1)=n$.

Subcase-(i): when $v_f(0) = n$ and $v_f(1) = n + 1$.

Define $f: V\left(L(B_{n,n})\right) \to \{0,1\}$ as follows:
$f(e_0) = 0$.
Now, $n - 1$ vertices must be labeled with 0 and remaining $n + 1$ with 1. To consider all different possibilities of labeling the variable j is defined as j Number of vertices having label 1 from the vertices $e_1, e_2, \ldots, e_n = 1 +$ Number of vertices having label 0 from the vertices $e_{n+1}, e_{n+2}, \ldots, e_{2n}$. It is noted that $1 \le j \le n$.

Without loss of generality $f(e_i)$ is defined as:

$$f(e_i) = \begin{cases} 1; & 1 \le i \le j \\ 0; & j+1 \le i \le n \\ 0; & n+1 \le i \le n+j-1 \\ 1; & n+j \le i \le 2n \end{cases}$$

In view of above defined labeling pattern $e_f(1) = j(n-j) + (j-1)(n-(j-1)) + n + 1 = 2nj + 2j - 2j^2$.

$L(B_{n,n})$ is cordial if and only if : $|e_f(0) - e_f(1)| \le 1$
: $e_f(0) = e_f(1) = \frac{n(n+1)}{2}$ because $|E\left(L(B_{n,n})\right)| = n(n + 1)$
: $2nj + 2j - 2j^2 = \frac{n^2+n}{2}$
: $n^2 + (1-4j)n + 4j^2 - 4j = 0$.

Now, Discriminant of the equation $n^2 + (1-4j)n + 4j^2 - 4j = 0$ is $8j + 1$.
$L(B_{n,n})$ is cordial if and only if $8j + 1$ is a perfect square number.
It is known that the integer j is a triangular number if and only if $8j + 1$ is a perfect square and a number is triangular if and only if it is of the form $\frac{k(k+1)}{2}$ for some $k \in \mathbb{N}$.

$L(B_{n,n})$ is cordial if and only if : $j = \frac{k(k+1)}{2}$ for some $k \in \mathbb{N}$
: $n = k^2 - 1$ or $(k+1)^2 - 1$ for $k \in \mathbb{N}$
: $n = (t+1)^2 - 1$ for $t \in \mathbb{N}$.

Subcase-(ii): when $v_f(0) = n + 1$ and $v_f(1) = n$.

Define $f: V\left(L(B_{n,n})\right) \to \{0,1\}$ as follows:
$f(e_0) = 0$.
Now, n vertices must be labeled with 0 and remaining n with 1. To consider all different possibilities of labeling the variable j is defined as j Number of vertices having label 1 from the vertices $e_1, e_2, \ldots, e_n =$ Number of vertices having label 0 from the vertices $e_{n+1}, e_{n+2}, \ldots, e_{2n}$. It is noted that $1 \le j \le n$.

Without loss of generality $f(e_i)$ is defined as:

$$f(e_i) = \begin{cases} 1; & 1 \le i \le j \\ 0; & j+1 \le i \le n \\ 0; & n+1 \le i \le n+j \\ 1; & n+j+1 \le i \le 2n \end{cases}$$

In view of above defined labeling pattern $e_f(1) = 2j(n-j) + n = 2nj - 2j^2 + n$.

$L(B_{n,n})$ is cordial if and only if : $|e_f(0) - e_f(1)| \le 1$
: $e_f(0) = e_f(1) = \frac{n(n+1)}{2}$ because $|E\left(L(B_{n,n})\right)| = n(n + 1)$
: $2nj - 2j^2 + n = \frac{n^2+n}{2}$
: $n^2 + (-4j-1)n + 4j^2 = 0$.

Now, Discriminant of the equation $n^2 + (-4j-1)n + 4j^2 = 0$ is $8j + 1$.
$L(B_{n,n})$ is cordial if and only if $8j + 1$ is a perfect square number.

It is known that the integer j is a triangular number if and only if $8j + 1$ is a perfect square and a number is triangular if and only if it is of the form $\frac{k(k+1)}{2}$ for some $k \in \mathbb{N}$.

$L(B_{n,n})$ is cordial if and only if : $j = \frac{k(k+1)}{2}$ for some $k \in \mathbb{N}$
: $n = k^2$ or $(k+1)^2$ for $k \in \mathbb{N}$
: $n = t^2$ for $t \in \mathbb{N}$.

Hence, $L(B_{n,n})$ is cordial if and only if $\mathrm{n} = \mathrm{t}^2$ or $\mathrm{n} = (\mathrm{t}+1)^2 - 1$ for $\mathrm{t} \in \mathbb{N}$.

Illustration 2.2 *Cordial labeling of the graph* $L(B_{8,8})$ *is shown in the Figure* 3.

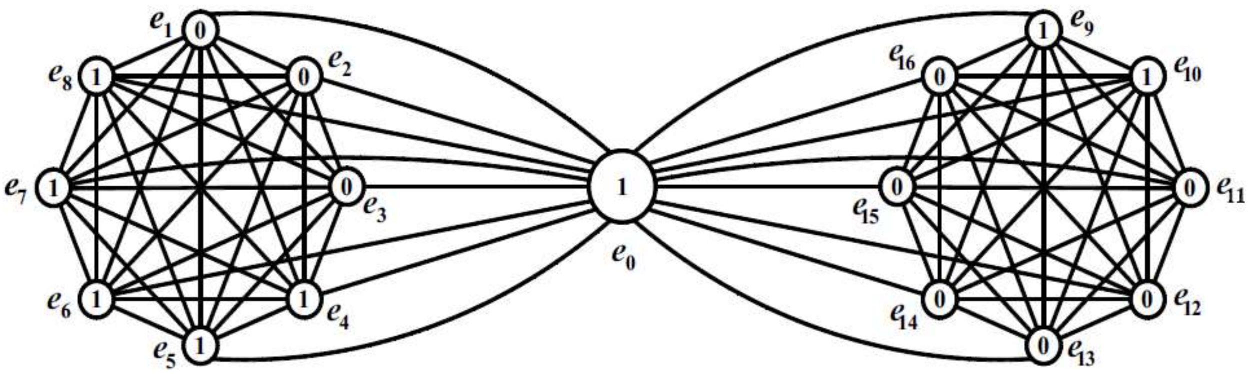

Figure 3: Cordial labeling of $L(B_{8,8})$.

3 PRODUCT CORDIAL LABELING AND TOTAL PRODUCT CORDIAL LABELING OF LINE GRAPH OF BISTAR

Definition 3.1 For a graph $G = (V(G); E(G))$, a vertex labeling function $f : V(G) \rightarrow \{0,1\}$ induces an edge labeling function $f^* : E(G) \rightarrow \{0,1\}$ defined as $f^*(e = uv) = f(u)f(v)$. Then the function f is called a *product cordial labeling* of graph G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits product cordial labeling is called a *product cordial graph*.

The concept of product cordial labeling was introduced by Sundaram et al.[5] in 2004. In the same paper they have proved that all trees, helm H_n and $P_m \cup P_n$ are product cordial. Vaidya and Barasara[8] have proved that the graphs $L(M(P_n))$ (only for odd n), $L(T_n)$ (only for even n) and $L(ACr_n)$ are product cordial.

Definition 3.2 For a graph $G = (V(G); E(G))$, a vertex labeling function $f : V(G) \rightarrow \{0,1\}$ induces an edge labeling function $f^* : E(G) \rightarrow \{0,1\}$ defined as $f^*(e = uv) = f(u)f(v)$. Then the function f is called a *total product cordial labeling* of graph G if $|(v_f(0) + e_f(0)) - (e_f(1) + v_f(1))| \leq 1$. A graph which admits total product cordial labeling is called a *total product cordial graph*.

The concept of total product cordial labeling was introduced by Sundaram et al.[6] in 2006. In the same paper they have proved the following results:

- All trees are total product cordial.
- Every product cordial graph of even order or odd order and even size is total product cordial.

Theorem 3.3 *$L(B_{n,n})$ is a product cordial graph.*
Proof: Let $B_{n,n}$ be the bistar with vertex set $\{u_0, v_0, u_i, v_i : 1 \leq i \leq n\}$ where u_0, v_0 are apex vertices and u_i, v_i are pendant vertices for all $1 \leq i \leq n$. Let $\{e_0 = u_0v_0,\ e_i = u_0u_i,\ e_{n+i} = v_0v_i : 1 \leq i \leq n\}$ be the edge set of $B_{n,n}$.

Then $V(L(B_{n,n})) = \{e_0, e_1, e_2, \dots, e_n, e_{n+1}, e_{n+2}, \dots, e_{2n}\}$. Hence, $|V\left(L(B_{n,n})\right)| = 2n + 1$ and $|E\left(L(B_{n,n})\right)| = n(n + 1)$.

Define vertex labeling $f : V\left(L(B_{n,n})\right) \rightarrow \{0,1\}$ as follows:

$f(e_0) = 1$.
$f(e_i) = 1;\ 1 \leq i \leq n$.
$f(e_{n+i}) = 0\ ; 1 \leq i \leq n$.

In view of the above defined labeling pattern $v_f(0) = n + 1$, $v_f(1) = n$ and $e_f(0) = e_f(1) = \frac{n(n+1)}{2}$.
Thus, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, $L(B_{n,n})$ is a product cordial graph.

Illustration 3.4 Product cordial labeling of the graph $L(B_{7,7})$ is shown in the Figure 4.

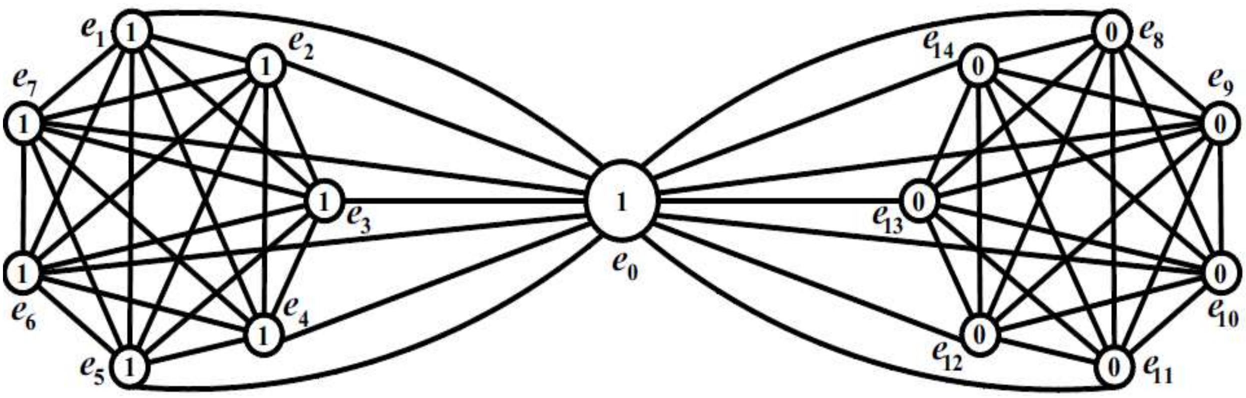

Figure 4: Product cordial labeling of $L(B_{7,7})$.

Theorem 3.5 *$L(B_{n,n})$ is a total product cordial graph.*
Proof: Let $B_{n,n}$ be the bistar with vertex set $\{u_0, v_0, u_i, v_i : 1 \leq i \leq n\}$ where u_0, v_0 are apex vertices and u_i, v_i are pendant vertices for all $1 \leq i \leq n$. Let $\{e_0 = u_0v_0,\ e_i = u_0u_i,\ e_{n+i} = v_0v_i : 1 \leq i \leq n\}$ be the edge set of $B_{n,n}$.
Then $V(L(Bn, n)) = \{e_0, e_1, e_2, \ldots, e_n, e_{n+1}, e_{n+2}, \ldots, e_{2n}\}$. Hence, $|V\left(L(B_{n,n})\right)| = 2n + 1$ and $|E\left(L(B_{n,n})\right)| = n(n + 1)$.

$L(B_{n,n})$ is a product cordial graph as proved in theorem 3.3 and every product cordial graph of even order or odd order and even size is total product cordial as proved in [6]. It is known that order of $L(B_{n,n})$ is odd number $2n + 1$ and size of $L(B_{n,n})$ is even number $n(n + 1)$.

Hence, $L(B_{n,n})$ is a total product cordial graph.

4 EDGE PRODUCT CORDIAL LABELING AND TOTAL EDGE PRODUCT CORDIAL LABELING OF LINE GRAPH OF BISTAR

Definition 4.1 For a graph $G = (V(G); E(G))$, the edge labeling function is defined as $f : E(G) \rightarrow \{0,1\}$ and induced vertex labeling function $f^* : V(G) \rightarrow \{0,1\}$ is given as $f^*(v) = \prod\{f(uv) : uv \in E(G)\}$.

Let $v_f(i)$ be the number of vertices of G having label i under f^* and $e_f(i)$ be the number of edges of G having label i under f for $i = 0, 1$. Then the function f is called an *edge product cordial labeling* of graph G if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. A graph which admits edge product cordial labeling is called an *edge product cordial graph*.

The concept of edge product cordial labeling was introduced by Vaidya and Barasara[7] in 2012. In the same paper they have proved that all trees, crown and armed crown are edge product cordial graphs.

Definition 4.2 For a graph $G = (V(G); E(G))$, the edge labeling function is defined as $f : E(G) \rightarrow \{0,1\}$ and induced vertex labeling function $f^* : V(G) \rightarrow \{0,1\}$ is given as $f^*(v) = \prod\{f(uv) : uv \in E(G)\}$. Then the function f is called a *total edge product cordial labeling* of graph G if $|(v_f(0) + e_f(0)) - (v_f(1) + (e_f(1))| \leq 1$. A graph which admits total edge product cordial labeling is called a *total edge product cordial graph*.

The concept of total edge product cordial labeling was introduced by Vaidya and Barasara[8] in 2013. In the same paper they have proved the following results:

- Every edge product cordial graph of either even order or even size admit total edge product cordial labeling.
- The cycle C_n is a total edge product cordial graph except for $n = 4$.
- The wheel W_n is a total edge product cordial graph.

Theorem 4.3 *$L(B_{n,n})$ is an edge product cordial graph.*
Proof: Let $B_{n,n}$ be the bistar with vertex set $\{u_0, v_0, u_i, v_i : 1 \leq i \leq n\}$ where u_0, v_0 are apex vertices and u_i, v_i are pendant vertices for all $1 \leq i \leq n$. Let $\{e_0 = u_0v_0,\ e_i = u_0u_i,\ e_{n+i} = v_0v_i : 1 \leq i \leq n\}$ be the edge set of $B_{n,n}$. Then $V(L(Bn, n)) = \{e_0, e_1, e_2, \dots, e_n, e_{n+1}, e_{n+2}, \dots, e_{2n}\}$. Hence, $|V\left(L(B_{n,n})\right)| = 2n + 1$ and $|E\left(L(B_{n,n})\right)| = n(n + 1)$.

Define edge labeling $f: E(L(B_{n,n})) \to \{0,1\}$ as follows:
$f(e_0e_i) = 1;\ 1 \leq i \leq n.$
$f(e_0e_{n+i}) = 0;\ 1 \leq i \leq n.$
$f(e_ie_j) = 1;\ i \neq j;\ 1 \leq i,j \leq n.$
$f(e_{n+i}e_{n+j}) = 0;\ i \neq j;\ 1 \leq i,j \leq n.$

In view of the above defined labeling pattern $e_f(0) = e_f(1) = \frac{n(n+1)}{2}$ and $v_f(0) = n + 1$, $v_f(1) = n$.
Thus, $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$.

Hence, $L(B_{n,n})$ is an edge product cordial graph.

Illustration 4.4 Edge product cordial labeling of the graph $L(B_{5,5})$ is shown in the Figure 5.

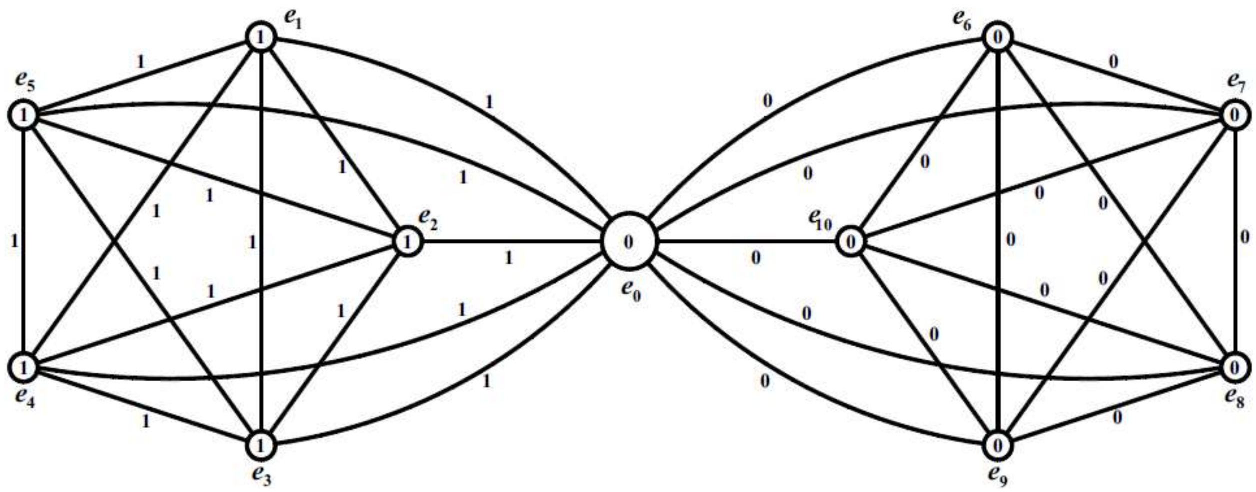

Figure 5: Edge product cordial labeling of $L(B_{5,5})$.

Theorem 4.5 *$B_{n,n}$ is a total edge product cordial graph.*
Proof: Let $B_{n,n}$ be the bistar with vertex set $\{u_0, v_0, u_i, v_i : 1 \leq i \leq n\}$ where u_0, v_0 are apex vertices and u_i, v_i are pendant vertices for all $1 \leq i \leq n$. Hence, $|V(B_{n,n})| = 2n + 2$ and $|E(B_{n,n})| = 2n + 1$.

$B_{n,n}$ is a tree and every tree with order greater than 2 is edge product cordial as proved in [7]. Thus, $B_{n,n}$ is edge product cordial. It is known that every edge product cordial graph of either even order or even size is total edge product cordial as proved in [8] and order of $B_{n,n}$ is even number $2n + 2$.

Hence, $B_{n,n}$ is a total edge product cordial graph.

Theorem 4.6 *$L(B_{n,n})$ is a total edge product cordial graph.*
Proof: Let $B_{n,n}$ be the bistar with vertex set $\{u_0, v_0, u_i, v_i : 1 \leq i \leq n\}$ where u_0, v_0 are apex vertices and u_i, v_i are pendant vertices for all $1 \leq i \leq n$. Let $\{e_0 = u_0v_0,\ e_i = u_0u_i,\ e_{n+i} = v_0v_i : 1 \leq i \leq n\}$ be the edge set of $B_{n,n}$. Then $V(L(Bn, n)) = \{e_0, e_1, e_2, \dots, e_n, e_{n+1}, e_{n+2}, \dots, e_{2n}\}$. Hence, $|V\left(L(B_{n,n})\right)| = 2n + 1$ and $|E\left(L(B_{n,n})\right)| = n(n + 1)$.

$L(B_{n,n})$ is an edge product cordial graph as proved in theorem 4.3 and every edge product cordial graph of either even order or even size is total edge product cordial as proved in [8]. It is known that size of $L(B_{n,n})$ is even number $n(n + 1)$.

Hence, $L(B_{n,n})$ is a total edge product cordial graph.

Remarks:

- $B_{n,n}$ is a tree and every tree is cordial, product cordial and total product cordial. Hence, $B_{n,n}$ is cordial, product cordial and total product cordial.
- $B_{n,n}$ is a tree with minimum order 4 and every tree with order greater than 2 is edge product cordial. Hence, $B_{n,n}$ is edge product cordial.
- $B_{n,n}$ is total edge product cordial.

5 CONCLUSION

Bistar $B_{n,n}$ is cordial as proved by Cahit[2], product cordial as proved by Sundaram et al.[5], total product cordial as proved by Sundaram et al.[6], edge product cordial as proved by Vaidya and Barasara[7] and total edge product cordial as proved in theorem 4.5 while it is proved that the line graph of bistar $B_{n,n}$ is cordial if and only if $n = t^2$ or $n = (t+1)^2 - 1$ for $t \in \mathbb{N}$, product cordial, total product cordial, edge product cordial and total edge product cordial. Thus, cordiality is not invariant under line graph for bistar $B_{n,n}$ but product cordiality, total product cordiality, edge product cordiality and total edge product cordiality are invariant under line graph for bistar $B_{n,n}$.

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