

MANIPULABILITY AND STIFFNESS INDEX OF 6 UPS MANIPULATOR

S Varun Kumar¹, S Jaya Krishna², Prof. G Satish Babu³

¹*Department of mechanical engineering, Jawaharlal Nehru Technological university Hyderabad.*

Email id: somisettyvarunkumar@gmail.com

²*Department of mechanical engineering, Holy mary Institute of Technology and Science*

Email id: yellowjupiter@gmail.com

³*Department of mechanical engineering, Jawaharlal Nehru Technological university Hyderabad.,*

Email id: satishbabug.jntuhm@gmail.com

Abstract— The performance evaluation is one of most important issues in the analysis and design of parallel manipulators. Characteristics such as manipulability and minimum singular value are used to determine the performance of the manipulators. The performance indices are used to eliminate the singularity and its near configurations. In this paper 6-UPS spatial parallel manipulator is considered and its performance indices such as condition number, manipulability and minimum singular value are determined for different structures. Based on the performance indices, the optimum configuration of the manipulator is determined. A MATLAB code is developed and used for the analysis. The results are graphically presented. **Key words:** Performance Index, Manipulability, Minimum Singular Value.

Keywords— Stiffness index, Manipulability, Jacobian, 6-UPS, workspace.

I. INTRODUCTION

The concept of manipulability measure was proposed by Tsuneo Yoshikawa [1]. This is a measure of manipulating ability of robotic mechanism in positioning and orienting the end effectors. Properties of the manipulability measure are also established by Yoshikawa. The utilization of this measure for determining the best postures of planar two links, PUMA and SCARA Robots were discussed by him. Khalil, W. Kleinfinger [2] presented a new geometric notation for the description of the kinematic of open-loop, tree and closed-loop structure robots. The method is derived from the well-known Denavit and Hartenberg (D-H) notation, which is powerful for serial robots but leads to ambiguities in the case of tree and closed-loop structure robots. The given method has all the advantages of D-H notation in the case of open-loop robots. Klein and Blaho [3] presented several dexterity measures for an optimizing posture for a given end-effector position and for intriguing the optimum link lengths of an arm. The four measures Determinant, Condition number, Minimum singular value and Joint range availability are determined for the entire reach of the planar three link revolute jointed manipulator. Timothy j.graettinger and bruceH.Krogh [4] collectively generated the concept of acceleration radius which is considered as the performance measure for robotic manipulators. The previously proposed manipulability indices are generalized as the acceleration radius its prime objective is to compute the index or to perform the optimization at discrete points in the workspace. Ming J. Tsai, Yee H. chiou [5] used the manipulability as a quantitative measure to find the closeness of a manipulator to singularity. Singularities can be reduced by the proper selection of joint parameters. By means of computer simulations the use of manipulability for singularity avoidance in joint rate control algorithm is demonstrated. Kucuk and Bingul [6] studied manipulability measure and condition number for the optimal robot design. The structures of the robot manipulators were compared based on the structural length index and global conditioning index. Alexander [7] in this paper has evaluated accuracy indices of planar parallel robots, such robots are widely used in industrial application where indices like exexterity, manipulability and global conditioning index are applied to translational and rotational motions of parallel robots. During their work, a simple geometric method was used for computing the exact local maximum position error and maximum orientation errors. These give the actuator displacement indices. Mansouri and Ouali [8] introduced a new performance index of robot manipulators i.e. the power manipulability, Which is fully homogeneous and constitutes a physically consistent system. This method has been applied for a 2-dof serial manipulator and a 3-dof parallel planar mechanism. Xiaojun Zhu [9] considered the manipulability measure of a multi-arm space robot. The manipulability measure of one arm can be influenced by the base and alternate arms, which effectsly affects the design optimization, the singularity shirking and the consistent control. The manipulability measure for a multi-arm space robot is more perplexing than that of a solitary arm space robot. Xin-Jun Liu et al. [10] used the evaluation criteria such as, the conditioning index, the stiffness to select the link lengths of 3-DOF spherical parallel manipulators and analyzed their operational performance. The atlases of the global conditioning index and the global stiffness index are obtained to optimize link lengths of 3-DOF SPMS.

II. KINEMATICS OF 6 UPS MANIPULATOR

2.1 Manipulability

It is important to formulate a quantitative measure of manipulation capability of the mechanical system. Yoshikawa have introduced the concept of kinematics manipulability is quantified as

$$M = \sqrt{\det(JJ^T)} \quad (2.1)$$

Where J is the jacobian matrix and depends on the instantaneous configuration of the manipulator defined by a joint vector, when m=n (that is when consider non-redundant manipulator) the manipulability M reduces to

$$M = |\det J(q)| \quad (2.2)$$

2.2 Stiffness Index

The stiffness condition number runs using the matrix K:

$$S = \|K^{-1}\| \cdot \|K\| = \|(J \cdot J^T)\| \cdot \|(J \cdot J^T)^{-1}\| \quad (2.3)$$

If the guiding chains of the machine between frame and working platform have different stiffness, the matrix K must be replaced by the matrix:

$$K_C = (J^T)^{-1} \cdot C \cdot J^{-1} \quad (2.4)$$

where the diagonal matrix C contains the stiffness of the single guiding chain. The reciprocal value of S is between $0 < 1/S \leq 1$; a singular pose is again characterized by $1/S=0$, whereas $1/S=1$ is the optimal (isotropic) index.

2.2 Kinematic Modeling

$$p_i = s + Qp'_i \quad i=1, \dots, 6 \quad (2.5)$$

Where $p'_i = [p'_{ix}, p'_{iy}, p'_{iz}]^T$. Subtracting vector b_i from both sides of eq. (2.5), one obtains

$$p_i - b_i = s + Qp'_i - b_i \quad i = 1, \dots, 6. \quad (2.6)$$

Where the left-hand side represents, in fact, a vector connecting point B_i to point A_i , along the i_{th} leg. Hence, taking the Euclidean norm of both sides of this equation leads to

$$\rho^2 = \|b_i - p_i\|^2 = (s + Qp'_i - b_i)^T (s + Qp'_i - b_i) \quad (2.7)$$

2.3.1 Inverse kinematics & Jacobian analysis

When eq. (2.7) is differentiated with respect to time, a set of linear equations relating the joint rates to the Cartesian velocities is obtained. Following the formalism proposed in Gosselin and Angeles (1990) for parallel manipulators, two Jacobian matrices A and B are obtained and the velocity equations can be written as

$$At = B\dot{\rho} \quad (2.8)$$

where t is the six-dimensional twist of the platform and $\dot{\rho}$ is the vector of joint velocities. These vectors are defined as

$$t = [s^T, \omega^T]^T, \quad \dot{\rho} = [\dot{\rho}_1, \dots, \dot{\rho}_6]^T, \quad (2.9)$$

in which the angular velocity of the platform is defined as ω and $\dot{s} = [\dot{x}, \dot{y}, \dot{z}]^T$ is the velocity of point O' . The aforementioned Jacobian matrices can then be written as

$$B = \text{diag}(\rho_1, \dots, \rho_6) \quad (2.10)$$

and

$$A = \begin{bmatrix} c_1^T \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ c_6^T \end{bmatrix}, \quad (2.11)$$

$$c_i = \begin{bmatrix} d_i \\ (Qp_i) \times d_i \end{bmatrix}, \quad i = 1, \dots, 6 \quad (2.12)$$

where d_i is the vector connecting point B_i to point P_i , i.e.,

$$d_i = p_i - b_i, \quad i = 1, \dots, 6. \quad (2.13)$$

Finally, the rotation matrix Q representing the orientation of the platform with respect to the base can be written using

$$\text{Euler angles as } Q = \begin{bmatrix} \cos \theta \cos \psi & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\ \cos \theta \sin \psi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \quad (2.14)$$

Where ψ , θ , ϕ are three Euler angles defined according to the convention (Qz, Qy, Qx). Other representations of the rotation could also be used (e.g., quaternions, linear invariants, dual representations), which would not affect the algorithm presented.



Fig 1 CAD model of 6-UPS manipulator

III. RESULTS AND DISCUSSION

The results obtained by using the MATLAB code for the performance indices: Manipulability and Stiffness index, are discussed in the following sub sections. The graphs for Stiffness index are obtained by considering a specified workspace of 6-UPS spatial manipulator. For the selected structures, the manipulability index values lie outside the specified workspace considered for other indices.

The Stiffness index of the manipulator is dependent on the end effector 's position and orientation with in the specified workspace of the manipulator. For the optimal design, the parameters: fixed platform size (b), moving platform size (p), twist angle(Ψ), tilt angle about Y axis(θ) and tilt angle about X axis(ϕ) are considered; it is implemented by evaluating the influence of these parameters on Stiffness index and Manipulability.

As a part of Optimal design the graphs showing the variation of dexterity measures: Stiffness index and Manipulability for different structures of the manipulator are developed. Only three structures are finally selected based on the optimum values of the dexterity measures. In each structure by fixing the twist and tilt angles the variation of the measures in a plane parallel to the base platform at different vertical reaches of the moving platform are developed.

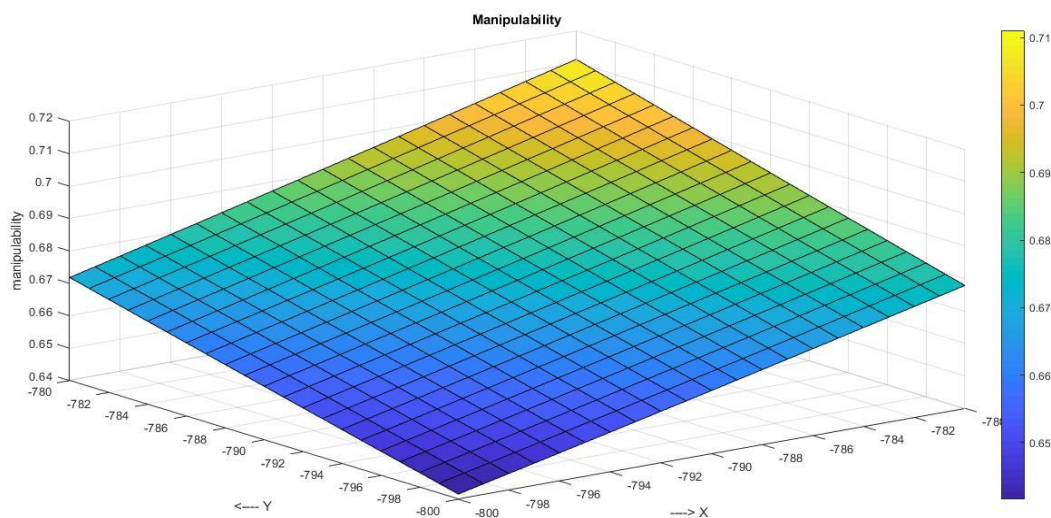


Fig.2 for $R_b=60, R_p=30$ & $z=20$, Twist angle $\Psi=10^0$, Tilt angles $\theta=10^0, \phi=10^0$

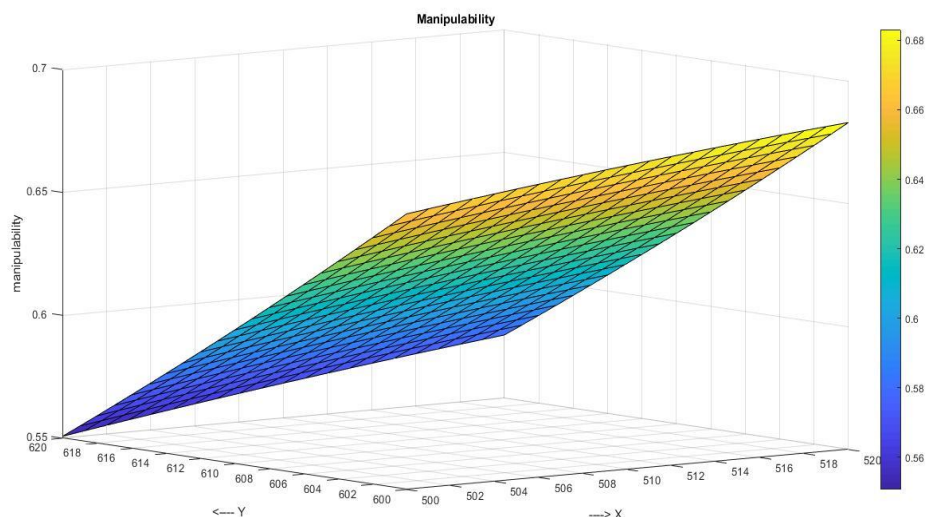


Fig.3 for $R_b=70, R_p=30$ & $z=35$, Twist angle $\Psi=8^\circ$, Tilt angles $\theta=9^\circ, \dot{\theta}=9^\circ$

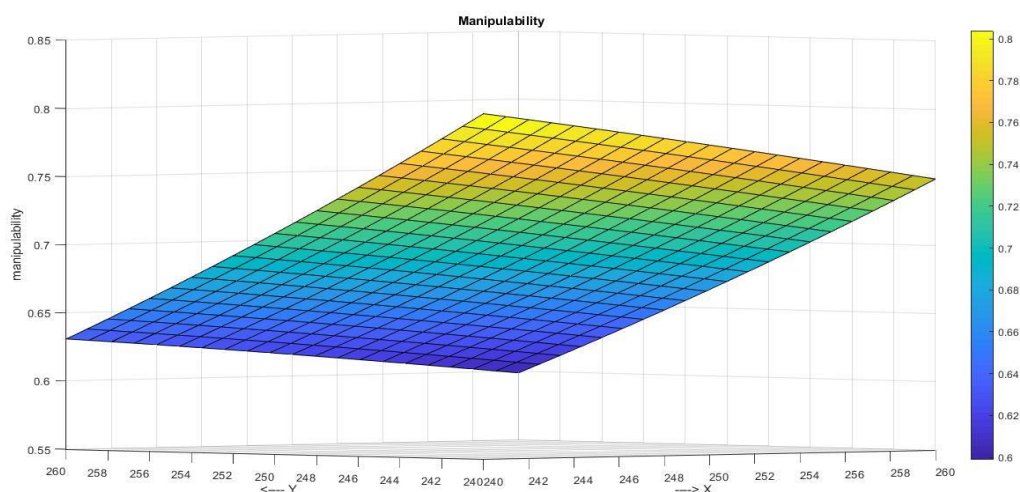


Fig.4 for $R_b=30, R_p=20$ & $z=28$, Twist angle $\Psi=9^\circ$, Tilt angles $\theta=7^\circ, \dot{\theta}=9^\circ$

The value of manipulability index lies between zero to one. The workspace in which the manipulability index values exists is entirely different from other indices.

For structure 1 specified by base platform and moving platform radius as 60mm and 30mm respectively with twist angle (Ψ)= 10° , tilt angle about Y axis(θ)= 10° and tilt angle about X axis($\dot{\theta}$)= 10° the value of Manipulability increases in the vertical reach range of 10mm to 20mm and the maximum value of Manipulability is 0.71105 which occurs at $x=-780$ mm, $y=-780$ mm and vertical reach of 20mm as shown in fig.2 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 2 specified by base platform and moving platform radius as 70mm and 30mm respectively with twist angle (Ψ)= 8° , tilt angle about Y axis(θ)= 9° and tilt angle about X axis($\dot{\theta}$)= 9° the value of Manipulability increases in the vertical reach range of 30mm and 35mm and the maximum value of Manipulability is 0.68307 which occurs at $x=520$ mm, $y=600$ mm and vertical reach of 35mm as shown in fig.3 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 3 specified by base platform and moving platform radius as 30mm and 20mm respectively with twist angle (Ψ)= 9° , tilt angle about Y axis(θ)= 7° and tilt angle about X axis($\dot{\theta}$)= 9° the value of Manipulability increases in the vertical reach range of 20mm to 28mm and the maximum value of Manipulability is 0.80389 which occurs at $x=240$ mm, $y=240$ mm and vertical reach of 28mm as shown in fig.4 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

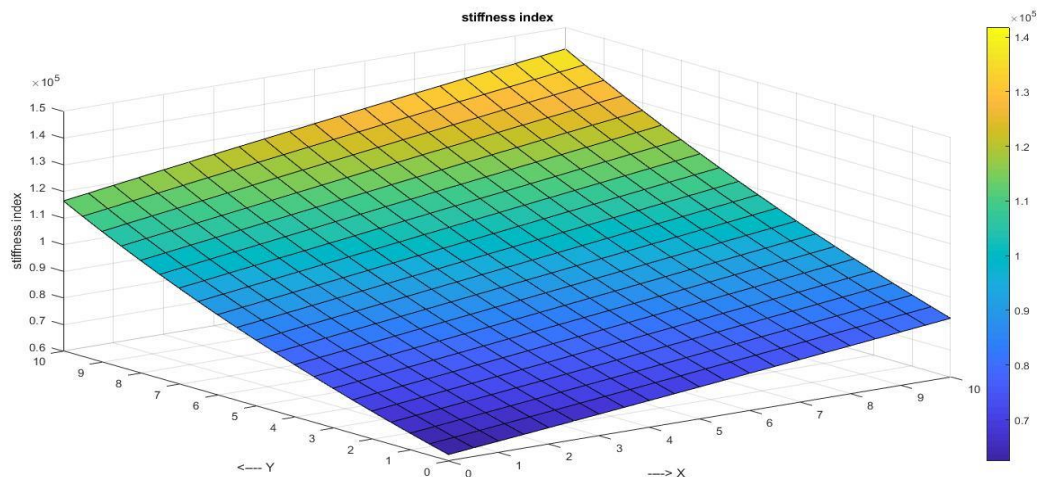


Fig.4.5 for $R_b=70, R_p=30$ & $z=20$, Twist angle $\Psi=10^0$, Tilt angles $\theta=10^0, \phi=10^0$

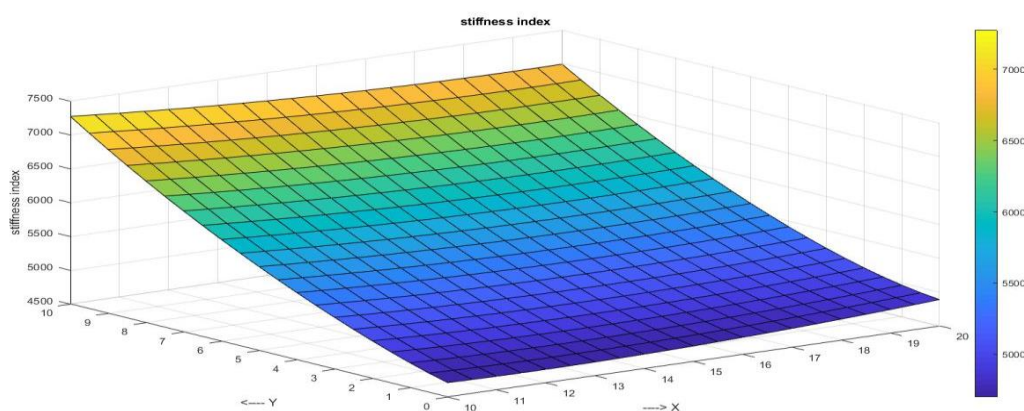


Fig.6 for $R_b=30, R_p=20$ & $z=35$, Twist angle $\Psi=8^0$, Tilt angles $\theta=9^0, \phi=9^0$

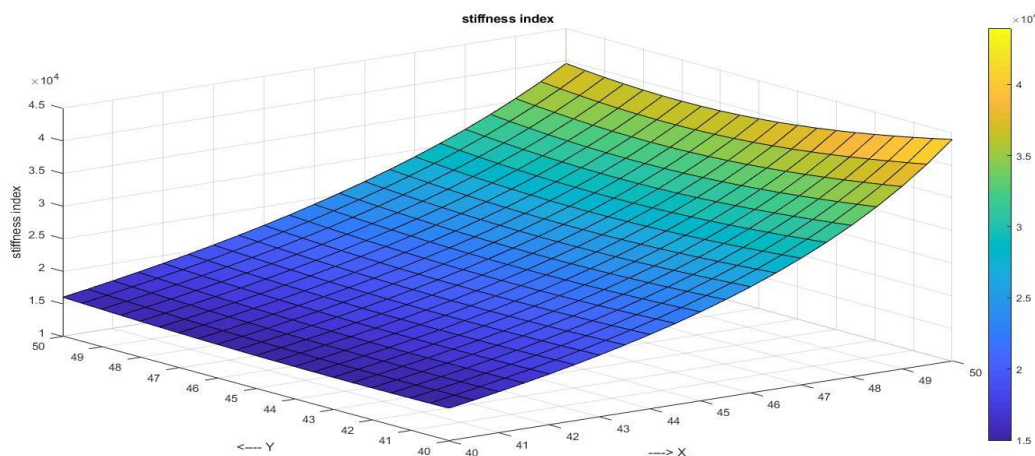


Fig.7 for $R_b=60, R_p=30$ & $z=28$, Twist angle $\Psi=9^0$, Tilt angles $\theta=7^0, \phi=9^0$

The variation of Stiffness index for four different structures of the 6-UPS spatial parallel manipulator is shown in fig.5 to 7. It is observed that, in every structure the Stiffness index is increasing with increase in vertical reach of the moving platform. Optimum values of the Stiffness index are obtained when the ratio of radius of base platform to moving platform is greater than or equal to 2.0.

For structure 1 specified by base platform and moving platform radius as 70mm and 30mm respectively with twist angle (Ψ)= 10^0 , tilt angle about Y axis(θ)= 10^0 and tilt angle about X axis(ϕ)= 10^0 the value of Stiffness index increases in the vertical reach range of 10mm to 20mm and the maximum value of Stiffness index is 141820.8689 which occurs at $x=10$ mm, $y=10$ mm and vertical reach of 20mm as shown in fig.5. At other values of vertical reach for this structure the obtained results show that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 2 specified by base platform and moving platform radius as 30mm and 20mm respectively with twist angle (Ψ)= 8° , tilt angle about Y axis(θ)= 9° and tilt angle about X axis($\dot{\theta}$)= 9° the value of Stiffness index increases in the vertical reach range of 30mm to 35mm and the maximum value of Stiffness index is 7276.489 which occurs at $x=10$ mm, $y=10$ mm and vertical reach of 35mm as shown in fig.6 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

For structure 3 specified by base platform and moving platform radius as 60mm and 30mm respectively with twist angle (Ψ)= 9° , tilt angle about Y axis(θ)= 7° and tilt angle about X axis($\dot{\theta}$)= 9° the value of Stiffness index increases in the vertical reach range of 20mm to 28mm and the maximum value of Stiffness index is 43961.8871 which occurs at $x=50$ mm, $y=40$ mm and vertical reach of 28mm as shown in fig.7 At other values of vertical reach for this structure the obtained results shows that the jacobian matrix is ill conditioned which indicates the singular locations of the moving platform.

IV. CONCLUSIONS

Present work has been focused on investigation of 6-UPS spatial parallel manipulators kinematics and performance evaluation based on the indices: Stiffness index and Manipulability. The following conclusions were drawn from the results.

- Using the kinematic model the loop closure equations are developed for 6-UPS manipulators.
- Jacobian matrices are derived for both 6-UPS and manipulators.
- All the performance measures considered in the present work are functions of the jacobian matrix.
- Graphs showing the variation of the performance measures in a specified workspace are developed.
- Maximum value of Stiffness index i.e.141820.08689 is obtained for the 6-UPS manipulator with structure having base radius to moving platform radius of 2.3, the orientation of the moving platform specified by $\Psi=10^\circ, \theta=10^\circ$ & $\dot{\theta}=10^\circ$ and the position in the workspace: $x=10$ mm, $y=10$ mm and $z=20$ mm. This posture will allow minimum deformation and maximum load carrying capacity.
- Maximum value of Manipulability index i.e.0.80389 is obtained for the 6-UPS manipulator with structure having base radius to moving platform radius of 1.5, the orientation of the moving platform specified by $\Psi=9^\circ, \theta=7^\circ$ & $\dot{\theta}=9^\circ$ and the position in the workspace: $x=240$ mm, $y=240$ mm and $z=28$ mm. This structure with the configuration specified above is useful for arbitrarily changing the position and orientation of the end effector. The structure with configurations having manipulability index value nearer to 1.0 are generally used in fast packaging applications.

REFERENCES

- [1] G.Sutherland and B.Roth, "A Transmission Index for Spatial Mechanisms", ASME Journal of Engineering for industry, 1973, pp. 589-597.
- [2] Ya-Qing Zheng, Xiong-Wei Liu, "Force Transmission Index Based Workspace Analysis Of A Six Dof Wire-Driven Parallel Manipulator," ASME journal, 2002.
- [3] Wen-Tung Chang and Chen-Chou Lin, "Force Transmissibility Performance of Parallel Manipulators," Journal of Robotic Systems, pp. 659-670 © Wiley Periodicals, Inc, 2003.
- [4] Geoffrey Pond and Juan A. Carretero. "Formulating Jacobian matrices for the dexterity analysis of parallel manipulators," Mechanism and Machine Theory 41, pp 1505-1519 (2006).
- [5] Daxing Zeng et al. "Performance Analysis and Optimal Design of a 3-dof 3-PRUR Parallel Mechanism," ASME the Journal of Mechanical Design, Vol. 130, 042307 pp 1-9, April 2008.
- [6] Xin-Jun LIU et al. "A new index for the performance evaluation of parallel manipulators: a study on planar parallel manipulators," IEEE, proceedings of the 7th world congress on Intelligent Control and Automation, pp 353-357, June 2008.
- [7] Xinjun LIU, Chao WU, Fugui XIE, "Motion/force transmission indices of parallel manipulators" Frontiers of Mechanical Engineering ; 6, 1; pp89-91 (2011).
- [8] Imed Mansouri and Mohammed Ouali, "The power manipulability-A new homogeneous performance index of robot manipulators," Robotics and Computer-Integrated Manufacturing 27, pp 434-449, 2011.
- [9] Amir Rezaei, Alireza Akbarzadeh, Payam Mahmoodi Ni "Position, Jacobian and workspace analysis of a 3-PSP spatial parallel manipulator" Elsevier journal of Robotics and Computer-Integrated Manufacturing 29 (2013) 158-173.
- [10] Metin Toz and Serdar Kucuk "Dexterous workspace optimization of an asymmetric six-degree of freedom Stewart-Gough platform type manipulator" Robotics and Autonomous Systems 61 (2013) 1516-1528.
- [11] Xiang Chen, Chao Chen and Xin-Jun Liu, "Evaluation of Force/Torque Transmission Quality for Parallel Manipulators", ASME Journal of Mechanisms and Robotics, Vol.7, pp. 1-9(2015).
- [12] Guanglei Wu, Stéphane Caro, Jiawei Wang "Design and transmission analysis of an asymmetrical spherical parallel manipulator" Elsevier journal of Mechanism and Machine Theory 94 (2017) 119-131.
- [13] Haitao Liu, Tian Huang, Andrés Kecskeméthy, Derek G. Chetwynd, Qing Li "Force/motion transmissibility analyses of redundantly actuated and overconstrained parallel manipulators" Elsevier journal of Mechanism and Machine Theory 109 (2017) 126-138.
- [14] Juan Diego Orozco-Muñiz, J. Jesús Cervantes-Sánchez, José M. Rico-Martínez "Dexterity indices for planar parallel manipulators," Elsevier journal of Robotics and Computer-Integrated Manufacturing 46 (2017) 144-155.