

NUMERICAL STUDIES ON MACH FLOW CHARACTERISTICS OVER BACKWARD FACING ROUNDED STEP THROUGH HYBRID RANS-LES

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Abstract— *The fluid flow behaviour, involving density, vorticity and Mach number distributions, over a backward facing rounded step is studied by developing a 2D numerical model associated with the hybrid RANS-LES/Spalart-Allmaras turbulent model, comprising a viscosity-like variable. The model incorporates fundamental issues like production, diffusion and destruction effects. Numerical simulations are performed with inflow free stream Mach number of 2.5 associated with free stream pressure and velocity of 15350 N/m² and 651.9 m/s², respectively. The simulation results of density, vorticity and Mach number are noticed to be realistically even and also along the expected lines throughout the whole flow regime. It is found that both vorticity intensity and Mach number are relatively more at the expansion fan near the lip of the separation and near the reattachment shock, however, the fluid density is comparatively low at the same. It results in comparatively weak shock generations. In addition, it is also observed that the recirculation vicinity i.e. dead air region has attained the least fluid density, vorticity and the Mach number because of inviscid rotation. Furthermore, both density field and Mach number help us in getting whether the fluid flow is compressible or incompressible in behaviour. Above and beyond, the gradual change i.e. expansion/contraction within the flow field causes the vorticity generation resulting in relatively sound, smooth, flawless and favourable flow behaviours. Eventually, the recirculation vicinity is noticeably very small and due to this there is reduction in reattachment length.*

Keywords— *Density, Vorticity, Mach number, Backward Facing, Rounded Step, Hybrid RANS-LES.*

I. INTRODUCTION

The flow over backward-facing step is one of the very vibrant perspectives and has increased unequivocal focus because of not just minimalism but for extensive industrial and scientific usages. Moreover, in applied aerodynamics, it is very extensively utilized to investigate so many complex structures together with separation and reattachment. In the arena of investigations on high Mach number flow, every time the backward facing step is rendered as a multifaceted system for ignition in a scramjet, where the recirculation zone has very important role in stabilizing the firing of the engine. The steps on the surfaces of hypersonic or supersonic flying machines (such as aeroplanes, aircrafts and spacecrafts, etc.) make the flow pattern extremely complicated and thus noteworthy researches are desperately necessary for appropriate refining of the dynamic design of the flying machines.

The experimental investigations on flow field along with the heat transfer, downstream of a rearward facing step in supersonic flow is performed by Smith [1]. The energy dissipation model of turbulence is introduced by Launder and Sharma [2], to examine the flow field within the vicinity of a spinning disc. Both experimental and theoretical investigations on backward facing step flow are also reported by Armaly et al. [3]. A one-equation turbulence model is used by Spalart and Allmaras [4], to analyse aerodynamic flow behaviours. The fundamental and yet comprehensive along with the illustrious discussions about the CFD is also reported by Anderson and Wendt [5]. Both DNS and LES are utilized by Neumann and Wengle [6], to investigate the passively controlled turbulent flow of backward-facing step. The numerical simulations of fluidic control for transonic cavity flows is also conducted by Hamed et al. [7]. The experimental investigations on fine structures of supersonic laminar along with turbulent flow over a backward-facing step by using Nano-based Planar Laser Scattering (NPLS) are also done by Chen et al. [8]. The numerical studies on the effects of inflow Mach number and step height on supersonic flows over a backward-facing step are carried out by Liu et al. [9]. The experimental studies on the separated flow behaviour behind a backward-facing step together with the passive disturbance are also conducted by Terekhov et al. [10].

From the aforesaid studies, to the best of author's understanding, it is realistic that there is not a single full numerical investigation on supersonic turbulent flow over a backward facing rounded step by means of hybrid RANS-LES technique. With this viewpoint, the current study exhibits the numerical studies on flow characteristics over a backward facing rounded step by considering the hybrid RANS-LES method. Furthermore, the numerical model also includes additional significant factors like production, diffusion and destruction terms above and beyond the normal issues concerning the current physical research problem. In addition, the specified model also introduces both compressibility as well as eddy viscous effects. The model is superbly demonstrated for the thorough numerical investigations on fluid

flow behaviours (relating to density, vorticity and Mach number distributions) for flow over a backward facing rounded step by presenting the inflow free stream velocity together with the corresponding Mach number as the important model parameters. Finally, the model predictions with reference to the stated important model parameters are along the expected lines as well. Eventually, the current case of fully supersonic fluid flow over the backward facing rounded step simulated through the hybrid RANS-LES i.e. Spalart-Allmaras turbulent model (involving a viscosity-like working variable) reveals that there are comparatively weak shock generations. Besides, the gradual expansion/contraction within the flow results in relatively uneven and favourable flow behaviours. Finally, the recirculation vicinity is observed to be very small and because of this there is reduction in reattachment length.

II. DESCRIPTION OF PHYSICAL PROBLEM

Backward facing rounded step possessing wide range of usages in industrial aerodynamics is studied in the current research. The geometric configuration accompanied by initial and boundary conditions are just the modified form of the backward facing sharp edge step. The setup used for testing this intricate geometry involves the placing of a rounded step of radius dimension is same as that of height from the upstream.

2.1. Geometric Model

Figure 1 elucidates the system structure for analysing the backward facing rounded step flow over rounded step geometry involving step height $H = 0.01125$ m, upstream distance from inlet to step $L_u = 0.1016$ m, downstream distance from rounded step to outlet $L_d = 0.2397$ m and the rounded step radius $H = 0.01125$ m. The distance from downstream to upper boundary layer $Z = 0.15875$ m, spanwise distance $L = 0.3048$ m along with the width $B = 0.025908$ m. Besides, both separation (S) and reattachment (R) points are likely to be witnessed from the numerical simulation.

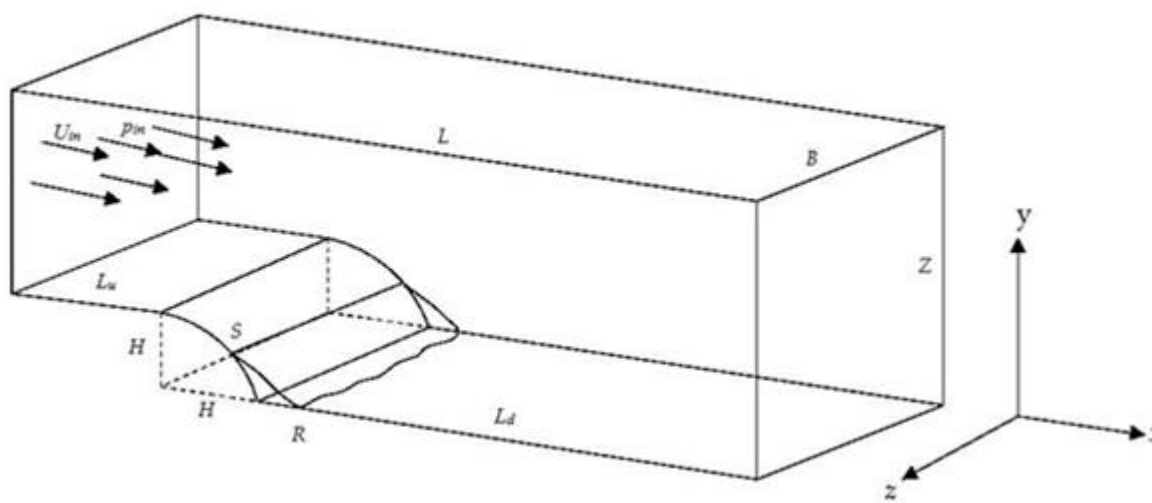


Fig 1. Flow specification of backward facing rounded step

2.2. Initial and Boundary Conditions

The inflow Mach number $Ma = 2.5$ Ma, corresponding to the specified inflow static pressure of about $p_{in} = 15350$ N/m² along with the free stream velocity of $U_{in} = 651.9$ m/s. The temperature to the left of the rounded step is maintained at $T_{in} = 169.2$ K.

For fully feeling the effects of turbulence, the Spalart-Allmaras one-equation Detached Eddy Simulation, DES (also otherwise called as hybrid RANS-LES) model is introduced.

The boundary conditions associated with the geometry shown in figure 2 are as mentioned underneath:

- Wave transmissive outflow pressure at $p = 15.35$ kPa, everywhere else for pressure relating to the present hybrid RANS-LES model.
- Temperature $T_{in} = 169.2$ K, everywhere else for temperature pertaining to the current hybrid RANS-LES model.
- Velocity $U_{in} = 651.9$ m/s at the inlet, no-slip wall at the lower boundary, slip wall at the upper boundary and zero velocity gradient at the outlet are set for the present hybrid RANS-LES model.

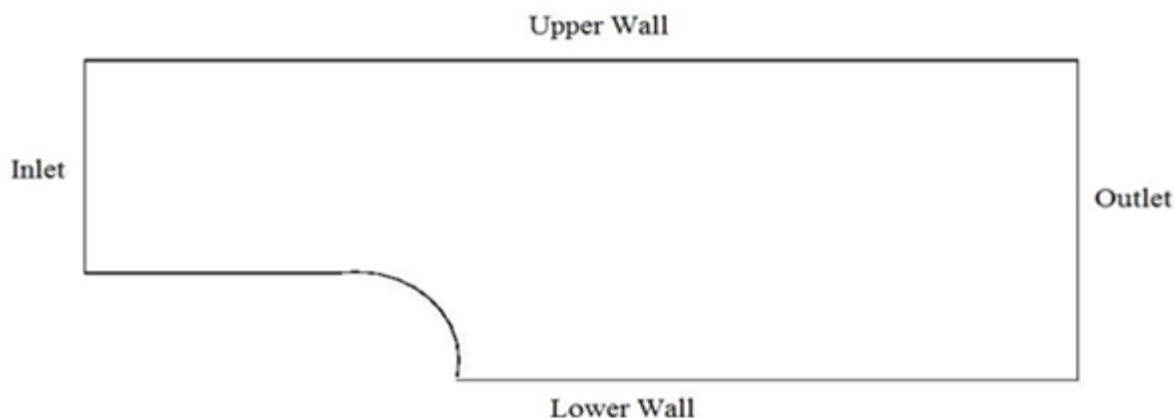


Fig 2. Backward facing rounded step boundary representation

III. MATHEMATICAL FORMULATION

3.1. Generalized Governing Transport Equations

A set of suitably generalized governing transport equations of mass, momentum and energy articulated in the conventional practice of Navier-Stokes equation for compressible flow concerning the effects of turbulence are described below.

$$\text{Continuity: } \frac{\partial \rho}{\partial t} + \frac{\partial(\rho \bar{u}_j)}{\partial x_j} = 0 \quad (1)$$

$$\text{Momentum: } \frac{\partial(\rho \bar{u}_i)}{\partial t} + \frac{\partial(\rho \bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} (2\mu S_{ij} + \tau_{ij}) \quad (2)$$

$$\text{Energy: } \frac{\partial(\rho E)}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_j (\rho E + p)) = \frac{\partial}{\partial x_j} \left((k + k_t) \frac{\partial \bar{T}}{\partial x_j} + (2\mu S_{ij} + \tau_{ij}) \bar{u}_i \right) + S_h \quad (3)$$

$$\text{Where, } \left. \begin{aligned} u_i &= \bar{u}_i + u'_i \\ p &= \bar{p} + p' \\ T &= \bar{T} + T' \end{aligned} \right\} \quad (4)$$

$$\text{Total energy, } E = e + k = h - \frac{p}{\rho} + \frac{v^2}{2} \quad (5)$$

The Reynolds stress term is modeled in terms of the eddy viscosity and is represented by: $\tau_{ij} = 2\mu_t (S_{ij} - S_{nn} \delta_{ij} / 3) - 2\rho k \delta_{ij} / 3$ (6)

The eddy viscosity is defined as a function of the turbulent kinetic energy k , and the turbulent dissipation rate ϵ , and is represented by: $\mu_t = c_\mu f_\mu \rho k^2 / \epsilon$ (7)

Furthermore, all the model terms/symbols/coefficients/functions have their usual meanings and values.

3.2. Hybrid RANS-LES Turbulence Modelling

The hybrid RANS-LES or Spalart–Allmaras turbulence model is a one-equation model for the eddy viscosity. This is also called as Detached Eddy Simulation (DES) model. The differential equation has been derived with pragmatism and concepts of dimensional analysis, Galilean invariance and certain constraint of the molecular viscosity. Grid resolution does not require to be very finer for this model, but, one can really capture the flow field with the related algebraic models.

The transport equation for the Spalart–Allmaras working variable i.e. viscosity-like variable ($\tilde{\nu}$) is represented by:

$$\frac{\partial(\rho \tilde{\nu})}{\partial t} + \tilde{u}_j \frac{\partial(\rho \tilde{\nu})}{\partial x_j} = c_{b1} \tilde{S} \rho \tilde{\nu} + \frac{1}{\sigma} \left[\frac{\partial}{\partial x_j} (\mu + \rho \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_j} + c_{b2} \frac{\partial \tilde{\nu}}{\partial x_j} \frac{\partial(\rho \tilde{\nu})}{\partial x_j} \right] - \rho c_{w1} f_w \left(\frac{\tilde{\nu}}{d} \right)^2 \quad (8)$$

Where, the eddy viscosity is represented by: $\mu_t = \rho \tilde{\nu} f_{v1} = \rho \nu_t$ (9)

Additionally, all the model terms/symbols/coefficients/functions have their usual meanings and values.

IV. NUMERICAL PROCEDURES

4.1. Numerical Scheme and Solution Algorithm

The previously talked over governing transport equations are transformed into an appropriately generalised form as mentioned aside: $\frac{\partial}{\partial t}(\rho \phi) + \nabla \cdot (\rho \mathbf{u} \phi) = \nabla \cdot (\Gamma \nabla \phi) + S$ (10)

The transformed governing transport equations are discretized by using a pressure based coupled framework pertaining to finite volume method (FVM) with the SIMPLER algorithm, where ϕ symbolises any conserved parameter and S is a source term. The developed pressure based, fully coupled solver is utilized to the predict flow characteristics of the associated flow parameters in relation to supersonic turbulent flow over a backward facing rounded step.

4.2. Choice of Grid Size, Time Step and Convergence Criteria

Figure 3 portrays that the whole computational domain is divided into different non-uniform zones and also the grids are relatively finer within the proximate of wall likely to have high gradient. In the current research work, the simulation of the RANS-LES turbulence model is performed within the entire computational domain. A complete grid-independence test is conducted to develop an appropriate spatial discretization, and the levels of iteration convergence criteria to be expended. As a result of this test, 175×175 number of non-uniform grids are used for the final simulation. Corresponding time step chosen in the current simulation is 0.000001 seconds. Although, it is checked with smaller grids of 210×210 in numbers, it is pragmatic that a finer grid system does not modify the results very greatly.

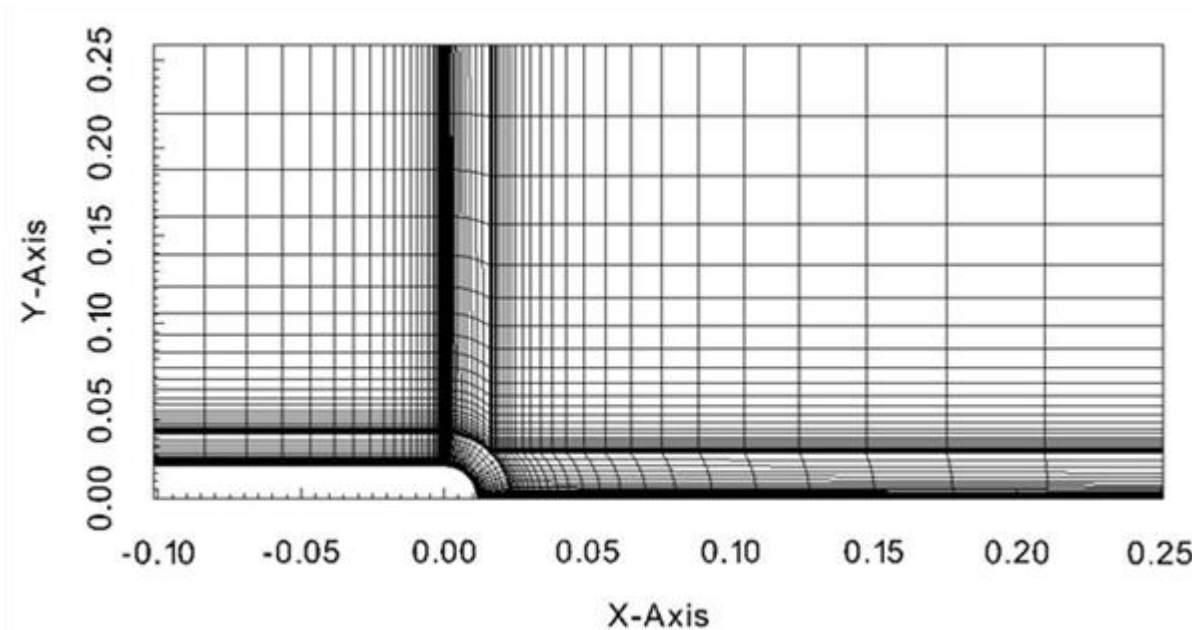


Fig 3. Mesh for backward facing rounded step

The convergence in inner iterations is confirmed while the relation $\left| \frac{\phi - \phi_{old}}{\phi_{max}} \right| \leq 10^{-4}$ is held good concurrently for all variables, where ϕ represents the field variable at a grid point at the present iteration level, ϕ_{old} represents the corresponding value at the previous iteration level, and ϕ_{max} is the maximum value of the variable at the current iteration level within the whole computational domain.

V. RESULTS AND DISCUSSIONS

With the aforementioned model conditions, the numerical simulations are performed for examining the fluid flow characteristics of the associated flow parameters on the subject of fully supersonic turbulent flow over a backward facing rounded step. The turbulence model taken into account for the present investigation work is the hybrid RANS-LES/Spalart-Allmaras model involving a viscosity-like variable ($\tilde{\nu}$). The hybrid RANS-LES turbulence model keeps up

the consistency in accuracy all over the entire flow field and hence results in quite superior and precise predictions. Thus, only the hybrid RANS-LES is chosen for the present research.

5.1. Density Distributions

It is very clear that the high Mach inflow is the root of density gradient inside the flow domain. Figure 4 exhibits the coloured density contour accompanied by the related vertical scale bar, presenting the density gradient along both expansion fan and reattachment regions. The density gradients at the vicinity of both expansion fan and reattachment regions are relatively low, directly indicating the presence of relatively weak shocks with less intensities at the stated regions. Besides, the recirculation vicinity (i.e. dead air region) has also experienced the least density because of inviscid rotation. The lowest density gradient can also be seen over the rounded step surface which is due to gradual expansion of the flow. Furthermore, the supersonic turbulent flow over the backward facing rounded step has got moderate density variations between the expansion fan and the reattachment shock wave regions. In other words, it is quite evident that due to the comparatively weak shock generations, density recovery behind the rounded step is relatively flawless enough for the sound and smooth fluid flow. In addition, the physics behind the density gradient caused by two parallel shocks can merely be unveiled from the coloured density field alongside the associated vertical scale bar, as demonstrated in figure 5. If the flow field gets density deviation more than five percent then the flow is taken as compressible which may be witnessed from the density field. Thus, the density field benefits us in getting whether the fluid flow is compressible or incompressible in behaviour.

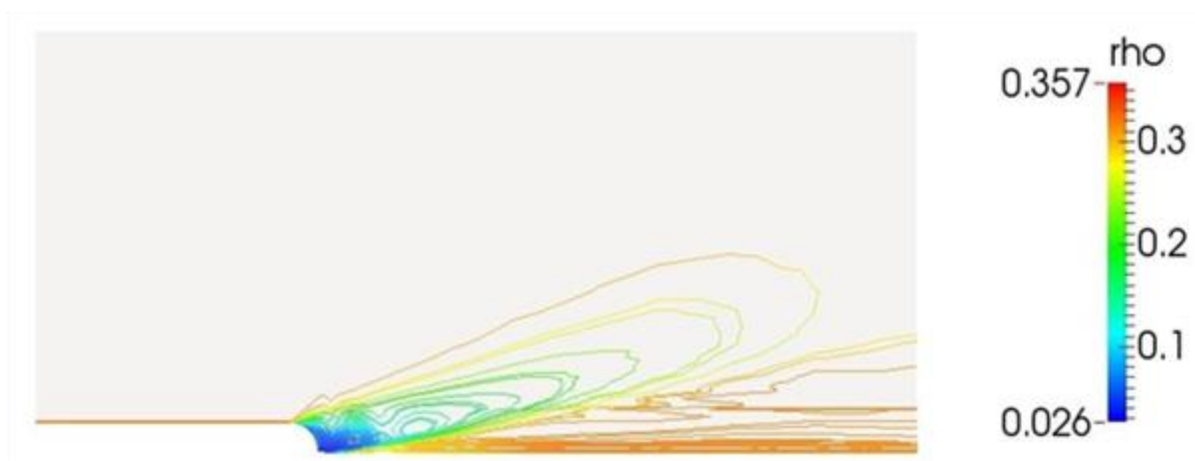


Fig 4. Density contour

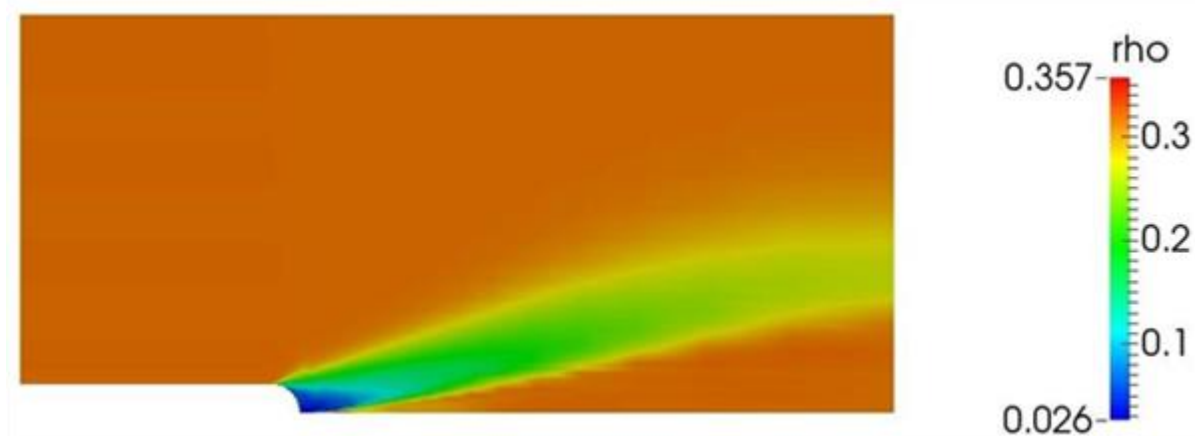


Fig 5. Density field

5.2. Vorticity Distributions

Figure 6 illustrates the coloured vorticity distribution alongside the associated vertical scale bar, inside the flow domain. It may be witnessed that the vorticity intensity is relatively more at the expansion fan close to the lip of the separation over and above near the reattachment shock. Because of viscous layer separation at the separation edge, the generation of lip shock has occurred. Furthermore, the interaction of shock and expansion fan has brought about the generation of vorticity. The vorticity generation is as a result of sudden change (i.e. expansion or contraction) inside the flow domain.

In addition, the vorticity generation becomes predominant owing to the turbulent boundary layer separation. It is also observed that the vorticity is having relatively more magnitude close to the shock regions. Thus, reducing the intensity of the shock is the only way for the reduction in vorticity magnitude.

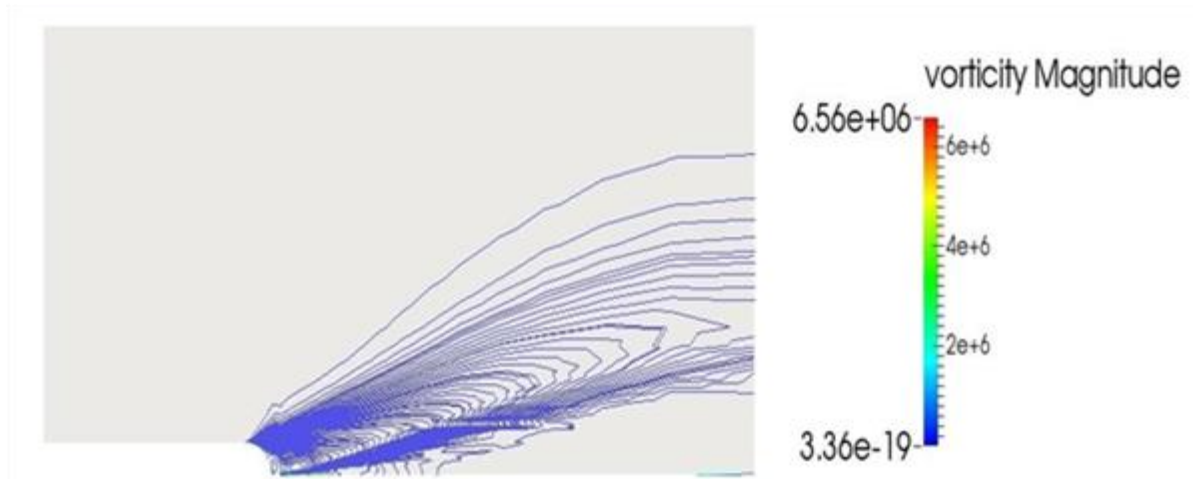


Fig 6. Vorticity distributions

5.3. Mach Number Distributions

Free stream Mach number (Ma) of about 2.5 approaches towards the rounded step. Figure 7 demonstrates the coloured Mach number contour alongside the related vertical scale bar, indicating the relative increase in Mach number at the vicinity of the expansion fan region, besides, the reattachment shock region has also experienced relatively more Mach number gradient. There is a gradual increase in Mach number which is also witnessed just ahead of the separation edge as well. Due to gradual flow field the Mach number gradient is found to be less near the lip of the step and relatively sound and smooth flow approaches towards the bottom wall, besides, at the point of reattachment gradual change in Mach number is also observed. Additionally, the recirculation vicinity (i.e. dead air region) has reached the least Mach number, almost near to zero, because of inviscid rotation. Furthermore, the supersonic turbulent flow over the backward facing rounded step has also experienced relatively more Mach number variations between the expansion fan and the reattachment shock wave regions. Besides, it is quite evident that due to the shock generation, Mach number recovery happens inside the redeveloped boundary layer close to the bottom wall just ahead of the reattachment point. But, after the reattachment shock the Mach flow occurs in its normal direction. Besides, the physics behind the Mach number gradient due to two parallel shocks can merely be unveiled from the coloured Mach number field alongside the associated vertical scale bar, as demonstrated in figure 8. If the Mach number is more than 0.3 then the fluid region is taken to be compressible which may be witnessed from the Mach number field. In other words, the Mach number field aids us in getting whether the fluid region is compressible or incompressible in behaviour. But, no such variations or additional unusual upshot is noticed close to the upper region of flow field. In overall, the flow features also provide favourable aspect in the Mach number contour (as demonstrated in figure 7). The recirculation vicinity is noticeably very small and due to this there is a reduction in reattachment length. Maximum reduction in the recirculation vicinity can also be found in the Mach number field (as illustrated in figure 8) as well.

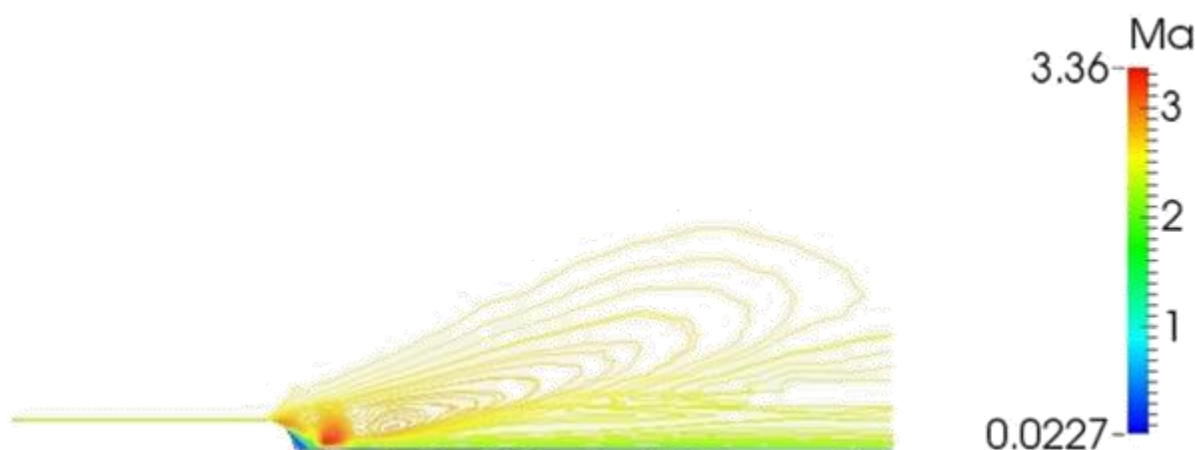


Fig 7. Mach number contour

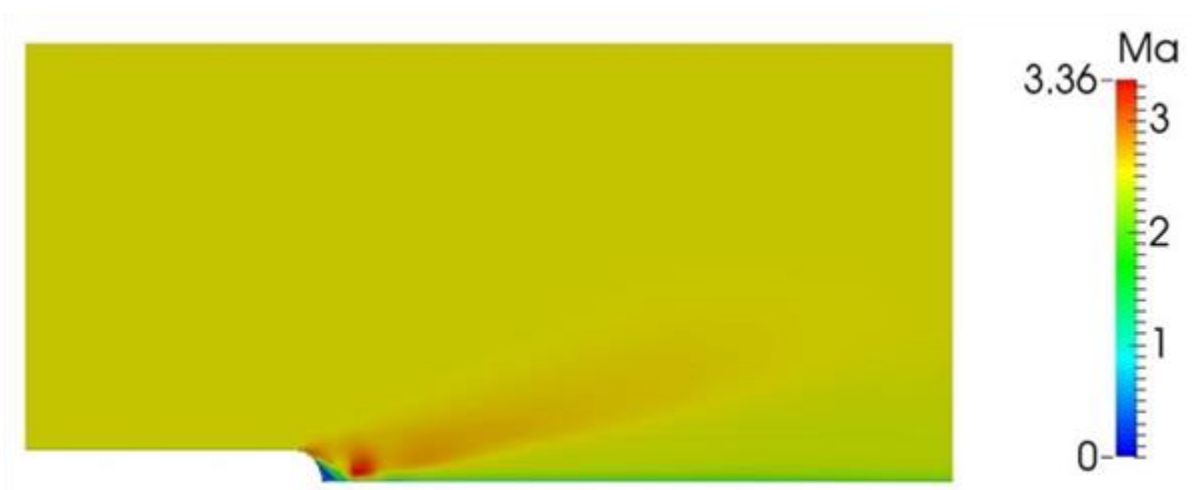


Fig 8. Mach number field

VI. CONCLUSIONS

In the present research, a two dimensional numerical model is developed using the hybrid RANS-LES/Spalart-Allmaras turbulent model involving a viscosity-like working variable ($\tilde{\nu}$), to study the fully supersonic compressible fluid flow characteristics over a backward facing rounded step. The numerical model also takes account of additional important issues namely production, diffusion and destruction factors, over and above the very usual features as well. The model incorporates the inflow free stream velocity and the accompanying Mach number as the key model parameters. Eventually, the model results with reference to the specified vital model parameters are also along the expected lines. Furthermore, the numerical model is witnessed to predict reasonably better and uneven results. Additionally, the simulation results demonstrate that the gradual expansion ensues over the rounded step leading to the delay in viscous layer separation. Certainly, it results in the realization of relatively shorter shear layer alongside the equally shorter recirculation or dead air region, approaching towards the bottom wall. Ultimately, it is also quite apparent about the presence of relatively weak shocks over the rounded step. Hence, it is observed that there are shrinking of shocks on account of the modelling exercises with the rounded step instead of the sharp edge step.

Acknowledgments

The author would like to thank the editor and the reviewers for their noble thoughts, valuable time and contributions for extending insightful reviews to the research article.

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