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# Collision of the Infinitesimal Mass with the First Primary in the CR4BP

Payal Singh[1], M. R. Hassan[2], Md. Aminul Hassan[3]

[1]Asst. Teacher, Maths, P.G.I. School, Bhagalpur, India, satyamshivam7707@gmail.com
[2]Department of Mathematics, S.M. College, T.M.B. University, Bhagalpur, India, hassansmc@gmail.com
[3]GTE, Bangalore, India, mahassan012@gmail.com

***Abstract*— The present paper deals with the possibility of collision of the fourth body with the first primary by using the method given by Bhatnagar [1, 2] in which it was found that in unperturbed case and in perturbed case**

***Keywords*— Collision, CR4BP, Infinitesimal Mass**

## I. Introduction

In this chapter, we have discussed about the conditions of collision of the infinitesimal mass with any of the three primaries. All the primaries are moving on different circular orbits about the centre of mass of the primaries at $P_1, P_2, P_3$ and the infinitesimal mass $P(x, y)$ is moving in the plane of motion of the primaries. Before collision with any primary, the infinitesimal mass moves on elliptic orbit about the centre of the primary. For the first time Levi-Civita [3] attempted to establish the condition of collision and proved that the invariant relation for collision orbits can be analytically continued from the one that corresponds to the two-body problem. Bhatnagar [1, 2] developed the condition of collision by equating to zero, the canonical variable $G$ i.e., he established the condition of collision by proving $G = 0$ in the unperturbed gravitational field of the primaries.

## II. Case of Collisions

Case I: Case of Collision when the Second Primary is a Triaxial Rigid body.

In our problem, the second primary is a triaxial rigid body so there must be some effect of triaxiality of the second primary on the motion of the infinitesimal mass and subsequent effect on the case of collision of the infinitesimal mass with the primaries is required. Thus, the condition of collision of the infinitesimal mass, when the second primary is a triaxial rigid body, is given by

$$G + \mu F(l, L, g, G, \mu, \sigma_1, \sigma_2, \varepsilon_0) = 0, \tag{1}$$

where $\mu, \sigma_1, \sigma_2, \varepsilon_0$ are sufficiently small parameters.

$$\Rightarrow na^2\left(\frac{1}{r_2^3} + \frac{\varepsilon_0}{2r_3^3} + \frac{3A}{r_2^5} - \frac{5Bn^2a^2}{4r_2^7}\right) + \mu F(l, L, g, G, \sigma_1, \sigma_2, \varepsilon_0) = 0, \tag{2}$$

where

$$n^2 = 1 + \frac{3}{2}(2\sigma_1 - \sigma_2), \qquad a = \frac{L}{n}\left[-2(nG + C_0)\right]^{-\frac{1}{2}},$$

$$A = \frac{1}{2}(2\sigma_1 - \sigma_2), \qquad B = 3(\sigma_1 - \sigma_2),$$

$$\sigma_1 = \frac{b_1^2 - b_3^2}{5R^2}, \qquad \sigma_2 = \frac{b_2^2 - b_3^2}{5R^2},$$

$$r_2^2 = 1 + a^2n^2, \qquad r_3^2 = 1 - \sqrt{3}an + a^2n^2,$$

$$\therefore F = -\frac{na^2}{\mu}\left(\frac{1}{r_2^3} + \frac{\varepsilon_0}{2r_3^3} + \frac{3A}{r_2^5} - \frac{5Bn^2a^2}{4r_2^7}\right) = \frac{a^2}{\mu}\left[\frac{5Ba^2}{4r_2^7}n^3 - n\left(\frac{3A}{r_2^5} + \frac{\varepsilon_0}{2r_3^3} + \frac{1}{r_2^3}\right)\right]. \tag{3}$$

By taking the value of $a, n, A, B, r_2, r_3$ and $\varepsilon_0$ in Equation (3), we can express $F$ as the function of $l, L, g, G, \sigma_1, \sigma_2, \mu, \varepsilon_0$ as in Hassan [4, 5] and Payal [6, 7] .

Case II: Case of Collision of the Infinitesimal mass with the First Primary when the Second and Third Primaries are Triaxial Rigid bodies.

Following the Equation (1), the condition of collision be written in this case as

$$G + \mu F(l, L, g, G, \mu, \sigma_1, \sigma_2, \sigma_1', \sigma_2', \varepsilon_0) = 0, \tag{4}$$

where $\mu, \sigma_1, \sigma_2, \sigma_1', \sigma_2', \varepsilon_0$ are sufficiently small parameters as in Hassan [4, 5] and Payal [6, 7].

$$\Rightarrow na^2\left[\frac{1}{r_2^3}+\frac{\varepsilon_0}{2r_3^3}+\frac{3A}{r_2^5}-\frac{5Bn^2a^2}{4r_2^7}+\frac{3A'}{2r_3^5}-\frac{5B'n^2a^2}{2r_3^5}-\frac{15B'}{8r_3^7}+\frac{5\sqrt{3}B'na}{2r_3^7}\right]+\mu F\left(l,L,g,G,\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'},\varepsilon_0\right)=0, \qquad (5)$$

where

$$\sigma_1^{'}=\frac{c_1^2-c_3^2}{5R^2}, \qquad \sigma_2^{'}=\frac{c_2^2-c_3^2}{5R^2},$$

$$A^{'}=\frac{\varepsilon_0}{2}\left(2\sigma_1^{'}-\sigma_2^{'}\right), \quad B^{'}=\frac{3\varepsilon_0}{2}\left(\sigma_1^{'}-\sigma_2^{'}\right).$$

$$\Rightarrow F=-\frac{na^2}{\mu}\left[\frac{1}{r_2^3}+\frac{\varepsilon_0}{2r_3^3}+\frac{3A}{r_2^5}-\frac{5Bn^2a^2}{4r_2^7}+\frac{3A^{'}}{2r_3^5}-\frac{5B^{'}}{6r_3^7}\left(4a^2n^2-4\sqrt{3}an+3\right)\right]. \qquad (6)$$

By putting the values of $a,n,A,B,A',B',r_2,r_3$ and $\varepsilon_0$ in Equation (6), we can express $F$ as the function of $l,L,g,G,\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'},\mu,\varepsilon_0$.

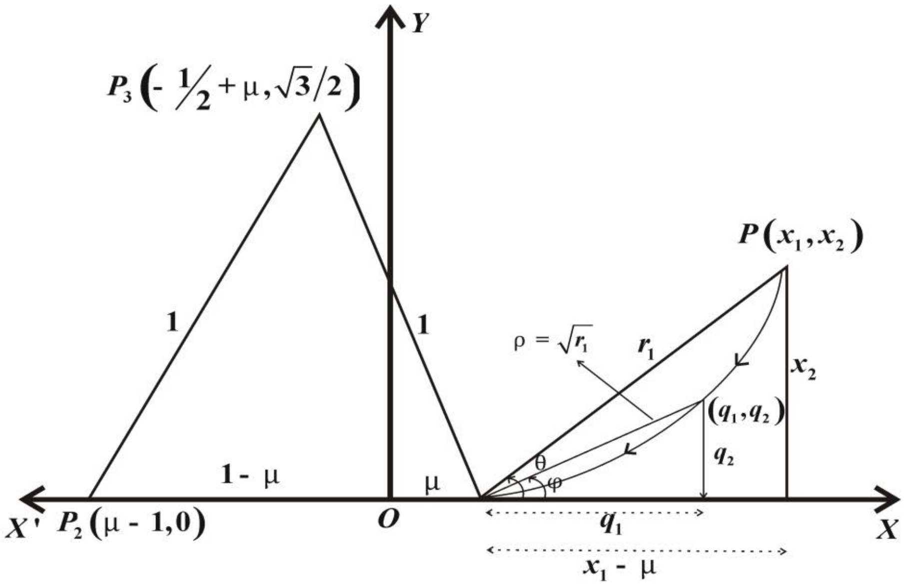


**Fig. 1**: Geometrical Configuration of Collision of Orbits

<u>Case III:</u> Case of collision of the infinitesimal mass with the First Primary when the Second Primary is an Oblate Spheroid and Third Primary is a Triaxial Rigid Body.

Following the Equations (1) and (4), the condition of collision of this case can be written as

$$G+\mu F\left(l,L,g,G,\mu,\sigma_1,\sigma_1^{'},\sigma_2^{'},\varepsilon_0\right)=0, \qquad (7)$$

where $\mu,\sigma_1,\sigma_1^{'},\sigma_2^{'},\varepsilon_0$ are sufficiently small parameters as in Hassan [4, 5] and Payal [6, 7].

$$\text{i.e., } na^2\left[\frac{1}{r_2^3}+\frac{3\sigma_1}{2r_3^5}-\frac{\varepsilon_0}{2r_3^3}+\frac{3A^{'}}{2r_3^5}-\frac{5B^{'}}{8r_3^7}\left(8n^2a^2-2\sqrt{3}na+3\right)\right]+\mu F\left(l,L,g,G,\sigma_1,\sigma_2,\sigma_1^{'},\sigma_2^{'},\varepsilon_0\right)=0, \qquad (8)$$

$$\Rightarrow F=-\frac{na^2}{\mu}\left[\frac{1}{r_2^3}+\frac{3\sigma_1}{2r_3^5}-\frac{\varepsilon_0}{2r_3^3}+\frac{3A^{'}}{2r_3^5}-\frac{5B^{'}}{8r_3^7}\left(8n^2a^2-2\sqrt{3}na+3\right)\right]. \qquad (9)$$

By putting the values of $a,n,\mu,\sigma_1,A',B'$ in the right hand side of Equation (9), we can express $F$ as the function of $l,L,g,G,\sigma_1,\sigma_1^{'},\sigma_2^{'},\mu,\varepsilon_0$.

By the method of analytic continuation, corresponding to the two-body problem, the Levi-Civita's [3] common condition of collision in all the above cases can be written as

$$\dot{\theta}+n=\rho f\left(\rho,\theta\right), \qquad (10)$$

$$\text{where } \tan\theta=\frac{x_2}{x_1-\mu}, \quad \rho=\sqrt{r_1}, \quad n^2=1+\frac{3}{2}\left(2\sigma_1-\sigma_2\right). \qquad (11)$$

Now,

$$\tan\theta=\frac{2q_1q_2}{q_1^2-q_2^2}=\frac{2\rho^2\cos\varphi\sin\varphi}{\rho^2\left(\cos^2\varphi-\sin^2\varphi\right)}=\frac{\sin 2\varphi}{\cos 2\varphi},$$

$$\therefore \tan\theta=\tan 2\varphi \Rightarrow \theta=2\varphi. \qquad (12)$$

Thus, the condition of collision reduces to

$$2\dot{\varphi}+n=\rho f\left(\rho,2\varphi\right),$$

$$\Rightarrow 2\frac{d\varphi}{dt}+n=\rho f\left(\rho,2\varphi\right),$$

$$\Rightarrow 2r_1\frac{d\varphi}{d\tau}\cdot\frac{d\tau}{dt}+nr_1=r_1\sqrt{r_1}\,f\left(\sqrt{r_1},2\varphi\right),$$

$$\Rightarrow 2\frac{d\varphi}{d\tau}+nr_1=r_1^{\frac{3}{2}}f\left(\sqrt{r_1},2\varphi\right).\qquad \text{as } \left[d\tau=r_1dt\right] \tag{13}$$

But $\tan\varphi=\frac{q_2}{q_1}\Rightarrow\varphi=\tan^{-1}\left(\frac{q_2}{q_1}\right)$ so

$$\frac{d\varphi}{d\tau}=\frac{q_1^2}{q_1^2+q_2^2}\left(\frac{q_1q_2^{'}-q_2q_1^{'}}{q_1^2}\right)=\left(\frac{q_1q_2^{'}-q_2q_1^{'}}{\rho^2}\right),$$

$$=\frac{1}{\rho^2}\left[q_1\frac{\partial K_0}{\partial Q_2}-q_2\frac{\partial K_0}{\partial Q_1}\right]=\frac{1}{\rho^2}\left[q_1\left(\frac{1}{4}Q_2-\frac{1}{2}\rho^2nq_1\right)-q_2\left(\frac{1}{4}Q_1+\frac{1}{2}\rho^2nq_2\right)\right],$$

$$=\frac{1}{\rho^2}\left[\left(q_1Q_2-Q_1q_2\right)-\frac{n\rho^2}{2}\left(q_1^2+q_2^2\right)\right],$$

$$\frac{d\varphi}{d\tau}=\frac{1}{4\rho^2}\left(q_1Q_2-q_2Q_1\right)-\frac{1}{2}n\rho^2,$$

$$=\frac{1}{4r_1}\left(n\frac{\partial W}{\partial\varphi}\right)-\frac{1}{2}nr_1,\qquad \left[\text{Using Equation (2)}\right]$$

$$2\frac{d\varphi}{d\tau}=\frac{1}{2}\frac{n}{r_1}\left(2G\right)-nr_1,\qquad \left[\text{Using Eqaution (2)}\right]$$

$$2\frac{d\varphi}{d\tau}+nr_1=n\frac{G}{r_1}. \tag{14}$$

From Equations (13) and (14),

$$\frac{nG}{r_1}=r_1^{\frac{3}{2}}f\left(\sqrt{r_1},2\varphi\right),$$

$$\Rightarrow nG=r_1^{\frac{5}{2}}f\left(\sqrt{r_1},2\varphi\right),$$

$$G-\frac{\rho^5}{n}f\left(\rho,2\varphi\right)=0. \tag{15}$$

Since Equation (15) corresponds to the Equations (1), (4) and (7), hence it is easy to say that the periodic orbits of collision must exist in all the above cases with different perturbations.

## III. CONCLUSIONS

In order to prove the existence of periodic orbits of collision in the Circular Restricted Four-body Problem, we have discussed the main problem into three sections. In the first section, conditions of periodic orbits of collision have been developed in Case I, Case II and Case III by the method of Bhatnagar [1, 2]. The condition given in Case I represents the condition of collision orbit when the second primary is a triaxial rigid body and the other primaries are spherical, Case II represents the condition of collision orbit when the second and third primary primaries are triaxial rigid bodies and Case III represents the condition of collision orbit when second primary is an oblate spheroid and third primary is a triaxial rigid body.

In the second section, we have extended the Levi-Civita's condition for collision orbit by the method of analytic continuation corresponding to the condition of collision orbit in the two-body problem. It is found that our cases of collision orbit are similar to the collision orbit given by Levi-Civita [3]. Thus, in our cases also, collision orbits exist.

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