

Review on Modelling of Beam-column Joints in Steel Frame Building

Dr. Akil Ahmed¹, Mohd Nadeem² and Prof. Khalid Moin³

¹Department of Civil Engineering, Jamia Millia Islamia, New Delhi, 110025, India.

²Department of Civil Engineering, Research Scholar, Jamia Millia Islamia, New Delhi, 110025, India.

³Department of Civil Engineering, Jamia Millia Islamia, New Delhi, 110025, India.

Abstract: *In analysis and design of a steel frame building, the structural engineers model the frame structure in anticipated software. Mostly in the modeling, the beam-column joint has not been considered semi-rigid due to its complex geometry, lack of knowledge, high computational cost and its unknown behaviour. Furthermore, the semi-rigid joint depends on the behaviour of moment rotation curve ($M-\theta$). To understand the behavior, the semi-rigid joint is classified into different types. Mostly, the joint is assumed as pinned or fully rigid as a result beam-column joint is uneconomical, and has low ductility and less energy dissipation. The pinned connection implies that no moment can be transferred through beam to the column. It means that, the beam-column joint has no rotational stiffness, transfer the axial, and shear force through beam to column. For fully rigid joint, rotational behaviour is there and some moment is transferred to the column through beam. The behaviour of beam-column joint has finite stiffness and therefore it may be categorized as a semi-rigid joint. In this review paper, a study is carried out on the different types of semi-rigid joint, its moment rotation behaviour and different methods have been used for the analysis and design such as empirical, mechanical, and component methods, and finite element-based software example ABQUAS, ANYSIS, SAP2000 and STAAD Pro.*

Key words: *Steel frame, semi-rigid joint, moment rotation*

1. Introduction

Past few decades, the researchers had conducted numerous experimental tests on semi-rigid joint for steel frame structure. The experiment results data are collected and compiled them in the data bank. The results of these experiments are presented with the various types of beam to column joints and selection of the geometry, material, weld and bolt properties for each joint classified as semi-rigid or rigid joint as well as the behaviour of moment- rotation curve of beam-column as semi-rigid joint.

Wilson and Moore [1] in 1917 conducted studies on semi-rigid joints on riveted connection in steel frame structure. Since then, the studies begun to develop the moment rotational relationship of semi-rigid joint and noted their steel frame structure behavior. Young and Jackson [2, 3] in Canada mentioned these reports into the Steel Structures Research Committee. After that, there have been many experiments, mathematical expressions and simulation using softwares for the behaviour of semi-rigid joint in steel frame structure building. Young and Jackson and Rowan and Batho [2] and [4] proposed a graphical beam line method to evaluate each joint end restraint of beam-column in steel frame and through experimentally obtained the moment rotation curve. Rathbun in 1936 [5] proposed to analyze the steel frame structure with semi-rigid joint by slope-deflection method and the moment distribution method and the report of results was submitted to Committee and others. Monoforton and Division [6] in 1963 studied on the analysis of steel frame structure, to consider the beam stiffness matrix effect in the beam-column joint. Lionberger and Mechanics [7] in 1969, and Suko and Adams [8] proposed a method to analyses of the steel frame structure by dynamic analysis method and the joint were taken as elasto-plastic, which were modelled as equivalent springs. Moncarz and Gerstl [9] developed the basic stiffness matrix to analyze the modification of semi-rigid in steel frame structure based on this new approximated technique. Moncarz and Gerstl, Jones, Kirby, and Nethercot [9] [10] in 1981 proposed a revised review on analysis of steel frame with joint as semi-rigid joint and their studies were prolonged by Nethercot [11] in which he had recommended other procedures and improvements by assuming the basic matrix stiffness technique to the analysis of the semi-rigid joint for steel frame. Nethercot [11] published the first European data bank in 1985 for steel frame beam-column joint. In his studies, more than 700 experimental tests were conducted in between 1915 and 1985, and in addition of the connections examined by the Goverdhan databanks, it contained the T-stub beam-column joint with and without web angles. Eric M Lui and Chen [12] and Goto and Chen [13] in 1987 used small computers to solve the stiffness matrix for the analysis of semi-rigid connection in steel frame structure. Several mathematical expressions were proposed to find out the relationship between moment-rotation behaviour of the semi-rigid joint. From the results, it was observed that the effects of flexible joint had the influence on the strength, deflection and internal force distribution of steel framed structures, the study proposed a computer-based method that can be easily applied and simple in formulation for analytical method with numerical stability.

2. Methods of modelling of semi-rigid joint

The true behaviour of semi-rigid joint depends upon the moment rotation curves ($M-\theta$). Nowadays, numerous models can be applied to evaluate the true behaviour of moment rotation curve. The frequently used models are analytical, empirical, mechanical, and finite element method-based software. The most accurate behaviour of the beam-column joint

is determined from the experimental tests. Experimental test results uneconomical and time consumed for structural design practice and it is being restrained commonly for research purposes only.

Experimental Model

Wilson and Moore [1] conducted the first experiment to determine the rotational stiffness of semi-rigid joint in steel frame structure. Rowan and Batho [4] conducted beam-column joint tests that focused on the riveted joint until 1950. Since then, a high yield strength bolts HY80 of different grades of steel is being widely used in steel construction. Jones et al. [9] reviewed numerous research papers and collected 323 tests from 29 separate studies. A large number of tests has been conducted and reported. Goverdhan in 1983 developed the first databank in between 1950 to 1983 in US. Based on 230 experimental tests results on the behaviour of moment-rotation curves of semi-rigid joint of steel frame structure, he digitized them and formed a database of joint behaviour. In this databank different types of semi-rigid joint like double web angle, single web angle/plate connections, header plate, end plate and top and seat angle connections were considered [14].

Kishi, Chen, Matsuoka, and Nomachi [15] prepared the world data bank from 1936 to 1986 to collect the experimental tests which were carried out, more than 303 tests results compiled and developed the Steel Connection Data Bank (SCDB) program. Abdalla and Chen [16] carried out the collected 46 more experimental tests results of semi-rigid joint of beam to column in steel structure frame. Semi-rigid joint included the joint with single angle, web cleat/plate, double angle web cleat connections, top and seat angle cleats connections with or without web angles, extended and flush end-plate connections and header-plate. These test data were collected in the SCDB. Weynand [17] and Weynand, Huter, Kirby and Cruz [18] developed SERICON data bank and included European test results only. In this data bank, tests results were included both single components connection as well as composite connections. It is further extended by Cruz et al. in the SERICON II database. This data bank was mostly used for the comparison of the results which were carried by mathematical, empirical, software technique etc. and intended on the evaluation of the semi-rigid connection performance to its geometrical and mechanical properties. Despite, it was used in structural design practice. However, the structural engineer has only a minimum chance for finding data from the data bank as the precise structural detail of the connection studied, due to the much diversity of types of joint, stiffening, coupling the global and structural analyses to beam-column joint. In steel frame analysis, rotational behaviour is necessary, each studied good agreement. It has been done with use of moment rotation curve. Faella, Piluso and Rizzano [19] detailed the panel zone and geometrical properties.

Takhirov and Popov [20], conducted two tests on bolted column beam semi-rigid joint in steel frame under seismic condition. 32 tests [21] were conducted on bolted T-stub beam-column connections made up of welded plates. 8 tests were conducted to evaluate the ductility of extended end-plate connections. Daz, Mart, Victoria and Querin carried the studies on the reviewed papers of the behaviour of the joints. In their paper, three areas were covered of steel beam-column joint research, analysis methods of semi-rigid joints, prediction methods for the mechanical behaviour of joints and the mathematical expression of the moment rotation curve. Daz et al. [22] discussed about the different method of analysis of semi-rigid joints, slope-deflection equation, moment distribution methods, matrix stiffness methods, to iterative methods and software analysis.

Coelho et al. [23] conducted experiment study on semi-rigid joint behaviour of high strength steel end-plate connections. A. Coelho, Bijlaard, et.al 2009 carried out the experimental tests study on the behaviour of high strength steel web shear panels in beam-column connections, in which they suggested other techniques and improvements by assuming the basic stiffness matrix for the analysis of semi-rigid joint of steel frame structure. Shi, Shi, Wang [24] conducted 5 experiments on joint by bolting with extended end-plate connections to provide an analytical model to obtain the moment-rotation curve and stiffness rotational of a joint. Piluso and Rizzano [25] carried out the study, under cyclic loading based on the experiment and modelling of bolted T-stubs beam-column joint and analysis of the frame. Munse et al. [26] carried out experimental tests in the University of Illinois. In their four specimens' tests, the beam-column section size was considered W18x50 and W12x65 respectively. The beam-column joint was consisted of angle L6x4x3/8x11-1/2 w, and high strength bolts to study their effects on beam-column joint behavior. The experiment test results displayed that the moment to- shear ratio, the performance of the double web angle connection was independent. Throughout the experiments, double web angle connection was found to be failed due to the angle of flexure, unlike the bolted connection. The position of center of rotation had moved its starting position in mid-depth of the beam and toward the compression flange. It was about 80% length of the angle from the tension end and developed the moment rotation relationships.

Liew, Chen, and Chen [27] carried out the many experimental tests at the University of Illinois, they obtained data from the previous experimental research. From the experimental test, they obtained the load deformation results and suggested a formula for the performance of moment rotation curve for beam-column connection with double angle. For this aim, Beaufoy et al. 1948 [28] considered division of an angle into many small elements to apply the characteristics load-deformation and in an additional tensile and compression tests on double web angle specimens. Sommer [29] carried out four experimental tests to evaluate the moment-rotation relationship of the double angle web connections. Furthermore, numerous experimental tests were performed on shear beam-column connection at the University of Toronto. It was concluded from the results that the behaviour of double web angle connections tested was much alike to that of header plate beam-column connection. However, compression flange of the beam in contact with the column flange were found

to be not similar. In compression zone, beam web developed more bending moment due to the stiffening effect, when bearing occurred in beam-column connection with double web angle. Azizinamini et al. [30] carried out the study on the performance of different types of connection under monotonic and cyclic loading. Many researchers recommended experimental results for strong references used in different analyzed methods. Azizinamini suggested an equation, to calculate the behaviour of moment rotation ($M-\theta_r$) curve of semi-rigid connection. Other method developed to calculate the cumulative fatigue of bolted angle, beam-column connections under cyclic loading.

McMullin and Astaneh-Asl [31] carried out the study on double web angle beam-column joints with seven full scale tests at the University of California, Berkeley. The true behaviour of moment-rotation and shear-rotation curves was obtained under shear and moment from the tested angle connections. Experimental tests were conducted with the varying properties of geometrical angles, diameter of bolts, and number of bolts. The tested connection of double web angle specimens was connected with the bolting to the web of beam. The specimen welded to column flange with length of weld 216 mm to 660 mm and 6 mm to 8 mm weld sizes were considered. Double web angle in steel frame beam-column connection could be categorized into three distinct parts throughout their experimental tests and observed the behaviour of loading, shear beam, a tee-range and a compression zone.

Owens and Moore [32] reviewed a number of tests on double web angle, end plate connection to beam-column joint, which were carried out by the Building Research Establishment. Furthermore, they had shown numerous methods of displacement analysis, which were conducted in the Steel Construction Institute. Eleven numbers of experiment tests were studied for the prediction of the behaviour of double web angle beam-column joints and carried the axial tensile forces. In the beam-column connection, ten tests specimens of angle were considered with bolted connection between the beam web and column flange respectively. While in the other test specimen angle was bolted to a column web. A test set-up was arranged with inverted tee section to simulate the performance of double web angle connections through different axial loads. The other four important failure modes that the author précised were for double web angle in beam-column joints, such as bolt drawing of cleat web, the failure behaviour of the cleat web, the bearing failure of the beam web and lastly, the breakage of web cleat near to the heel. Now taking an account on the experimental test results, the following assumptions were made:

- i) The axial tensile force has greater resistance as compared to the end plate, which is provided by the double web angle in beam-column joints.
- ii) In the bolted connection, high deformations occur due to the prying forces in the bolts, where the axial tensile loads were expressively more than those to normally encountered.
- iii) The design of double web angle in beam-column connections were directed by axial tensile force which were not likely to be essential, for tall buildings they might affect the end plate connections.

Astaneh –Asl and Ho [33] carried out the studies on the two categories of double web angle connections by testing with different specimen properties and observed the actual cyclic behavior of double web angle connections. The tested specimen angle was connected to column flange and A325 bolts were used in the beam web. In other experiments, the double angle specimens were welded of size 6 mm to 8 mm thick to a flange column and bolted to a beam web. High ductility had been sufficiently shown by double angle connections in the experimental tests. Unsymmetrical behaviour of hysteresis curve showed very small stiffness and strength by pulling action, but larger strength and stiffness by pushing of column by the beam. Failure of bolt and angle in bolted connections occurred due to yielding shear and angle of rupture. In spite of beam-column connection by bolting and welding failure took place due to the rupture of welds with a minor degree of yielding in connected angle. Due to cyclic axial loading, beam-column connection weakened in shear strength which was more severe in bolted-bolted connections, but not much observed in bolted welded connections. In the connection the prying effect action should be considered due to the joint flexibility by angle.

Shen and Astaneh [34] performed the study on the behaviour of bolted angle connections, under cyclic loads and their failure modes along with deformation patterns. Garlock, Ricles and Sause [35] investigated the connection strength, stiffness and capacity of energy absorption, and the low cyclic fatigue life of semi-rigid joint. The geometry of the bolted connections was considered with top and seat angle under cyclic loading to study the performance of size angle and bolt gage distance. Tsai and Popov [36] conducted three tests on large-size beam end-plate moment to column flanges under severe cyclic loading. Such connections were considered in the erection of low-rise steel frames. Stewart [37] carried out the studies on eight beam-to-column connection with flush end-plate specimens and tested under monotonic and cyclic loading. These test specimens showed a stable cyclic behaviour up to a defined rotation limit, and their response of the test specimens indicated that this type of connection could be considered to seismic design. Sumner and Murray [38] carried out on eleven full-scale cyclic loading test of column - beam, extended end-plate connection. The experimental results revealed that the extended endplate strong moment connections can be detailed and designed in earthquake resistance moment connection in steel frames. As the strong connection, strong column and weak beam theory should be applied. The requirement of the moment connections in strength, stiffness and ductility for seismic regions, provide four bolt unstiffened extended plate and the eight-bolt stiffened extended end-plate moment connections.

Mathematical models

In traditional analysis and design of steel frame structure, the behavior of beam-column joint is considered ideally pinned or perfectly rigid. However, numerous experimental studies have clearly shown that the actual joint behaviour has been nonlinear due to gradual yielding of joint. The component connections were consisted of angles, plates and bolts, etc. A beam-to-column connection is generally subjected to axial force, shear force and bending moment. However, in the analysis of joint the axial and shear deformations are small and have been neglected, only flexural deformation is considered. The rotational deformations have been considered for semi-rigid joint due to flexural action. In the analysis of semi-rigid joint in steel frame the moment rotation ($M-\theta_r$) curve have been incorporated. The moment rotation relationship is usually considered in the analysis by using mathematical functions. The joint behaviour can be expressed in term of moment rotation $M-\theta_r$. Mathematically, this can be expressed in the form:

$$M = f(\theta_r)$$

Or

$$\theta_r = g(M)$$

Numerous mathematical expressions have been suggested for appropriate moment-rotation curves from the experimental data, such as linear, bilinear, trilinear, multilinear, power, logarithmic, exponential and polynomial functions.

Linear model

Rathbun [39] proposed a linear model simplest equation as below but it is less accurate and overestimates the joint stiffness at finite rotation. However, this model is easy to apply the moment rotation behavior, utilizes the initial stiffness k_i to represent the behaviour of beam-column connection $M = K_i\theta$

Bilinear model

Tarpy and Cardinal [40] Melchers and Kaur (1982) [41] and Lui and Chen [42] also recommended the bilinear model, the initial slope of moment-rotation curve is exchanged by a lower line at a certain changed moment.

Trilinear model

Shi et al. [24] and Pirmoz et al. [43] proposed a tri-linear curve of moment-rotation relations. For transition of elastic to plastic behaviors, the characteristic feature of the curve i.e., at an intermediate zone of stiffness incorporated in the analysis. However, steady reduction of stiffness was observed in the experiment result.

Multilinear model

Moncarz and Gerstl [9] proposed a tri-linear curve with five parameters- the rotational stiffness of the first yielding moment, plastic moment, post-yielding rotational, and plastic rotational stiffness of the beam-column joint. In addition, Razzaq [42] proposed a piecewise multilinear model in which the nonlinear moment-rotation curve. It is considered approximately a series of straight-line segments.

Nonlinear model

Ramberg and Osgood [44] proposed the first nonlinear model for connection to incorporate moment rotation ($M-\theta_r$) curve in the analysis. Generally, these three parameters depend on rotational stiffness, rotation of the joint, and shape factor. The curve develops bilinear effect to perfectly elastic to plastic behaviour. The shape factor tends to infinite where the point of plastic moment of beam-column joint is equal to the required location of the moment.

Cubic B spline model

Jones et al. (1981) suggested an equation, which provides the segmented line as cubic B spline formulation as follows

$$\theta = \theta_i + \sum_{j=0}^n b_j (|M - M_j|)^3$$

Where $|M - M_j| = \{M - M_j \text{ or } M > M_j\}$ and $\{0 \text{ for } M < M_j\}$

Where, θ_i is initial rotation ($\theta_i = 0$) and 'n' is the number of points, which lies in between two basic portions of the moment-rotation curve., M_j is higher moment of the j^{th} part of the moment rotation curve, and b_j is the curve fitting constant which can be determine by least-squares method.

Empirical models

Empirical models are represented by mathematical expressions, which related to moment-rotation curve and found from experimental data. Different parameters have been considered to geometrical and mechanical properties of beam-to-column connections. The formula was derived from the regression data analyses and can be obtained in various ways such as: experimental testing, finite element (FE) models in parametric analyses and mechanical models. These global models fit a skeleton curve through the experimentally obtained key parameters such as initial stiffness, moment

capacity, post-hardening stiffness and rotation capacity as it represents the full behaviour of moment-rotation curves in the connections. In the steel frame structure, column to beam joint is an important component. To overcome, this problem a number of experiments, empirical relation, and software results are available. Nowadays, we know the mathematical expression and with finite element-based software the column- beam joint can be modelled.

Polynomial model

Frye and Morris [45] proposed a polynomial odd function model, and based on the method of least squares, the constant of the polynomial was evaluated. The mathematical developed model, M - θ_r relationship is as follows

$$\theta_r = C_1(KM)^1 + C_2(KM)^3 + C_3(KM)^5$$

Where, M and θ_r are the moment and rotation, respectively; C_1 , C_2 , and C_3 are curve-fitting parameters; K is a standardized parameter, which is depends on geometrical parameters such as size of the connected member and plate thickness. The disadvantage of this odd polynomial function is that its first derivative, which correspond to the rotational stiffness, may be adopted either positive or negative values. The negative values of rotational stiffness are completely unacceptable in structure. In the analysis of frame structures, negative stiffness cannot be accepted due to numerical difficulties.

Picard et al. [46] presented a similar model as of Frye and Morris (1975) [45] as shown below:

$$\theta_r = C_1(KM) - C_2(KM)^2 + C_3(KM)^5$$

In this nonlinear model equation, the disadvantage of the polynomial equation is that obtained results have negative slopes in certain ranges of parameters, but in actual it is not possible and not reliable with the behavior of connections.

Power model

In steel frame structure, the beam-column connections were considered in various ways. To determine the behaviour of moment rotation curve, many power models have been proposed such as two or three parameters are required in their functions for the beam-column connections.

Batho and Lash [48] attempted the fit experimental tests to mathematical expressions. They used a general form of power function that relates the applied bending moment and joint rotation.

$$M = C\theta^{0.412}$$

Where 'C' is a constant that depends on the beam-column joint and dimensions, to evaluate the rotational behaviour of end plate connections

Krishnamurthy conducted a broad parametric study of 168 connections and 559 loading conditions by means of a two-dimensional (2D) finite elements model. This numerical model had been previously validated against thirteen experimental tests on end plate connections. Krishnamurthy and Graddy [49] studied on 2D and 3-dimensional joint model with finite element method (FEM) method. The column to beam joint, end plate connection with bolt used, modelled with an eight-node brick element for the prediction of elastic linearly in end plate connections response. In the analysis, he included both contact and pretension bolts and simulated. However, due to the limited use of computer technology at that time, the 3D structural models were used to extend a correlation factor between (2D) and 3D results to allow the estimate of the good 2D results and values from the minimum expense of 2 Dimensional results.

Richard and Abbott [50] suggested three types of parameter which is power model to represent the behaviour of moment-rotation curve of joint under monotonic loading. The model equation represented by the following equation:

$$M = \frac{R_{ki}\theta_r}{\left[1 + \left(\frac{\theta_r}{\theta_0}\right)^n\right]^{n-1}}$$

Where, 'n' shape parameter, R_{ki} is joint stiffness (initial), M_u is the moment capacity. Since the tangent stiffness R_{ki} of the joint, the relative rotation θ_r can be evaluated directly without the iterations and above equation may be used an effective tool for directing a second-order nonlinear analysis of beam-column joint as a semi-rigid.

Frye and Morris's polynomial model and power model require the previous information of the initial stiffness, ultimate moment capacity of a joint and whole estimation of the moment-rotation curve response. Therefore, its prediction to the response depends on the two essential values and that should be calculated by using different model equation.

$$R_{ki} = \frac{dM}{d\theta_r} = \frac{R_{ki}}{\left[1 + \left(\frac{\theta_r}{\theta_0}\right)^n\right]^{\frac{(n+1)}{n}}}$$

In Frye and Morris's polynomial model, equation is improved for related with the evolution of negative slope or tangent stiffness of joint, when its differentiation is considered within specific ranges.

Ang and Morris [51] suggested by using of a standardized Ramberg-Osgood model equation to the behaviour of moment-rotation curve for different five types of semi-rigid joint - the single web, double web-angle, header plate, top-and seat-angle, and angle connections. The developed equation model is given as

$$\frac{\theta}{\theta_0} = \frac{M}{M_0} \left[1 + \left(\frac{M}{M_0} \right)^{n-1} \right]$$

Where $M_0 = K_0 \theta$

Where n , θ_0 , and M_0 are constants that depend on the geometry of the section and used beam-column connection.

Logarithmic models

Wu and Chen [52] proposed a logarithmic relation model with including three parameters.

$$\frac{M}{M_u} = n \cdot \ln \left(1 + \frac{\theta}{n\theta_0} \right)$$

Where, M_u is the ultimate moment capacity of connection, “ n ” is the shape parameter, and $\theta_0 = M_u / K_i$. where, K_i is the initial stiffness of the connection. In their model, the slope of the curve tends to zero. Considering the zero plastic stiffness, the relationship is given as:

$$C = \frac{359 \times 10^{-6} (P_f)^{2.227}}{(h_b)^{2.616} (t_{wb})^{0.501} (t_{fb})^{0.038} (d_b)^{0.849} (g_b)^{0.519} (b_{ep})^{0.218} (t_{ep})^{1.539}}$$

Lee and Moon [53] recommended a logarithmic relation model with taking two parameters of the nonlinear moment rotation curve of beam-column connection with angles.

$$M = \alpha [\ln(n \cdot 10^3 \cdot \theta + 1)]^n$$

Where, α and n are shape parameters for the curve. Demerit of the Lee-Moon model is that its parameters hold no physical meaning. In addition, it did not consider the initial stiffness and plastic stiffness of joint clearly.

Exponential model

Lui and Chen [54], proposed an exponential moment-rotation relationship as follows:

$$M = M_0 + \sum_{j=1}^n C_j \left[1 - \exp\left(\frac{|\theta|}{2j\alpha}\right) \right] + K_p \theta$$

Where, M_0 is the initial moment, K_p is the plastic stiffness, ‘ α ’ is the scale factor for numerical stability, and C_j is a parameter evaluated from linear regression analysis and the experimental based data ‘ m ’ is the number of curve fitting.

Kishi et al. [55] modified the Lui-Chen [54] model equation and in addition to linear model component of the moment-rotation curve as follows:

$$M = M_0 + \sum_{j=1}^m C_j \left[1 - \exp\left(\frac{|\theta|}{2j\alpha}\right) \right] + \sum_{k=1}^n D_k H(\theta - \theta_k)$$

Where, D_k and C_j are the parameters of curve-fitting and evaluated by linear regression; θ_k is the initial rotation of the k_{th} linear component of the moment-rotation. However, the exponential models of Liu-Chen and Kishi-Chen model represent the moment-rotation curves with more accuracy, by using numerous parameters and use of series[56].

Yee and Melchers [57] proposed a four-parameter exponential model as follows:

$$M = M_p \left\{ 1 - \exp \left[\frac{(R_{ki} - K_p + C_j \theta_r)}{M_p} \right] \right\} + K_p \theta_r$$

Where, M_p is the plastic moment capacity, K_i is the initial stiffness joint, K_p is the strain stiffness hardening, and C_j is a constant depending on slope of the fitting curve. In the bolted extended end-plate eve connections, dimensions were considered to develop a moment rotation relationship as expressed above. The model established the moment-rotation curves taking into consideration, deformation characteristics and the possibility of failure modes of connection. In this model, the deformation of connections was observed as bending in end-plate and column flange, extension of bolt; shear deformation in column web panel and column web compression. Furthermore, bolt failure in tension, plastic mechanism formed in end-plate and column flange, web crippling, shear yielding in the column and column web buckling were some modes of failure.

Johnson and law [58] proposed a similar method, to considered four parameters, in exponential model expression to evaluate the initial joint stiffness and plastic moment capacity of flush end-plate beam-column connections.

Chisala [59] developed a three-parameter exponential model expression as given below:

$$M = (M_0 + K_p \theta) \left[1 - \exp \left(\left(\frac{-K_i \theta}{M_0} \right) \right) \right]$$

Where, K_i and K_p are the elastic and plastic stiffness respectively, and M_0 is the initial moment i.e., point of first intersection of tangent in the desired curve. In the model of Chisala, the three parameters were included, independent to each other and observed results were acceptable. Thus, Chisala's model was able to overcome the inadequacies of the earlier models.

Shoore and Mofid [60] extended the Chisala [59] model and developed a five-parameter model in exponential form as given below:

$$M = \alpha K_p \theta + (M_0 + (1 - \alpha) K_p \theta) \left[1 - e^{\left(\frac{-(k_i - \alpha k_p)(1 + \beta \theta) \theta}{M_0} \right)} \right]$$

In the above expression, the parameters K_i , K_p and M_0 are same as in the Chisala's model and α , β are the shape parameters of the curve. The above expression will same as of Chisala's model, if the values of α and β are zero. Livesley [61] also proposed same procedures to analyze the semi-rigid joint of steel frame. A linear moment rotation (M - θ .) relationship was considered in the analysis and the linear semi-rigid joint factor which is $G = \theta/M$ was taken into modifying the stiffness matrix of beam.

Finite element model

Finite element method (FEM) of analysis is most suitable to evaluate the rotational behaviour moment rotation curve for the semi-rigid joint in steel frame structure. To study, the moment rotation curve of a connection in steel frame structure are complex due to its geometrical and material nonlinearities. FEM based software like ABAQUAS, ANSYSIS, etc., made possible to understand the result of the joint after the software analysis under the different loading condition. These results of analysis were associated with experimental tests, and helping in calibration with the finite element models-based result. The obtained results from experiments agreed with the software results. Generally, FEM analysis of beam-column joint in model consideration are strain-hardening, large deformations, plasticity, effects of instability, contacts between pre-stressed bolts and plates, the representation for large strain and/or displacements.

Bose et al. [62] conducted the first study on column to beam joints connected by welding in steel frame structure analyzed by finite element method considering the strain hardening, plasticity, and buckling. The FEM results agreed to the available experimental results after comparison. Since then, numerous researches have been done to investigate the joint behaviour through FEM. Kukreti et al. [63] also proposed the same studies on the analysis of beam-column joint with end plate, by 2D FM and 3D FM model for the estimated the behaviour of moment-rotation curve. Since the model in 3D FEM of the analysis was difficult and more complex while in 2D analysis by finite element method, considering plane stress rectangular elements and they have conducted to obtain the basic estimated equations. Levy (1971) developed the 3D subparametric elements for the flush end plate, bolt of shank, head of bolt, the flush end plate connected by welding to the beam flanges. However, eight-noded plane stress rectangular hybrid elements were considered for the web of beam which was connecting to the end plate of column.

Patel and Chen [64] carried out the study by using different nonlinear FEM program at the University of California, Berkeley to analyze the stiffness matrix, the stress distribution of panel zones, and three different types of moment connections of plastic zones, 2D plane stress isoparametric finite elements considering flange moment connections. Bolted model connection was used that contained pretension and shear load, modeled with two bar elements. In comparison with experimental test results to the finite element analysis gives good agreement for only welded flange connections. Chasten Lu and Driscoll [65] carried out by FEM applying shell elements for the end-plate and beam flanges and plane stress elements to the beam web. For evaluating the prying force, the end-plate and column flange were modelled with unstiffened large extended endplate of connection with eight bolts at the tension flange.

Zhu et al. [66] proposed empirical and effective methods to evaluating the dynamic increase factor to increase the applied loads on portal steel frame with semi-rigid connection. To calculate the dynamic responses, a nonlinear static alternate path analysis has been used. In their first study, the polynomial models of the extended end plate and the top and seat column to beam modified, and suggested a polynomial model of the flush end plate connection. It showed that the results were acceptable with experimental results. In other hand, in beam model combining effect had been considered with nonlinear spring elements, plastic hinges to incorporate the effect of connection flexibility and material nonlinearity. To obtain the dynamic increase factor depend on a combination of the push down analysis and nonlinear dynamic analysis, a new step-by-step analysis procedure was established and proposed a function of the maximum ratio value of energy demand to energy capacity of an affected beam. They recommended by curve fitting data points already given by the different analysis, and five types of semi-rigid connections with different column removal scenarios.

Bursi and Leonelli [67] conducted an analysis to the elasto-plastic response of end plate in beam-column connections by using finite element based commercial software ABAQUS. The beam-column connection model in 3D nonlinear finite element was used and considered twenty-node with second-order hybrid solid elements and basic contact elements. Also bolt model with shanks and head by using standard two-node beam elements, web and flange of beam, and welding thick

part were modeled with rigid solid elements. Sherbourne and Bahaari (1994 and 1997) [68] and [69] carried out a study on the behaviour of steel bolted end-plate joints by Finite element method. In this model, plate elements were considered to end plate, column flanges, beam flanges, webs, and stiffeners and by using six spar elements bolts were modelled. In this model, boundaries were taken between column flange and end-plate back and it may make or break contact by 3D interface elements. Citipitioglu et al. [70] studied on different types of bolted 3D models, column to beam connections with angles, bolting contact between connecting parts with effect of friction and confirmed that the frictional effect on the initial stiffness of the connection was insignificant. However, they observed it slightly more on the plastic region. The pre-tensional bolt effect was same to that of friction, while it might be modifying the ultimate moment capacity of the connection by 25%.

Gantes and Lemonis [71] proposed a FEM model of the steel frame beam-column connection from T-stub by bolting. The Geometric and material nonlinearities and also contact and friction were implemented. The finite element (FE) analysis results validated with experimental results data. The effective length bolt was taken in the model and analyzed the steel frame to obtain the results on its behaviour. A 3D finite element elasto-plastic beam-column joint model [72] was studied on their structural behaviour of butt-type steel bolted joints. The FEM results demonstrated the bolted joints nominal capacities and calculated the obtained results by FEM and they were acceptable with the AISC specification. A parametric study [73] was conducted on the behaviour of column to beam joint with bolted extended end-plate. The 3D FE modelling of the joint and results were calibrated with experimental tests results. In the analysis, material nonlinearities, discontinuities, large displacements and geometric discontinuities of joint were considered while they are in contact. The experimental data were compared with numerical results for the moment-rotation curves, flush end plate displacements and bolting forces.

Abolmaali et al. [74] carried out the studies on Ramberg-Osgood and three-parameter power model to estimate the behaviour of moment-rotation ($M-\theta$) curve, with flush end-plate beam-column connections with one row of bolts in tension and compression flanges. A finite element model (FEM) of the connected region along with the connected beam and column is proposed to load deformation analyses including the material, geometric, and contact nonlinearities. The FEM model was validated with experiments test results obtained and noted in literature during the 1980s, for flush end-plate joint of steel frame structure. A matrix of test cases was developed by variable geometry of flush end-plate joint within their acceptable ranges. The moment rotation ($M-\theta_r$) data of these tests results had been obtained by FEM analyses, after that, curve fitted to Ramberg-Osgood and three-parameter power model equations to find the defined parameters from these equations. Lastly, predicted equations were obtained for parameters of the model equations. It depends on the varying geometry of the flush end-plate beam-column connections. Akbar Pirmoz et al. [75] carried out studies on numerous 3D FE models with geometric and mechanical properties considered as parameters. In this research, the behaviour of beam-column joint with bolted top-seat angle and web angles was studied considering the combined effect of moment and shear force. In this model, components of connection like beam, column, angles and bolts were modelled by solid elements. The contacts between surfaces were simulated with surface-to-surface contact elements. The experimental results associated with FE analyzed results were found satisfactorily agreed.

Mohamadi et al. [76] proposed the numerous parametric results analyses on the initial rotational stiffness of connection by bolted flush end-plate splice beam. Finite element software was used for modelling by considering 20-noded brick elements, material behaviour, geometrical discontinuities and large displacements. These models were validated for three case studies as given in the literature with the obtained results and compared them in good agreement. Lemonis et al. [77] developed method to determine the moment rotation curve of steel frame beam-to-column joints based on the component method. In this work, some cases were observed, including bolted connections with end-plates and connections with angles. The results were found with good agreement when they were compared with experimental tests results and advanced FE models in terms of strength, stiffness and rotational capacity of joint. Dai et al. [78] carried out a simulation study for 10 fire tests on restrained steel beam to column connections as a semi-rigid of five different types of beam-column connection. 3D solid elements were considered in modelling the basic structural members. The results showed satisfactorily good agreement between observed experimental result and numerical results.

Godrich et al. [79] carried out the study on the beam to column connection for the partial strength of rotation capacity. This was numerically modelled by means of CDM model. A FE 3D model which included the Lemaitre formula for the estimation of the end plate failed in ductility. Therefore, the overall moment-rotation behaviour from the initial stiffness was obtained just before initiation of fracture to get high degree of correctness and was calibrated with experimental tests results. Khodaie et al., [80] proposed the finite element models to predict the behaviour of semi-rigid connections in steel frame accurately, i.e. the results were close to the experimental observations. A well-calibrated finite element model can reduce the number of experiments that needs to be studied in order to prediction of certain behaviour in a parametric sense. However, the FE models incorporating semi-rigid connections require intensive computational effort and time which may be impractical for the analysis of large structures. Wang et al. [81] studied on the seismic behaviors of steel frame for semi-rigid joint end-plate connections, model in ABAQUS software, a competent and specific finite element FE model well-known, subjected to cyclic loadings. In this element types, material cyclic component models, and contact models for bolts, end plate and members have been taken. In this, geometric nonlinearity and material nonlinearity were considered. The results of several models had been verified by typical quasi-static tests of end-plate connections and included both hysteretic curves and failure modes. The result exhibited that, this type of connection is a strong tool of finding the behaviors of beam-column connections of steel frame.

So many research papers have recently been published on the moment rotation behaviour by finite element software – Pirmoz et al., 2009; Yang and Tan, 2012, Mohamadi-Shooreh. 2013; Prinz et al., 2014), [82], [83]; Brunesi et al., 2014 (seismic and cyclic loads); Wang et al., 2013; Fang et al; Rahbari et al., 2014, (high loading rates); and Li et al., 2012 (elevated temperatures); Pakala et.al., 2012; Selamat and Garlock, 2014, Pakala, Kodur and Dwaikat [84]; Selamat and Garlock [85], and so on. These recent articles highlighted the giant potential of numerical techniques for assessing joint behaviour, not only under monotonic loads, but also under extreme loading conditions. However, advances in computational power are keeping in pace with erudite contact algorithms and solvers which also involve a giant amount of computational resources. Therefore, carrying out FE simulations is still a very time-consuming task primarily reserved for research purposes, rather than as a practical design method.

Ahmed [86] carried out the studies on the failure mechanisms for top-seat-angle steel connections and developed a formulation depend on three basic mechanisms of failure, taking shear deformations and bending, also considered effects of prying force action on the top angle and stiffness of the tension bolts to calculate rationally the ultimate moment capacity M_u of the joint, and four parameters in power model which were developed. The results of those FE analyses with experimental failure strategies of the connections were obtained to propose failure mechanisms of top-seat-angle column to beam connections in the study. Pertinence of the proposed formulation is evaluated by comparing moment-rotation ($M-\theta_r$) curves and ultimate moment capacities with those conducted by experiments and proposed by finite element analyses, power model of three-parameter. It is clear in this research that given formulation and Kishi-Chen's method both achieved close approximated moment rotation $M-\theta_r$ curves of all given joints except a few cases of Kishi-Chen model, and ultimate moment M_u by the suggested formulation is more rational than that calculated by Kishi-Chen's method.

Fakih et al. [87] studied on review of research papers on the behaviour of semi-rigid joint for beam to column by using different software like ABAQUAS, ANSYS etc. In this paper, review studies were presented beam-to-column connections with simulation models and can be taken as reference in future examination particularly in analysis and design of steel frame structure. Linear analysis of behaviour of the beam-column connections have been very complex because of there is no instant solution for it. This article provides the foundation for the design and quick analysis, also in addition economically and acquiring the vital strength of beam-column joint of steel frame with many methods like experiments, mechanical and statistical analysis. Numerous academic, research and literature in the field of beam-column joints in steel structure, an overview of current events have made the construction of the system workable in the field of civil engineering.

Mechanical model

The beam-column joint in steel frame may be developed by mechanical model methods. This lies between the finite element models and mathematical models. In mechanical models, the beam-column joint has been considered by a spring's elements of properties as a rigid and deformable element with specified load-deformation characteristics. The joint in steel frame structure are classified in different types. These beam-column joints give the sufficient deformation and different failure of modes. To simulate the joint for the model, accumulate all of the springs to produce the moment-rotation curve of complete joint and develop a system for each elements of beam-column joint by using experimental or numerical, analytical model method. The main feature of mechanical models is that they simplify the process of dealing with the complex behaviour of a connection. Furthermore, the degree of accuracy and applicability of this model without rigorous computations compared with the detailed finite element models make it the best useful practical tool to analyses a semi-rigid joint.

Stefano et al. [88] suggested a mechanical model to simulate the behavior of semi-rigid connection with double-angle subjected to more amplitude cyclic loading. They observed that the angle behavior is mainly dependent on the bending response of the angle leg adjacent to column flange. Beam element with distributed plasticity approach was taken with a kinematic hardening in the material model. However, the model has been used for predicting only the cyclic behavior of the double-angle connection. Abdollahzadeh et al. [89] conducted to evaluate moment-rotation ($M-\theta_r$) curve for connections. The Component methods were used for the analysis. The connection modeled according to different mechanical method in which springs were considered while in components with specific mechanical properties of the connection such as strength, stiffness, and rigid bars. Load -displacement relationship was considered for bi-linear and trilinear-linear curve. The joint behaviour were considered through accumulating the stiffness of different components and also study in laboratory test under the monotonic loading of the joint of I beam-to-CFT column with end plate by bolting. It should be clear that those bolts which were penetrated into the column depth created continuity between flanges of the column. The proposed method had been compared with the experimental test and the advanced FE model in terms of stiffness, strength, and rotational capacity. However, their results showed satisfactorily good agreement.

Prabha et al. [90] studied on the Frye-Morris polynomial model. In this study, they found out the behaviour of the moment-rotation curve of 8 types of angle beam-column connections. They proposed revised equation which was based on the Frye-Morris procedure, and in addition, an air gap as a parameter was considered. The suggested equation is verified with the experimental results and Frye-Morris model and achieved good agreement. In Frye-Morris model method, it was observed the disadvantage that it gives the negative slope which was not acceptable in the joint stiffness. Finally, the Frye-Morris model for double web-angle connection is also modified based on different parametric studies. 10 mm nominal air-gap distance is provided between beam and column assembling the beam-column joint for ease of

fabrication. Rassati et al. [91] proposed a method of advanced component-based model of beam-column connection for cyclic behaviors of partially restrained joint. 22 models have been verified to reproduce complex 3D connection behaviors in steel frame and were not appropriate in practical applications due to huge input data and extra computation.

3. Conclusion

This review paper summarizes the behaviour of semi-rigid joint in steel frame building depending on the moment rotation curve. Several types of connection have been modelled for evaluating the moment rotation curve to analyses the beam-column joint. The researchers published moment rotation curve behaviour by analytical, experimental, empirical, mechanical and finite element method. In traditional design, the beam-column joint is assumed as ideally pinned or fully rigid, but the actual behavior of the joint is somewhere in between and that is semi-rigid joint. The semi-rigid joint is modelled on the basis of moment rotation curve. By using FEM based software, the joint is modelled with bolted connections and the response is noted. Bolted connection is a common design in beam-column connections; however, extra considerations should be taken when designing bolted beam-column connections in accordance with the Eurocode and other codes. In the design of semi-rigid joint, there are some gaps of knowledge that are needed to be incorporated in the design codes for the beam-column connections. This could be completed with comprehensive future research with FEM based software to beam - column connections under different types of load behaviour, types of connections.

References:

- [1] W. W. M. Wilson and H. F. H. Moore, "Tests to determine the rigidity of riveted joints of steel structures," Bull. Illinois, Eng. Exp. Station. no. 104, p. 68, 1917.
- [2] C. R. Young and K. B. Jackson, "The relative rigidity of welded and riveted connections," Can. J. Res., vol. 11, no. 1, pp. 62–100, 1934.
- [3] S. S. R. Committee and others, "First Report, Department of Scientific and Industrial Research." 1931.
- [4] H. C. Rowan and C. Batho, "The analysis of the moments in the members of a frame having rigid or semi-rigid connections. Second Report Steel Structures Research Committee." London HMSO, 1934.
- [5] J. C. Rathbun, "Elastic Properties of Riveted Connections," Trans. Am. Soc. Civ. Eng., vol. 101, no. 1, pp. 524–563, 1936.
- [6] G. Monforton, T. W.-J. of the S. Division, and undefined 1963, "Matrix analysis of semi-rigid connected frames," cedb.asce.org.
- [7] S. Lionberger, W. W.-J. of the E. Mechanics, and undefined 1969, "Dynamic Response of Frames with Nonrigid Connectors," cedb.asce.org.
- [8] M. Suko and P. F. Adams, "Dynamic analysis of multibay multistory frames," J. Struct. Div., vol. 97, no. 10, pp. 2519–2533, 1971.
- [9] P. D. Moncarz and K. H. Gerstl, "Steel Frames with Nonlinear Connections," J. Struct. Div., vol. 107, no. 8, pp. 1427–1441, 1981.
- [10] S. W. Jones, P. A. Kirby, and D. A. Nethercot, "The analysis of frames with semi-rigid connections - A state-of-the-art report," J. Constr. Steel Res., vol. 3, no. 2, pp. 2–13, 1983.
- [11] D. A. Nethercot, Steel beam to column connections-a review of tests data. CIRIA, Construction Industry Research and Information Assoc., 1985.
- [12] E. M. Lui and W. F. Chen, "Steel frame analysis with flexible joints," J. Constr. Steel Res., vol. 8, pp. 161–202, 1987.
- [13] Y. Goto and W.-F. Chen, "On the computer-based design analysis for the flexibly jointed frames," J. Constr. Steel Res., vol. 8, pp. 203–231, 1987.
- [14] A. Goverdhan, "A collection of experimental moment-rotation curves and evaluation of prediction equations for semi-rigid connections," 1983.
- [15] N. Kishi, W. F. Chen, K. G. Matsuoka, and S. G. Nomachi, "Moment-rotation relation of top- and seat-angle with double web-angle connections," in Proceedings of the State-of-the-Art Workshop on Connections and the Behavior, Strength and Design of Steel Structures, 1987.
- [16] K. M. Abdalla and W. F. Chen, "Expanded database of semi-rigid steel connections," Comput. Struct., vol. 56, no. 4, pp. 553–564, 1995.
- [17] K. Weynand, "SERICON-Databank on Joints in Building Frames. COST PROJECT C1," in Proc. of the First State of the Art Workshop, 1992.
- [18] K. Weynand, M. Huter, P. A. Kirby, L. A. P. Simões da Silva, and P. J. S. Cruz, "SERICON-Databank on Joints in Building Frames," in Proceedings of the 1st COST C1 Workshop, 1992.

- [19] C. Faella, V. Piluso, and G. Rizzano, *Structural steel semirigid connections: theory, design, and software*, vol. 21. CRC press, 1999.
- [20] S. . Takhirov and E. Popov, "Bolted large seismic steel beam-to-column connections Part 2: numerical nonlinear analysis," *Eng. Struct.*, vol. 24, no. 12, pp. 1535–1545, 2002.
- [21] A. M. G. Coelho, F. S. K. Bijlaard, N. Gresnigt, and L. S. da Silva, "Experimental assessment of the behaviour of bolted T-stub connections made up of welded plates," *J. Constr. Steel Res.*, vol. 60, no. 2, pp. 269–311, 2004.
- [22] C. Daz, P. Mart, M. Victoria, and O. M. Querin, "Review on the modelling of joint behaviour in steel frames," *Journal of Constructional Steel Research*, vol. 67, no. 5, pp. 741–758, 2011.
- [23] A. Coelho, F. Bijlaard, H. K.-E. Structures, and undefined 2009, "Experimental behaviour of high-strength steel web shear panels," Elsevier.
- [24] Y. Shi, G. Shi, and Y. Wang, "Experimental and theoretical analysis of the moment–rotation behaviour of stiffened extended end-plate connections," *J. Constr. Steel Res.*, vol. 63, no. 9, pp. 1279–1293, 2007.
- [25] V. Piluso and G. Rizzano, "Experimental analysis and modelling of bolted T-stubs under cyclic loads," *J. Constr. Steel Res.*, vol. 64, no. 6, pp. 655–669, Jun. 2008.
- [26] W. H. Munse, W. G. Bell, Chesson E Jr., and E. Chesson, "Behavior of riveted and Bolted Beam-to-Column connections," *J Struct Div ASCE*, vol. 85, no. 3 ST, pp. 29–50, 1959.
- [27] J. Y. R. Liew, W. F. Chen, and H. Chen, "Advanced inelastic analysis of frame structures," *J. Constr. Steel Res.*, 2000.
- [28] L. A. Beaufoy and A. Moharram, "Derived moment-angle curves for web-cleat connections," in *Preliminary Publication, International Association for Bridge and Structural Engineering*, 3 rd Congress, 1948.
- [29] W. H. Sommer, "Behavior of Welded Header Plate Connections Master's Thesis," Univ. Toronto, Ontario, 1969.
- [30] A. Azizinamini, J. H. Bradburn, and J. B. Radziminski, "Static and cyclic behaviour of steel beam–column connections. Structural Research Studies," *Civ. Eng. Dep. Univ. South Carolina*, 1985.
- [31] K. M. McMullin and A. Astanesh-Asl, *Analytical and experimental studies of double-angle framing connections. Structural Engineering, Mechanics, and Materials*, Department of Civil ..., 1988.
- [32] G. W. Owens and D. B. Moore, "The robustness of simple connections," *Struct. Eng.*, vol. 70, no. 3, pp. 37–46, 1992.
- [33] A. Astanesh-Asl and I. Ho, "Behavior and design of angle connections subjected to cyclic axial force and shear," in *Structural engineering in natural hazards mitigation*, 1993, pp. 1232–1237.
- [34] J. Shen and A. Astanesh-Asl, "Hysteretic behavior of bolted-angle connections," *J. Constr. Steel Res.*, vol. 51, no. 3, pp. 201–218, 1999.
- [35] M. M. Garlock, J. M. Ricles, and R. Sause, "Cyclic load tests and analysis of bolted top-and-seat angle connections," *J. Struct. Eng.*, vol. 129, no. 12, pp. 1615–1625, 2003.
- [36] K. Tsai and E. P. Popov, "Cyclic Behavior of End-Plate Moment Connections," *J. Struct. Eng.*, vol. 116, no. 11, pp. 2917–2930, 2007.
- [37] R. W. Stewart, "Analysis of frames with elastic joints," *Trans. Am. Soc. Civ. Eng.*, vol. 114, no. 1, pp. 17–24, 1947.
- [38] E. A. Sumner and T. M. Murray, "Behavior of Extended End-Plate Moment Connections Subject to Cyclic Loading," *J. Struct. Eng.*, vol. 128, no. 4, pp. 501–508, Apr. 2002.
- [39] J. C. Rathbun, "Elastic Properties of Riveted Connections," *Trans. Am. Soc. Civ. Eng.*, vol. 101, no. 1, pp. 524–563, 1936.
- [40] A. N. T. Ihaddoudène et al., "Mechanical model for determining the critical load of plane frames with semi-rigid joints subjected to static loads," *Eng. Struct.*, vol. 30, no. 1, pp. 109–117, 2017.
- [41] R. E. Melchers and D. Kaur, "The behaviour of frames with flexible connections," in *Proceedings of the Eighth Australasian Conference on the Mechanics of Structures and Materials*, ACMSM8 Organising Committee, University of New Castle, New South Wales, August 1982., 1982.
- [42] E. M. Lui and W. F. Chen, "Analysis and behaviour of flexibly-jointed frames," *Eng. Struct.*, vol. 8, no. 2, pp. 107–118, 1986.
- [43] A. Pirmoz, M. M. Ahmadi, E. Valadi, V. Farajkhah, and S. R. Balanoji, "Performance of the PR connections under combined Axial-tension and moment loading," *Electron. J. Struct. Eng.*, vol. 10, pp. 66–73, 2010.
- [44] W. Ramberg and W. R. Osgood, "Description of stress-strain curves by three parameters," 1943.

- [45] M. J. Frye and G. A. Morris, "ANALYSIS OF FLEXIBILITY CONNECTED STEEL FRAMES," Can. J. Civ. Eng., vol. 2, no. 3, pp. 280–291, Sep. 1975.
- [46] A. Picard, Y.-M. Giroux, and P. Brun, "Discussion: Analysis of flexibly connected steel frames," Can. J. Civ. Eng., vol. 3, no. 2, pp. 350–352, 1976.
- [47] W. M. G. Ho et al., "Moment-Rotation Relations of Semirigid Connections with Angles," J. Constr. Steel Res., vol. 41, no. 7, pp. 179–192, 1991.
- [48] C. Batho and S. D. Lash, "Further investigations on beam and stanchions connections, Including connections encased in concrete; Together with laboratory investigations on a full-scale steel frame," Final Rep. Steel Struct. Res. Comm., pp. 276–363, 1936.
- [49] N. Krishnamurthy and D. E. Graddy, "Correlation between 2-and 3-dimensional finite element analysis of steel bolted end-plate connections," Comput. Struct., vol. 6, no. 4–5, pp. 381–389, 1976.
- [50] R. M. Richard and B. J. Abbott, "Versatile Elastic-Plastic Stress-Strain Formula," J. Eng. Mech. Div., vol. 101, no. 4, pp. 511–515, 1975.
- [51] K. M. Ang and G. A. Morris, "Analysis of three-dimensional frames with flexible beam–column connections," Can. J. Civ. Eng., vol. 11, no. 2, pp. 245–254, Jun. 1984.
- [52] F. H. Wu and W. F. Chen, "A design model for semi-rigid connections," Eng. Struct., vol. 12, no. 2, pp. 88–97, 1990.
- [53] S.-S. Lee and T.-S. Moon, "Moment–rotation model of semi-rigid connections with angles," Eng. Struct., vol. 24, no. 2, pp. 227–237, 2002.
- [54] E. M. Lui and W. F. Chen, "Analysis and behaviour of flexibly-jointed frames," Eng. Struct., vol. 8, no. 2, pp. 107–118, 1986.
- [55] N. Kishi, W. F. P. Chen, and N. Kishi, "Data base of steel beam-to-column connections," J. Struct. Engng, ASCE, vol. 115, no. 1, pp. 105–119, 1986.
- [56] W. Chen and N. Kishi, "Semirigid Steel Beam-to-Column Connections: Data Base and Modeling," J. Struct. Eng., vol. 115, no. 1, pp. 105–119, Jan. 1989.
- [57] Y. L. Yee and R. E. Melchers, "Moment-Rotation Curves for Bolted Connections," J. Struct. Eng., vol. 112, no. 3, pp. 615–635, Mar. 1986.
- [58] R. P. Johnson and C. L. C. Law, "Semi-rigid joints for composite frames," in Proc. Int. Conf. on Joints in Structural Steelwork, 1981, p. 3.
- [59] M. L. Chisala, "Modelling M- ϕ curves for standard beam-to-column connections," Eng. Struct., vol. 21, no. 12, pp. 1066–1075, 1999.
- [60] M. R. Mohamadi-Shoore and M. Mofid, "New modeling for moment–rotation behavior of bolted endplate connections," Sci. Iran., vol. 18, no. 4, pp. 827–834, 2011.
- [61] R. Livesley, Matrix Methods of Structural Analysis: Pergamon International Library of Science, Technology, Engineering and Social Studies. 2013.
- [62] S. K. Bose, G. M. McNeice, and A. N. Sherbourne, "Column webs in steel beam-to-column connexions part I—Formulation and verification," Comput. Struct., vol. 2, no. 1–2, pp. 253–279, 1972.
- [63] A. Kukreti, T. Murray, A. A.-J. of C. Steel, and U. 1987, "End-plate connection moment-rotation relationship," Elsevier.
- [64] K. V Patel and W. F. Chen, "Nonlinear analysis of steel moment connections," J. Struct. Eng., vol. 110, no. 8, pp. 1861–1874, 1984.
- [65] C. P. Chasten, L.-W. Lu, and G. C. Driscoll, "Prying and shear in end-plate connection design," J. Struct. Eng., vol. 118, no. 5, pp. 1295–1311, 1992.
- [66] Y. F. Zhu, C. H. Chen, Y. Yao, L. M. Keer, and Y. Huang, "Dynamic increase factor for progressive collapse analysis of semi-rigid steel frames," Steel Compos. Struct., vol. 28, no. 2, pp. 209–221, 2018.
- [67] O. S. Bursi and L. Leonelli, "A finite element model for the rotational behavior of end plate steel connections," SSRC Proc. 1994 Annu. Task Gr. Tech. Sess., pp. 162–175, 1994.
- [68] A. N. Sherbourne and M. R. Bahaari, "3D simulation of end-plate bolted connections," J. Struct. Eng., vol. 120, no. 11, pp. 3122–3136, 1994.
- [69] A. N. Sherbourne and M. R. Bahaari, "Finite element prediction of end plate bolted connection behavior. I: Parametric Study," J. Struct. Eng., vol. 123, no. 2, pp. 157–164, 1997.

- [70] A. M. Citipitioglu, R. M. Haj-Ali, and D. W. White, "Refined 3D finite element modeling of partially-restrained connections including slip," *J. Constr. Steel Res.*, vol. 58, no. 5–8, pp. 995–1013, 2002.
- [71] C. J. Gantes and M. E. Lemonis, "Influence of equivalent bolt length in finite element modeling of T-stub steel connections," *Comput. Struct.*, vol. 81, no. 8–11, pp. 595–604, 2003.
- [72] S. H. Ju, C. Y. Fan, and G. H. Wu, "Three-dimensional finite elements of steel bolted connections," *Eng. Struct.*, vol. 26, no. 3, pp. 403–413, 2004.
- [73] Y. I. Maggi, R. M. Gonçalves, R. T. Leon, and L. F. L. Ribeiro, "Parametric analysis of steel bolted end plate connections using finite element modeling," *J. Constr. Steel Res.*, vol. 61, no. 5, pp. 689–708, 2005.
- [74] A. Abolmaali, J. H. Matthys, M. Farooqi, and Y. Choi, "Development of moment-rotation model equations for flush end-plate connections," *J. Constr. Steel Res.*, vol. 61, no. 12, pp. 1595–1612, 2005.
- [75] A. Pirmoz, A. S. Daryan, A. Mazaheri, and H. E. Darbandi, "Behavior of bolted angle connections subjected to combined shear force and moment," *J. Constr. Steel Res.*, vol. 64, no. 4, pp. 436–446, 2008.
- [76] M. Mohamadi-Shooreh, M. M.-J. of C. Steel, and undefined 2008, "Parametric analyses on the initial stiffness of flush end-plate splice connections using FEM," Elsevier.
- [77] M. Lemonis, C. G.-J. of C. S. Research, and undefined 2009, "Mechanical modeling of the nonlinear response of beam-to-column joints," Elsevier.
- [78] X. H. Dai, Y. C. Wang, and C. G. Bailey, "Numerical modelling of structural fire behaviour of restrained steel beam-column assemblies using typical joint types," *Eng. Struct.*, vol. 32, no. 8, pp. 2337–2351, 2010.
- [79] L. Gödrich, Z. Sokol, and F. Wald, "Numerical Design Model of Joints," *Des. Fabr. Econ. Met. Struct.*, no. Wald 2001, pp. 303–308, 2013.
- [80] S. Khodaie, M. R. Mohamadi-shooreh, and M. Mofid, "Parametric analyses on the initial stiffness of the SHS column base plate connections using FEM," *Eng. Struct.*, vol. 34, pp. 363–370, 2012.
- [81] M. Wang, Y. Shi, Y. Wang, and G. Shi, "Numerical study on seismic behaviors of steel frame end-plate connections," *J. Constr. Steel Res.*, vol. 90, pp. 140–152, 2013.
- [82] M. R. Mohamadi-Shooreh and M. Mofid, "Prediction of the yielding moment of flush endplate splice connections using finite element modeling," *Sci. Iran.*, vol. 20, no. 2, pp. 270–277, 2013.
- [83] G. S. Prinz, A. Nussbaumer, L. Borges, and S. Khadka, "Experimental testing and simulation of bolted beam-column connections having thick extended endplates and multiple bolts per row," *Eng. Struct.*, vol. 59, pp. 434–447, 2014.
- [84] P. Pakala, V. Kodur, and M. Dwaikat, "Critical factors influencing the fire performance of bolted double angle connections," *Eng. Struct.*, vol. 42, pp. 106–114, 2012.
- [85] S. Selamat and M. E. Garlock, "Fire resistance of steel shear connections," *Fire Saf. J.*, vol. 68, pp. 52–60, 2014.
- [86] A. Ahmed, "Prediction of moment–rotation characteristic of top- and seat-angle bolted connection incorporating prying action," *Int. J. Adv. Struct. Eng.*, vol. 9, no. 1, pp. 79–93, 2017.
- [87] K. Al Fakih, S. C. Chin, and S. I. Doh, "Finite Element Analysis of Beam to Column Bolted Connection – A Review," *Indian J. Sci. Technol.*, vol. 10, no. 8, pp. 1–9, 2017.
- [88] M. De Stefano, A. De Luca, and A. Astaneh-Asl, "Modeling of Cyclic Moment-Rotation Response of Double-Angle Connections," *J. Struct. Eng.*, vol. 120, no. 1, pp. 212–229, Jan. 1994.
- [89] G. Abdollahzadeh, S. Y. Gharavi, and M. H. Beigy, "Evaluation of the moment-rotation curve of I beam-to-CFT column connection with endplate, using mechanical modeling," *Int. J. Steel Struct.*, vol. 15, no. 4, pp. 911–922, 2015.
- [90] P. Prabha, S. Rekha, V. Marimuthu, M. Saravanan, G. S. Palani, and M. Surendran, "Modified Frye–Morris polynomial model for double web-angle connections," *Int. J. Adv. Struct. Eng.*, vol. 7, no. 3, pp. 295–306, 2015.
- [91] G. A. Rassati, R. T. Leon, and S. Noè, "Component modeling of partially restrained composite joints under cyclic and dynamic loading," *J. Struct. Eng.*, vol. 130, no. 2, pp. 343–351, 2004.