

SOME PROPERTIES ON DEGREES OF DOMINATING SETS USING EUCLIDEAN DIVISION ALGORITHM WITH DIVISOR 5 FOR INTERVAL GRAPH G

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Abstract : *The aim of this paper is to put forth some revelations regarding the dominating sets of Euclidian division algorithms with divisor five. A connected dominating set is used as back bone for communications and vertices that are not in this set communicate by passing message through neighbors that are in the set. In this some special classes of dominating sets are chosen for this purpose and their properties on degrees of dominating sets using Euclidean division algorithm with divisor 5 for interval graph G*

Index terms: *Dominating sets, degree, Euclidian division algorithm, interval graph, divisor.*

I.INTRODUCTION

Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as traffic control, ecology, biology, computer science and particularly useful in cyclic scheduling and computers storage allocation problems etc. Have a representation of graph with intervals or arcs can be helpful in combinatorial problems of the graph, such as isomorphism testing and finding maximum independent set and clique graphs. Interval graphs have a wide variant of applications to varies branches of science and technology. Among the various applications of the theory of domination and vertex degrees the most often discussed in a communication networks. This leads to the introduction of the concepts of dominating sets, division algorithms with divisors. A graph is said to be connected if there is a path between every pair of vertices, otherwise it is said to be disconnected graph. The number of edges incident with a vertex V is called degree of V and is denoted by $d(v)$.

II.PRELIMINARIES

Let $I = \{ I_1, I_2, I_3, I_4, \dots, I_n \}$ be an interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$ for $i = 1, 2, 3, \dots, n$. Here a_i is called the left end point and b_i is called the right end point. Without loss of generality, one can assume that, all end points of the intervals are distinct numbers between 1 and $2n$. An interval of degree one is called a pendent interval[1]. The intervals are named in the increasing order of their right end points. The graph G is an interval graph if there is one-to-one correspondence between the vertex set V and the interval family I. Two vertices of G are joined by an edge if and only if their corresponding intervals in I intersect. That is if $I_i = [a_i, b_i]$ and $I_j = [a_j, b_j]$, then I_i and I_j will intersect if $a_i < b_j$ or $a_j < b_i$.

Let $G = (V, E)$ be a graph, a set $D \subseteq V$ is a dominating set of G if every vertex in $V \setminus D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set[4,5]. A minimum dominating set with minimum cardinality among all the dominating sets of a graph G is said to be Minimum dominating set[2,3]. If $V_1, V_2, \dots, V_n$ are the n vertices of G, then the sequence $(d_1, d_2, \dots, d_n)$ where $d_i = \deg(v_i)$ is the degree sequence of G. According Euclid's division algorithm[8,10] if we have two positive integers a and b, then there exists unique integers q and r which satisfies the condition $a = bq + r$ where $0 \leq r < b$.

III.MAIN THEOREMS

THEOREM 1:

Let $I = \{ I_1, I_2, I_3, I_4, \dots, I_n \}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i , $i \neq \{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I. suppose $n = 5q + r$, where $r = 0, 1, 2, 3, 4$ and q is any positive integer. If $r = 0$, then the Minimum dominating set $D = \{ V_{5m-2} / m \in N, 1 \leq m \leq q \}$ and the domination number $\gamma(G) = q$ and then the following results are true.

$$(i) \gamma = \frac{n}{5} \quad (ii) \sum_{v \in D} d(v) = 4q \quad (iii) \sum_{v \in D} d(v) = 4\gamma \quad (iv) \sum_{v \in D} d(v) = \frac{4n}{5}.$$

PROOF: Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i , $i \neq \{n, n-1\}$ intersects the next two intervals only and let $G(V, E)$ be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where $V = \{V_1, V_2, V_3, V_4, \dots, V_n\}$.

By Euclidean division algorithm, $n = 5q + r$ where $r = 0, 1, 2, 3, 4$ and q is any integer, n is a number of vertices in G .

Suppose $r = 0$ then we have $n = 5q + 0 \Rightarrow n = 5q \Rightarrow q = \frac{n}{5}$

Then we have $D = \{V_{5m-2} / m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-2}\} \text{ and } \gamma = \gamma(G) = q$$

$$(i) \gamma = q = \frac{n}{5}$$

$$\text{Therefore } \gamma = \frac{n}{5}$$

$$(ii) \sum_{v \in D} d(v) = d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-2})$$

$$= 4 + 4 + 4 + \dots + 4$$

$$= 4(1 + 1 + 1 + \dots + 1)$$

$$= 4q$$

$$\sum_{v \in D} d(v) = 4q$$

$$(iii) \sum_{v \in D} d(v) = 4q = 4\gamma \quad \text{since } \gamma = q$$

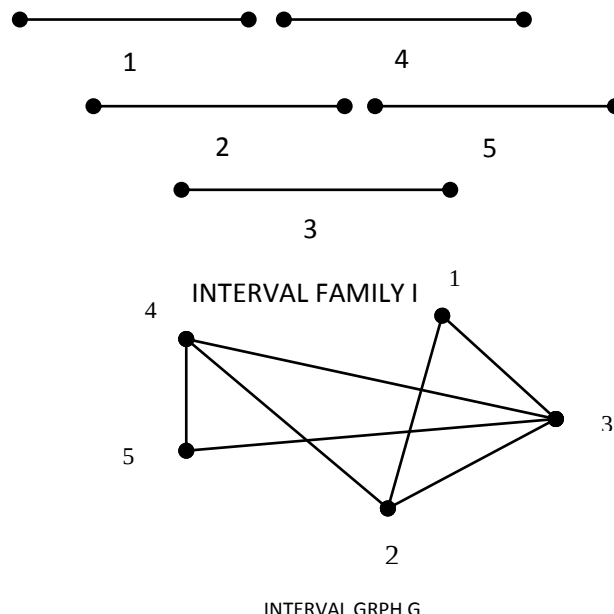
$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma$$

$$(iv) \sum_{v \in D} d(v) = 4q = 4\left(\frac{n}{5}\right) = \frac{4n}{5}$$

$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n}{5}$$

Practical problem:

Let $I = \{I_1, I_2, I_3, I_4, I_5\}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



Here $n = 5 = 5 \times 1 + 0$

This is of the form $n = 5q + r$ and then $q = 1$ and $r = 0$.

If $r = 0$ then the Minimum dominating set $D = \{3\}$, since theorem 1.

And the domination number $\gamma = 1 = q$

And $\sum_{v \in D} d(V) = d(3) = 4$

$$(i) \gamma = \frac{n}{5} = \frac{5}{5} = 1$$

Therefore $\gamma = 1$

$$(ii) \sum_{v \in D} d(V) = 4q = 4 \times 1 = 4 \quad \text{Since } q=1$$

$$\text{Therefore } \sum_{v \in D} d(V) = 4$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma = 4 \times 1 = 4 \quad \text{Since } \gamma = 1$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4$$

$$(iv) \sum_{v \in D} d(v) = \frac{4n}{5} = \frac{4 \times 5}{5} = 4$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4$$

Hence theorem is verified at $r = 0$.

THEOREM 2:

Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i , $i \neq \{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I . Suppose $n = 5q + r$, where $r = 0, 1, 2, 3, 4$ and q is any positive integer. If $r = 1$, then the Minimum dominating set $D = \{V_{5m-2}, V_n \mid m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+4}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 2 \quad (iii) \sum_{v \in D} d(v) = 4\gamma - 2 \quad (iv) \sum_{v \in D} d(v) = \frac{4n+6}{5}.$$

PROOF: Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$ be any finite interval family such that every interval I_i , $i \neq \{n, n-1\}$ intersects the next two intervals only and let $G(V, E)$ be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where $V = \{V_1, V_2, V_3, V_4, \dots, V_n\}$.

By Euclidean division algorithm, $n = 5q + r$ where $r = 0, 1, 2, 3, 4$ and q is any integer, n is a number of vertices in G .

Suppose $r = 1$ then we have $n = 5q + 1$ and $q = \frac{n-1}{5}$

Then we have $D = \{V_{5m-2}, V_n \mid m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_n\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-3}, V_n\} \quad \text{and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-1}{5} + 1 = \frac{n-1+5}{5} = \frac{n+4}{5}$$

$$\text{Therefore } \gamma = \frac{n+4}{5}$$

$$\begin{aligned} (ii) \sum_{v \in D} d(V) &= d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-3}) + d(V_n) \\ &= 4 + 4 + 4 + \dots + 4 + 2 \\ &= 4(1 + 1 + 1 + \dots + 1) + 2 \\ &= 4q + 2 \end{aligned}$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4q + 2$$

$$\begin{aligned} (iii) \sum_{v \in D} d(v) &= 4q + 2 \\ &= 4(\gamma - 1) + 2 \quad \text{Since } \gamma = q + 1 \\ &= 4\gamma - 4 + 2 \\ &= 4\gamma - 2 \end{aligned}$$

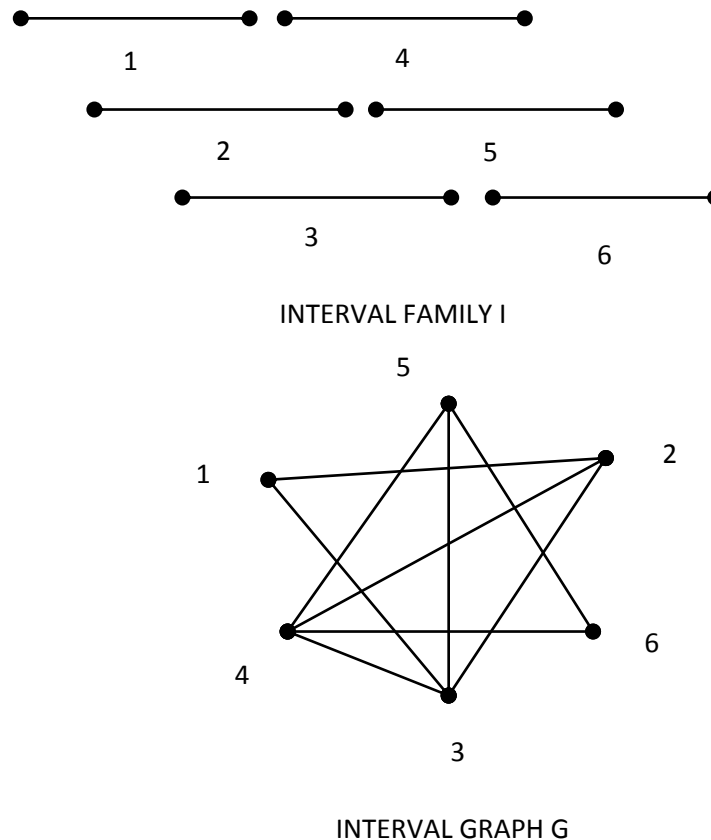
$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma - 2$$

$$\begin{aligned} (iv) \sum_{v \in D} d(v) &= 4q + 2 \\ &= 4\left(\frac{n-1}{5}\right) + 2 \\ &= \frac{4n-4}{5} + 2 = \frac{4n-4+10}{5} \end{aligned}$$

$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n+6}{5}$$

Practical problem:

Let $I = \{ I_1, I_2, I_3, I_4, I_5, I_6 \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



Here $n = 6 = 5 \times 1 + 1$

This is of the form $n = 5q + r$ and then $q = 1, r = 1$.

If $r = 1$, then the Minimum dominating set $D = \{ 3, 6 \}$ and $\gamma = 2$ since theorem 2.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(6) = 4 + 2 = 6$$

$$(i) \gamma = \frac{n+4}{5} = \frac{6+4}{5} = \frac{10}{5} = 2$$

Therefore $\gamma = 2$

$$(ii) \sum_{v \in D} d(V) = 4q + 2 = 4 \times 1 + 2 = 6$$

$$\text{Therefore } \sum_{v \in D} d(V) = 6$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma - 2 = 4 \times 2 - 2 = 8 - 2 = 6$$

$$\text{Therefore } \sum_{v \in D} d(v) = 6$$

$$(iv) \sum_{v \in D} d(v) = \frac{4n+6}{5} = \frac{4 \times 6 + 6}{5} = \frac{30}{5} = 6$$

$$\text{Therefore } \sum_{v \in D} d(v) = 6$$

Hence theorem is verified at $r = 1$.

THEOREM 3:

Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i , $i \neq \{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I . suppose $n = 5q + r$, where $r = 0, 1, 2, 3, 4$ and q is any positive integer. If $r = 2$, then two cases will arise

Case(i): The Minimum dominating set $D = \{V_{5m-2}, V_{n-1} / m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+3}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 3 \quad (iii) \sum_{v \in D} d(v) = 4\gamma - 1 \quad (iv) \sum_{v \in D} d(v) = \frac{4n+7}{5}.$$

Case(ii): The Minimum dominating set $D = \{V_{5m-2}, V_n / m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+3}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 2 \quad (iii) \sum_{v \in D} d(v) = 4\gamma - 2 \quad (iv) \sum_{v \in D} d(v) = \frac{4n+2}{5}.$$

PROOF: Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$ be any finite interval family such that every interval I_i , $i \neq \{n, n-1\}$ intersects the next two intervals only and let $G(V, E)$ be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where $V = \{V_1, V_2, V_3, V_4, \dots, V_n\}$.

By Euclidean division algorithm, $n = 5q + r$ where $r = 0, 1, 2, 3, 4$ and q is any integer, n is a number of vertices in G .

Suppose $r = 2$ then we have $n = 5q + 2$ and $q = \frac{n-2}{5}$

Case (i): if $r = 2$, Then $D = \{V_{5m-2}, V_{n-1} / m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_{n-1}\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-4}, V_{n-1}\} \text{ and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-2}{5} + 1 = \frac{n-2+5}{5} = \frac{n+3}{5}$$

$$\text{Therefore } \gamma = \frac{n+3}{5}$$

$$(ii) \sum_{v \in D} d(V) = d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-4}) + d(V_{n-1})$$

$$= 4 + 4 + 4 + \dots + 4 + 3$$

$$= 4(1 + 1 + 1 + \dots + 1) + 3 \quad (\text{since } \gamma = q + 1)$$

$$= 4q + 3$$

$$\sum_{v \in D} d(v) = 4q + 3$$

$$(iii) \sum_{v \in D} d(v) = 4q + 3$$

$$= 4(\gamma - 1) + 3 \quad \text{since } \gamma = q + 1$$

$$= 4\gamma - 4 + 3$$

$$= 4\gamma - 1$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma - 1$$

$$(iv) \sum_{v \in D} d(v) = 4q + 3$$

$$= 4\left(\frac{n-2}{5}\right) + 3$$

$$= \frac{4n-8}{5} + 3$$

$$= \frac{4n-8+15}{5}$$

$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n+7}{5}$$

Case (ii): if $r = 2$, Then $D = \{V_{5m-2}, V_n \mid m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_n\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-4}, V_n\} \quad \text{and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-2}{5} + 1 = \frac{n-2+5}{5} = \frac{n+3}{5}$$

$$\text{Therefore } \gamma = \frac{n+3}{5}$$

$$(ii) \sum_{v \in D} d(V) = d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-4}) + d(V_n)$$

$$= 4 + 4 + 4 + \dots + 4 + 2$$

$$= 4(1 + 1 + 1 + \dots + 1) + 2$$

$$= 4q + 2$$

$$\sum_{v \in D} d(v) = 4q + 2$$

$$(iii) \sum_{v \in D} d(v) = 4q + 2$$

$$= 4(\gamma - 1) + 2 \quad \text{since } \gamma = q + 1$$

$$= 4\gamma - 4 + 2$$

$$= 4\gamma - 2$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma - 2$$

$$(iv) \sum_{v \in D} d(v) = 4q + 2$$

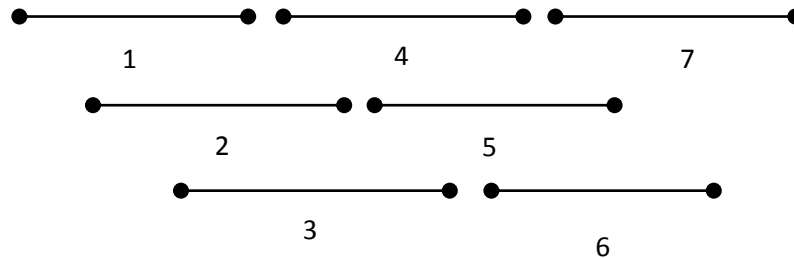
$$= 4\left(\frac{n-2}{5}\right) + 2$$

$$= \frac{4n-8}{5} + 2 = \frac{4n-8+10}{5}$$

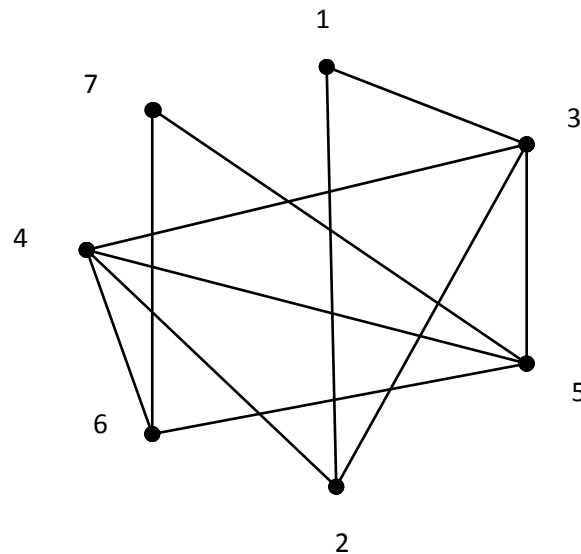
$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n+2}{5}$$

Practical problem:

Let $I = \{ I_1, I_2, I_3, I_4, I_5, I_6, I_7 \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here $n = 7 = 5 \times 1 + 2$

This is of the form $n = 5q + r$ and then $q = 1, r = 2$.

If $r = 2$, then there will two cases arise

Case(i): $D = \{ 3, 6 \}$ and $\gamma = 2$ since theorem 3.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(6) = 4 + 3 = 7$$

$$(i) \gamma = \frac{n+3}{5} = \frac{7+3}{5} = \frac{10}{5} = 2$$

Therefore $\gamma = 2$

$$(ii) \sum_{v \in D} d(V) = 4q + 3 = 4 \times 1 + 3 = 7$$

$$\sum_{v \in D} d(V) = 7$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma - 1 = 4 \times 2 - 1 = 8 - 1 = 7$$

Therefore $\sum_{v \in D} d(v) = 7$

$$(iv) \sum_{v \in D} d(v) = \frac{4n+7}{5} = \frac{4 \times 7 + 7}{5} = \frac{35}{5} = 7$$

Therefore $\sum_{v \in D} d(v) = 7$

Case(ii): $D = \{3, 7\}$ and $\gamma = 2$, since theorem 3.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(7) = 4 + 2 = 6$$

$$(i) \gamma = \frac{n+3}{5} = \frac{7+3}{5} = \frac{10}{5} = 2$$

Therefore $\gamma = 2$

$$(ii) \sum_{v \in D} d(V) = 4q + 2 = 4 \times 1 + 2 = 6$$

Therefore $\sum_{v \in D} d(V) = 6$

$$(iii) \sum_{v \in D} d(v) = 4\gamma - 2 = 4 \times 2 - 2 = 8 - 2 = 6$$

Therefore $\sum_{v \in D} d(v) = 6$

$$(iv) \sum_{v \in D} d(v) = \frac{4n+2}{5} = \frac{4 \times 7 + 2}{5} = \frac{30}{5} = 6$$

Therefore $\sum_{v \in D} d(v) = 6$

Hence theorem is verified at $r = 2$.

THEOREM 4:

Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i , $i \neq \{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I . suppose $n = 5q + r$, where $r = 0, 1, 2, 3, 4$ and q is any positive integer. If $r = 3$, then three cases will arise

Case(i): The Minimum dominating set $D = \{V_{5m-2}, V_{n-2} / m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+2}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 4 \quad (iii) \sum_{v \in D} d(v) = 4\gamma \quad (iv) \sum_{v \in D} d(v) = \frac{4n+8}{5}$$

Case(ii): The Minimum dominating set $D = \{V_{5m-2}, V_{n-1} / m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+2}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 3 \quad (iii) \sum_{v \in D} d(v) = 4\gamma - 1 \quad (iv) \sum_{v \in D} d(v) = \frac{4n+3}{5}$$

Case(iii): The Minimum dominating set $D = \{V_{5m-2}, V_n / m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+2}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 2 \quad (iii) \sum_{v \in D} d(v) = 4\gamma - 2 \quad (iv) \sum_{v \in D} d(v) = \frac{4n-2}{5}$$

PROOF: Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$ be any finite interval family such that every interval I_i , $i \neq \{n, n-1\}$ intersects the next two intervals only and let $G(V, E)$ be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where $V = \{V_1, V_2, V_3, V_4, \dots, V_n\}$.

By Euclidean division algorithm, $n = 5q + r$ where $r = 0, 1, 2, 3, 4$ and q is any integer, n is a number of vertices in G .

Suppose $r = 3$ then we have $n = 5q + 3$ and $q = \frac{n-3}{5}$

Case (i): if $r = 3$, Then $D = \{V_{5m-2}, V_{n-2} / m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_{n-2}\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-5}, V_{n-2}\} \text{ and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-3}{5} + 1 = \frac{n-3+5}{5} = \frac{n+2}{5}$$

$$\text{Therefore } \gamma = \frac{n+2}{5}$$

$$(ii) \sum_{v \in D} d(v) = d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-5}) + d(V_{n-2})$$

$$= 4 + 4 + 4 + \dots + 4 + 4$$

$$= 4(1 + 1 + 1 + \dots + 1)$$

$$= 4(q + 1)$$

$$\sum_{v \in D} d(v) = 4q + 4 = 4(q + 1)$$

$$(iii) \sum_{v \in D} d(v) = 4q + 4$$

$$= 4(\gamma - 1) + 4 \quad \text{since } \gamma = q + 1$$

$$= 4\gamma - 4 + 4$$

$$= 4\gamma$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma$$

$$(iv) \sum_{v \in D} d(v) = 4q + 4$$

$$= 4\left(\frac{n-3}{5}\right) + 4$$

$$= \frac{4n-12}{5} + 4$$

$$= \frac{4n-12+20}{5}$$

$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n+8}{5}$$

Case (ii): if $r = 3$, Then $D = \{V_{5m-2}, V_{n-1} / m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_{n-1}\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-5}, V_{n-1}\} \text{ and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-3}{5} + 1 = \frac{n-3+5}{5} = \frac{n+2}{5}$$

$$\text{Therefore } \gamma = \frac{n+2}{5}$$

$$(ii) \sum_{v \in D} d(v) = d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-4}) + d(V_{n-1})$$

$$= 4 + 4 + 4 + \dots + 4 + 3$$

$$= 4(1 + 1 + 1 + \dots + 1) + 3$$

$$= 4q + 3$$

$$\sum_{v \in D} d(v) = 4q + 3$$

$$\begin{aligned} (iii) \sum_{v \in D} d(v) &= 4q + 3 \\ &= 4(\gamma - 1) + 3 \quad \text{since } \gamma = q + 1 \\ &= 4\gamma - 4 + 3 \\ &= 4\gamma - 1 \end{aligned}$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma - 1$$

$$\begin{aligned} (iv) \sum_{v \in D} d(v) &= 4q + 3 \\ &= 4\left(\frac{n-3}{5}\right) + 3 \\ &= \frac{4n-12}{5} + 3 \\ &= \frac{4n-12+15}{5} \end{aligned}$$

$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n+3}{5}$$

Case (iii): if $r = 2$, Then $D = \{V_{5m-2}, V_n \mid m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_n\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-5}, V_n\} \quad \text{and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-3}{5} + 1 = \frac{n-3+5}{5} = \frac{n+2}{5}$$

$$\text{Therefore } \gamma = \frac{n+2}{5}$$

$$\begin{aligned} (ii) \sum_{v \in D} d(v) &= d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-4}) + d(V_n) \\ &= 4 + 4 + 4 + \dots + 4 + 2 \\ &= 4(1 + 1 + 1 + \dots + 1) + 2 \\ &= 4q + 2 \end{aligned}$$

$$\sum_{v \in D} d(v) = 4q + 2$$

$$\begin{aligned} (iii) \sum_{v \in D} d(v) &= 4q + 2 \\ &= 4(\gamma - 1) + 2 \quad \text{since } \gamma = q + 1 \\ &= 4\gamma - 4 + 2 \\ &= 4\gamma - 2 \end{aligned}$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma - 2$$

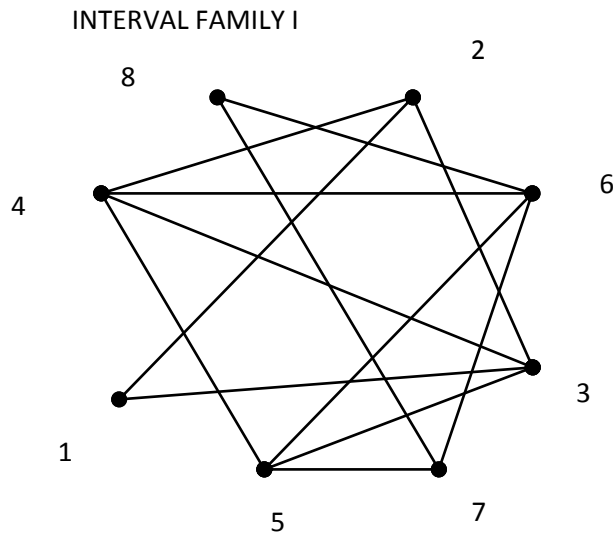
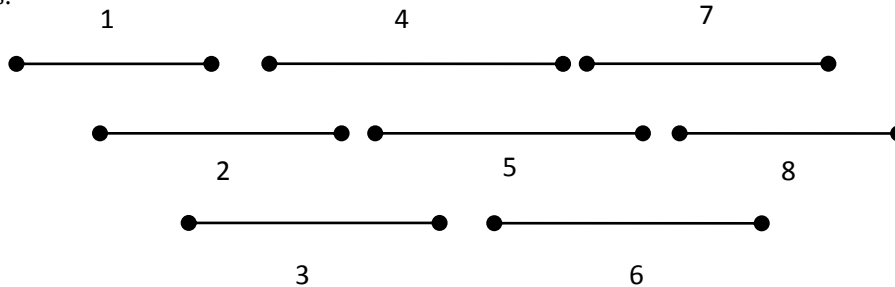
$$\begin{aligned} (iv) \sum_{v \in D} d(v) &= 4q + 2 \\ &= 4\left(\frac{n-3}{5}\right) + 2 \\ &= \frac{4n-12}{5} + 2 \end{aligned}$$

$$= \frac{4n-12+10}{5}$$

$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n-2}{5}$$

Practical problem:

Let $I = \{ I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8 \}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL GRAPH G

Here $n = 8 = 5 \times 1 + 3$

This is of the form $n = 5q + r$ and then $q = 1, r = 3$.

If $r = 3$, then there will three cases arise.

Case(i): $D = \{ 3, 6 \}$ and $\gamma = 2$ since theorem 4.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(6) = 4 + 4 = 8$$

$$(i) \gamma = \frac{n+2}{5} = \frac{8+2}{5} = \frac{10}{5} = 2$$

Therefore $\gamma = 2$

$$(ii) \sum_{v \in D} d(V) = 4q + 4 = 4 \times 1 + 4 = 8$$

$$\text{Therefore } \sum_{v \in D} d(V) = 8$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma = 4 \times 2 = 8$$

$$\text{Therefore } \sum_{v \in D} d(v) = 8$$

$$(iv) \sum_{v \in D} d(v) = \frac{4n+8}{5} = \frac{4 \times 8 + 8}{5} = \frac{40}{5} = 8$$

$$\text{Therefore } \sum_{v \in D} d(v) = 8$$

Case(ii): $D = \{3, 7\}$ and $\gamma = 2$ since theorem 4.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(7) = 4 + 3 = 7$$

$$(i) \gamma = \frac{n+2}{5} = \frac{8+2}{5} = \frac{10}{5} = 2$$

$$\text{Therefore } \gamma = 2$$

$$(ii) \sum_{v \in D} d(V) = 4q + 2 = 4 \times 1 + 3 = 7$$

$$\sum_{v \in D} d(V) = 7$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma - 1 = 4 \times 2 - 1 = 8 - 1 = 7$$

$$\text{Therefore } \sum_{v \in D} d(v) = 7$$

$$(iv) \sum_{v \in D} d(v) = \frac{4n+3}{5} = \frac{4 \times 8 + 3}{5} = \frac{35}{5} = 7$$

$$\text{Therefore } \sum_{v \in D} d(v) = 7$$

Case(iii): $D = \{3, 8\}$ and $\gamma = 2$ since theorem 4.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(8) = 4 + 2 = 6$$

$$(i) \gamma = \frac{n+2}{5} = \frac{8+2}{5} = \frac{10}{5} = 2$$

$$\text{Therefore } \gamma = 2$$

$$(ii) \sum_{v \in D} d(V) = 4q + 2 = 4 \times 1 + 2 = 6$$

$$\sum_{v \in D} d(V) = 6$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma - 2 = 4 \times 2 - 2 = 8 - 2 = 6$$

$$\text{Therefore } \sum_{v \in D} d(v) = 6$$

$$(iv) \sum_{v \in D} d(v) = \frac{4n-2}{5} = \frac{4 \times 8 - 2}{5} = \frac{30}{5} = 6$$

$$\text{Therefore } \sum_{v \in D} d(v) = 6$$

Hence theorem is verified at $r = 3$.

THEOREM 5:

Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$, $\forall n \geq 5$ be any finite interval family such that every interval I_i , $i \neq \{n-1, n\}$ intersects next two intervals only and let G be an interval graph corresponding to an interval family I . Suppose $n = 5q + r$, where $r = 0, 1, 2, 3, 4$ and q is any positive integer. If $r = 4$, then two cases will arise

Case(i): The Minimum dominating set $D = \{V_{5m-2}, V_{n-2} / m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+1}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 4 \quad (iii) \sum_{v \in D} d(v) = 4\gamma \quad (iv) \sum_{v \in D} d(v) = \frac{4n+4}{5}.$$

Case(ii): The Minimum dominating set $D = \{V_{5m-2}, V_{n-1} / m \in N, 1 \leq m \leq q\}$ and the domination number $\gamma(G) = q + 1$ and then the following results are true.

$$(i) \gamma = \frac{n+1}{5} \quad (ii) \sum_{v \in D} d(v) = 4q + 3 \quad (iii) \sum_{v \in D} d(v) = 4\gamma - 1 \quad (iv) \sum_{v \in D} d(v) = \frac{4n-1}{5}.$$

PROOF: Let $I = \{I_1, I_2, I_3, I_4, \dots, I_n\}$ be any finite interval family such that every interval I_i , $i \neq \{n, n-1\}$ intersects the next two intervals only and let $G(V, E)$ be an interval graph if there is one to one correspondence between the vertex set V and the interval family I where $V = \{V_1, V_2, V_3, V_4, \dots, V_n\}$.

By Euclidean division algorithm, $n = 5q + r$ where $r = 0, 1, 2, 3, 4$ and q is any integer, n is a number of vertices in G .

Suppose $r = 4$ then we have $n = 5q + 4$ and $q = \frac{n-4}{5}$

Case (i): if $r = 4$, Then $D = \{V_{5m-2}, V_{n-2} / m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_{n-2}\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-5}, V_{n-2}\} \text{ and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-4}{5} + 1 = \frac{n-4+5}{5} = \frac{n+1}{5}$$

$$\text{Therefore } \gamma = \frac{n+1}{5}$$

$$\begin{aligned} (ii) \sum_{v \in D} d(v) &= d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-5}) + d(V_{n-2}) \\ &= 4 + 4 + 4 + \dots + 4 + 4 \\ &= 4(1 + 1 + 1 + \dots + 1) \\ &= 4(q + 1) \end{aligned}$$

$$\sum_{v \in D} d(v) = 4q + 4 = 4(q + 1)$$

$$\begin{aligned} (iii) \sum_{v \in D} d(v) &= 4q + 4 \\ &= 4(\gamma - 1) + 4 \quad \text{since } \gamma = q + 1 \\ &= 4\gamma - 4 + 4 \\ &= 4\gamma \end{aligned}$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma$$

$$\begin{aligned} (iv) \sum_{v \in D} d(v) &= 4q + 4 \\ &= 4\left(\frac{n-4}{5}\right) + 4 \\ &= \frac{4n-16}{5} + 4 \\ &= \frac{4n-16+20}{5} \end{aligned}$$

$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n+4}{5}$$

Case (ii): if $r = 4$, Then $D = \{V_{5m-2}, V_{n-1} / m \in N, 1 \leq m \leq q\}$ and $\gamma(G) = q + 1$

$$D = \{V_3, V_8, V_{13}, \dots, V_{5q-2}, V_{n-1}\}$$

$$D = \{V_3, V_8, V_{13}, \dots, V_{n-5}, V_{n-1}\} \text{ and } \gamma = q + 1$$

$$(i) \gamma = q + 1 = \frac{n-4}{5} + 1 = \frac{n-4+5}{5} = \frac{n+1}{5}$$

$$\text{Therefore } \gamma = \frac{n+1}{5}$$

$$(ii) \sum_{v \in D} d(v) = d(V_3) + d(V_8) + d(V_{13}) + \dots + d(V_{n-4}) + d(V_{n-1})$$

$$= 4 + 4 + 4 + \dots + 4 + 3$$

$$= 4(1 + 1 + 1 + \dots + 1) + 3$$

$$= 4q + 3$$

$$\sum_{v \in D} d(v) = 4q + 3$$

$$(iii) \sum_{v \in D} d(v) = 4q + 3$$

$$= 4(\gamma - 1) + 3 \quad \text{since } \gamma = q + 1$$

$$= 4\gamma - 4 + 3$$

$$= 4\gamma - 1$$

$$\text{Therefore } \sum_{v \in D} d(v) = 4\gamma - 1$$

$$(iv) \sum_{v \in D} d(v) = 4q + 3$$

$$= 4\left(\frac{n-4}{5}\right) + 3$$

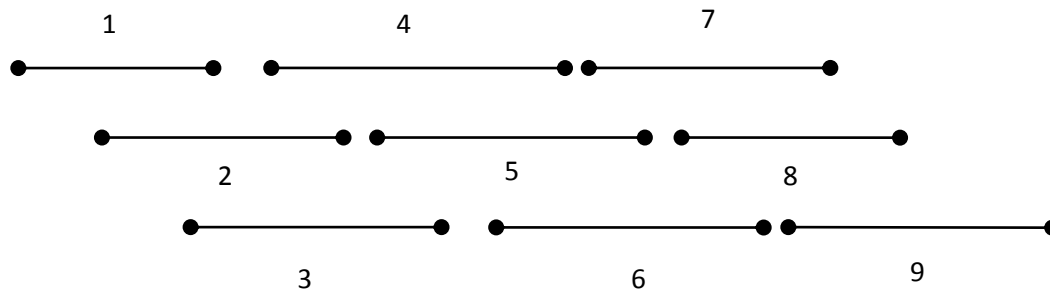
$$= \frac{4n-16}{5} + 3$$

$$= \frac{4n-16+15}{5}$$

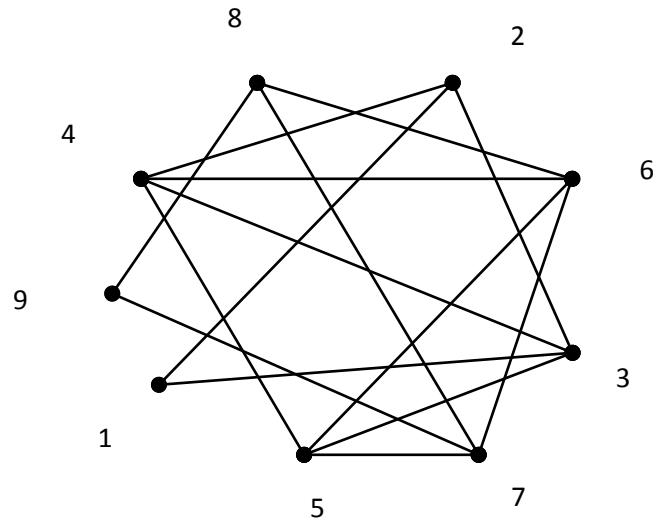
$$\text{Therefore } \sum_{v \in D} d(v) = \frac{4n-1}{5}$$

practical problem:

Let $I = \{I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9\}$ be an interval family and let G be an interval graph corresponding to an interval family I is as follows:



INTERVAL FAMILY I



INTERVAL GRAPH G

Here $n = 9 = 5 \times 1 + 4$

This is of the form $n = 5q + r$ and then $q = 1, r = 4$.

If $r = 4$, then there will two cases arise

Case(i): $D = \{ 3, 7 \}$ and $\gamma = 2$ since theorem 5.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(7) = 4 + 4 = 8$$

$$(i) \gamma = \frac{n+1}{5} = \frac{9+1}{5} = \frac{10}{5} = 2$$

Therefore $\gamma = 2$

$$(ii) \sum_{v \in D} d(V) = 4q + 4 = 4 \times 1 + 4 = 8$$

$$\sum_{v \in D} d(V) = 8$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma = 4 \times 2 = 8$$

$$\text{Therefore } \sum_{v \in D} d(v) = 8$$

$$(iv) \sum_{v \in D} d(v) = \frac{4n+4}{5} = \frac{4 \times 9 + 4}{5} = \frac{40}{5} = 8$$

$$\text{Therefore } \sum_{v \in D} d(v) = 8$$

Case(ii): $D = \{ 3, 8 \}$ and $\gamma = 2$ since theorem 4.

$$\text{And } \sum_{v \in D} d(V) = d(3) + d(8) = 4 + 3 = 7$$

$$(i) \gamma = \frac{n+1}{5} = \frac{9+1}{5} = \frac{10}{5} = 2$$

Therefore $\gamma = 2$

$$(ii) \sum_{v \in D} d(V) = 4q + 3 = 4 \times 1 + 3 = 7$$

$$\text{Therefore } \sum_{v \in D} d(V) = 7$$

$$(iii) \sum_{v \in D} d(v) = 4\gamma - 1 = 4 \times 2 - 1 = 8 - 1 = 7$$

$$\text{Therefore } \sum_{v \in D} d(v) = 7$$

$$(iv) \sum_{v \in D} d(v) = \frac{4n-1}{5} = \frac{4 \times 9 - 1}{5} = \frac{35}{5} = 7$$

$$\text{Therefore } \sum_{v \in D} d(v) = 7$$

Hence theorem is verified at $r = 4$.

IV.CONCLUSIONS

The study of the interval graph G corresponding to an interval family I, the dominating set and the vertex degree of G towards Euclidean division algorithm of interval graph G has been providing this sufficient stimulation uptaining some in depth knowledge of various concepts of the graphs. It is hoped that encouragement provided by this study of these interval graphs will be a good domination numbers with divisor 5 using Euclidean algorithm for further research for finite interval graphs.

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