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FORWARD DYNAMICS OF 3 LINK ROBOT

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Abstract*— As the robot manipulators are widely used in today's hi-tech industrialization for carrying out various tasks, the precise control of the wrist position is most important. For conceptualizing the structure, characteristics and functionality of robot under different operating conditions, virtual modelling and simulation becomes necessary. As robot manipulator's dynamics forward as well as inverse are nonlinear in nature and are computationally intensive, a virtual model has been developed for the same. The dynamics based on Lagrangian-Euler form for 3DOF RRR robot manipulator is presented.

Keywords*— 3-DOF, Lagrangian-Euler, Robot Arm, Dynamics, PID

I. INTRODUCTION

Robotic system possess highly nonlinear and complex characteristic, which has attracted the attention of researchers to design an efficient controller for improving the performance of the robot. The dynamics of robot arm is denoted by the mathematical expressions of robot arm motion. These expressions illustrate the dynamic behaviour of the robot and associate the manipulator's motion to the efforts involved. Such formulations are crucial for simulating the manipulator motion. The design of relevant control equations for a robot arm determines the kinematic design and a robot arm structure.

The paper is structured as follows. The dynamics based on Lagrangian-Euler method is discussed in section II. Section III describes the virtual model in Simulink which is a tool box in MATLAB, developed using the S-function. Section IV represents the response of controller followed by conclusion.

II. DYNAMICS

The dynamics of robot arm is denoted by the mathematical expressions of motion of a robot manipulator. These demonstrates the behaviour of the robot dynamically. Such formulations are crucial for simulating the manipulator in workspace prior to the actual implementation. The derivation of relevant control equations for a robot arm was determined by the kinematics and structure of a robot arm. The many researcher proposed various methods based on different principles leading to different structure of equation of motion for robot manipulator. Some of them are, D'Alembert Principle (G-D) [1], Lagrange-Euler (L-E) [2, 3], Newton-Euler (N-E) [4], and Recursive Lagrangian (R-L) [5]. L-E equations of motion reported to be well structured but computationally extensive. The N-E equations of motion have less computational requirement i.e. less multiplications, additions and storage requirement, but it has a degenerated structure, which gives the insight of the resulting dynamics. Using the D'Alembert Principle, the expressions of motion derived, maintains the structure with less penalty in computation.

Using the basic physical laws in Lagrange-Euler method, the dynamic model of a 3 DOF revolute joints (RRR) manipulator has been presented that was observed to be simple and systematic [6, 7]. The equations expressed were of non-linear 2nd order coupled differential equations. The formulations does not include the dynamics of the electric motor and drive friction.
The Lagrangian of a manipulator

$$L(\Theta,\dot{\Theta}) = k(\Theta,\dot{\Theta}) - u(\Theta) \tag{1}$$

Where $\dot{\Theta}$ = joint velocity vector

The vectors of manipulator's kinetic energy and potential energy are indicated by $k(\Theta,\dot{\Theta})$ and $u(\Theta)$ respectively.

The robot's motion can be stated as:

$$\frac{d}{dt}\frac{\partial l}{\partial \dot{\Theta}} - \frac{\partial l}{\partial \Theta} = \tau$$

or

$$\frac{d}{dt}\frac{\partial k}{\partial \dot{\Theta}} - \frac{\partial k}{\partial \Theta} + \frac{\partial u}{\partial \Theta} = \tau$$

where $'\tau'$ is the *n x 1* vector of actuator torque i.e 3x1 for 3DOF RRR manipulator.

In the general Lagrangian-Euler form [8], the equations of robot dynamic motions has been formulated in terms of the time derivate and position of the its joint angles as:

$$\tau = D(\Theta)\ddot{\Theta} + C(\Theta, \dot{\Theta}) + G(\Theta) \quad (2)$$

where τ = generalized torque applied at joint

$G(\Theta)$ is an n x 1 vector of gravity terms.

$D(\Theta)$ is the n x n mass matrix.

$C(\Theta, \dot{\Theta})$ is an n x 1 vector of centrifugal and Coriolis terms

More generalized form of dynamic equation of n degree of freedom mentioned as follows

$$\tau_i = \sum_{j=1}^{n} D_{ij}\ddot{q_j} + I_{i(actuator)}\ddot{q_i} + \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q_j}\dot{q_k} + D_i \quad (3)$$

where D_{ii} = the effective inertia at the joint i

D_{ij} = the coupling inertia between joints i and j

D_{ijk} = the Coriolis and centrifugal coefficients at the joint i.

D_i; = the gravitational force at the joint i

τ_i = the generalized torque at the joint i

$\dot{q_i}$ = the velocity of the joint i

$\ddot{q_i}$ = the acceleration of the joint i

$I_{i(actuator)}$ =Inertia of actuator (assumed as zero)

In state space, the dynamic parameters of manipulator are stated in terms of the instantaneous configuration in which the generalized joint velocities and coordinates are represented as state-variables. The dynamic parameters of the manipulator model can be obtained in Lagrangian formulation as described in subsequent subsections.

A. Inertia Tensor

The coefficients D_{kl} deals with the acceleration of the joint variables and can be computed as Eq. 4. When k=l, D_{kk} was associated with the k^{th} joint acceleration, where the acting torque τ_k exists. As for the situation when k≠l, the reaction torque caused by the k^{th} joint acceleration defined by $D_{kl,}$ and implying at k^{th} joint, or vice versa. As the inertia matrix was symmetric in nature, also

Trace(A) = Tr (A^{t}), it can be written as $D_{kl} = D_{lk.}$

$$D_{ij} = \sum_{p=\max(i,j)}^{n} Trace(U_{p\,j} J_p U_{p\,i}^{T}) \quad (4)$$

Where Jp = Pseudo inertia matrix calculated as follows

$$J_i = \begin{bmatrix} \frac{\left(-I_{xx} + I_{yy} + I_{zz}\right)_i}{2} & I_{ixy} & I_{ixz} & m_i\bar{x_i} \\ I_{ixy} & \frac{\left(I_{xx} - I_{yy} + I_{zz}\right)_i}{2} & I_{iyz} & m_i\bar{y_i} \\ I_{ixz} & I_{iyz} & \frac{\left(I_{xx} + I_{yy} - I_{zz}\right)_i}{2} & m_i\bar{z_i} \\ m_i\bar{x_i} & m_i\bar{y_i} & m_i\bar{z_i} & m_i \end{bmatrix}$$

where, I_{ii} = moment of inertia in ii axis

m_i = mass of link i

The coefficient D_{ik} was derived as follows.

$$D_{11} = Tr(U_{11}J_1U_{11}^{T}) + Tr(U_{21}J_2U_{21}^{T}) + Tr(U_{31}J_3U_{31}^{T}) \quad \ldots for\ i=1,\ j=1,\ p=1,2,3$$

$$D_{12} = Tr(U_{22}J_2U_{21}^{T}) + Tr(U_{32}J_3U_{31}^{T}) \quad \ldots for\ i=1,\ j=2,\ p=2,3$$

$$D_{13} = Tr(U_{33}J_3U_{31}^{T}) \ldots for\ i=1,\ j=3,\ p=3$$

$$D_{21} = Tr(U_{21}J_2U_{22}^T) + Tr(U_{31}J_3U_{32}^T) \quad \dots for\ i=2,\ j=1,\ p=2,3$$

$$D_{22} = Tr\left(U_{22}J_2U_{22}^T\right) + Tr\left(U_{32}J_3U_{32}^T\right) \quad \dots for\ i=2,\ j=2,\ p=2,3$$

$$D_{23} = Tr\left(U_{33}J_3U_{32}^T\right) \dots for\ i=2,\ j=3,\ p=3$$

$$D_{31} = Tr\left(U_{31}J_3U_{33}^T\right) \dots for\ i=3,\ j=1,\ p=3$$

$$D_{32} = Tr\left(U_{32}J_3U_{33}^T\right) \dots for\ i=3,\ j=2,\ p=3$$

$$D_{33} = Tr\left(U_{33}J_3U_{33}^T\right) \dots for\ i=3,\ j=3,\ p=3$$

From the above formulations it can be said that

$$D_{12}=D_{21},\ D_{13}=D_{31},\ D_{23}=D_{32}$$

With reference to the generalized joint coordinate's q*i*, the homogeneous transformation matrices were partially differentiated for the first time, were

Where ${}^{i-1}A_i$ = Transformation matrix.

The transformation matrix of the 3DOF RRR robot as shown in figure 1 are obtained as follows.

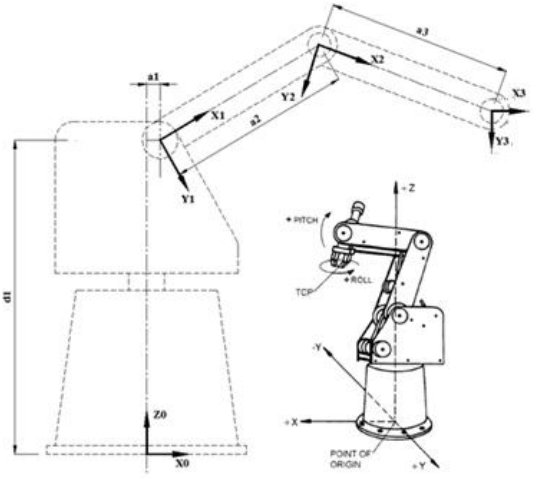


Figure 1. A 3DOF Robot Arm

$${}^0A_1 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & a_1C\theta_1 \\ S\theta_1 & 0 & C\theta_1 & a_1S\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Q_{Revolute} = Sparse matrix

$$Q_i = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By substituting the values of i and j, both of which varies from 1 to 3 for 3 DOF manipulator following formulation was obtained

$$U_{11}=\frac{\partial T_1}{\partial \theta_1}=\frac{\partial^0 T_1}{\partial \theta_1}=\frac{\partial^0 A_1}{\partial \theta_1}=QA_1 \quad for\ i=1, j=1$$

$U_{12}=0$ (j>i) *for* $i=1, j=2$

$U_{13}=0$ (j>i) *for* $i=1, j=3$

$$U_{21}=\frac{\partial T_2}{\partial \theta_1}=\frac{\partial^0 T_2}{\partial \theta_1}=\frac{\partial^0 A_2}{\partial \theta_1}$$

$$=\frac{^0A_1\,^1A_2}{\partial \theta_1}=Q\,^0A_1\,^1A_2 \ldots for\ i=2,\ j=1$$

$$U_{22}=\frac{\partial T_2}{\partial \theta_2}=\frac{\partial^0 T_2}{\partial \theta_2}=\frac{\partial^0 A_2}{\partial \theta_2}=\frac{^0A_1\,^1A_2}{\partial \theta_2}$$

$$=^0A_1\,Q\,^1A_2 \ \ldots for\ i=2,\ j=2$$

$U_{23}=0$ (j>i) *for* $i=2, j=3$

$$U_{31}=\frac{\partial T_3}{\partial \theta_1}=\frac{\partial^0 T_3}{\partial \theta_1}=\frac{\partial^0 A_3}{\partial \theta_1}$$

$$=\frac{^0A_1\,^1A_2\,^2A_3}{\partial \theta_1}=Q\,^0A_1\,^1A_2\,^2A_3 \ldots for\ i=3,\ j=1$$

$$U_{32}=\frac{\partial T_3}{\partial \theta_2}=\frac{\partial^0 T_3}{\partial \theta_2}=\frac{\partial^0 A_3}{\partial \theta_2}=\frac{^0A_1\,^1A_2\,^2A_3}{\partial \theta_2}$$

$$=^0A_1Q\,^1A_2\,^2A_3 \ \ldots for\ i=3,\ j=2$$

$$U_{33}=\frac{\partial T_3}{\partial \theta_3}=\frac{\partial^0 T_3}{\partial \theta_3}=\frac{\partial^0 A_3}{\partial \theta_3}$$

$$=\frac{^0A_1\,^1A_2\,^2A_3}{\partial \theta_3}=^0A_1\,^1A_2\,Q\,^2A_3 \ldots for\ i=3,\ j=3$$

Similarly with reference to the generalized joint coordinate's q*i*, the homogeneous transformation matrices were partially differentiated for the 2nd time, were

$$U_{ijk}=\frac{\partial U_{ij}}{\partial q_k}=\begin{cases}\dfrac{\partial(A_1A_2\ldots QA_j\ldots A_i}{\partial q_k} & for\ k<i\\ 0 & for\ k>i\end{cases} \qquad (5)$$

By substituting the values of i, j and k in equation (5) following formulations were obtained.

$$U_{111}=\frac{\partial U_{11}}{\partial \theta_1}=\frac{\partial\left(Q\,^0A_1\right)}{\partial \theta_1}=QQ\,^0A_1$$

$$U_{211}=\frac{\partial(U_{21})}{\partial \theta_1}=\frac{\partial\left(Q\,^0A_1\,^1A_2\right)}{\partial \theta_1}=QQ\,^0A_1\,^1A_2$$

$$U_{212}=U_{221}=\frac{\partial(U_{21})}{\partial \theta_2}=\frac{\partial\left(Q\,^0A_1\,^1A_2\right)}{\partial \theta_2}=Q\,^0A_1Q\,^1A_2$$

$$U_{222}=\frac{\partial(U_{22})}{\partial \theta_2}=\frac{\partial\left(^0A_1Q\,^1A_2\right)}{\partial \theta_2}=^0A_1QQ\,^1A_2$$

$$U_{311}=\frac{\partial(U_{31})}{\partial \theta_1}=\frac{\partial\left(Q\,^0A_1\,^1A_2\,^2A_3\right)}{\partial \theta_1}=QQ\,^0A_1\,^1A_2\,^2A_3$$

$$U_{312}=U_{321}=\frac{\partial(U_{31})}{\partial \theta_2}=\frac{\partial\left(Q\,^0A_1\,^1A_2\,^2A_3\right)}{\partial \theta_2}$$

$$=Q\,^0A_1Q\,^1A_2\,^2A_3$$

$$U_{313}=U_{331}=\frac{\partial(U_{31})}{\partial \theta_3}=\frac{\partial\left(Q\,^0A_1\,^1A_2\,^2A_3\right)}{\partial \theta_3}=Q\,^0A_1\,^1A_2Q\,^2A_3$$

$$U_{322}=\frac{\partial(U_{32})}{\partial \theta_2}=\frac{\partial\left(^0A_1Q\,^1A_2\,^2A_3\right)}{\partial \theta_2}=^0A_1QQ\,^1A_2\,^2A_3$$

$$U_{323}=U_{332}=\frac{\partial(U_{32})}{\partial\theta_3}=\frac{\partial\left({}^0A_1Q^1A_2{}^2A_3\right)}{\partial\theta_3}={}^0A_1Q^1A_2Q^2A_3$$

$$U_{333}=U_{333}=\frac{\partial(U_{33})}{\partial\theta_3}=\frac{\partial\left({}^0A_1{}^1A_2Q^2A_3\right)}{\partial\theta_3}={}^0A_1{}^1A_2QQ^2A_3$$

B. Centrifugal and Coriolis force N-vector

The coefficients D_{ikp} were associated to the joint velocity variables and can be computed as Eq. 6. The indices k and p represents the velocities of joints k and p that leads to induce a reaction torque at joint i. Thus, the index i associates with joint where the reaction torque was induced by joint velocity. In general, D_{ikk} are linked to the velocity of joint k (for k = p), where the centrifugal Coriolis torque exists. As for the case k≠p, the reaction torque induced by the velocity of joint k is defined by D_{ikp}, and acting at joint i, or vice versa.
C(q,q.) = [Ci], with components
Ci = $q^{\cdot}$T C(i) $q^{\cdot}$
Coupling (n x n) matrix C(i) =[D_{ijk}], with elements are calculated as follows:

$$D_{ijk}=\sum_{p=\max(i,j,k)}^{n}\operatorname{trace}(U_{pjk}J_pU_{pi}^T)\quad\text{... for i, j, k varies from 1 to 3} \tag{6}$$

By substituting the values of i, j and k, all of which varies from 1 to 3 following formulations was obtained.

$$D_{111}=Tr(U_{111}J_1U_{11}^T)+Tr(U_{211}J_2U_{21}^T)+Tr(U_{311}J_3U_{31}^T)\ \text{... for } i=1,\ j=1,\ k=1,\ p=1,2,3$$

$$D_{112}=Tr(U_{212}J_2U_{21}^T)+Tr(U_{312}J_3U_{31}^T)\ \text{... for } i=1,\ j=1,\ k=2,\ p=2,3$$

$$D_{113}=Tr(U_{313}J_3U_{31}^T)\ \text{... for } i=1,\ j=1,\ k=3,\ p=3$$

$$D_{121}=Tr(U_{221}J_2U_{21}^T)+Tr(U_{321}J_3U_{31}^T)\ \text{... for } i=1,\ j=2,\ k=1,\ p=2,3,$$

$$D_{122}=Tr(U_{222}J_2U_{21}^T)+Tr(U_{322}J_3U_{31}^T)\ \text{... for } i=1,\ j=2,\ k=2,\ p=2,3$$

$$D_{123}=Tr(U_{323}J_3U_{31}^T)\ \text{... for } i=1,\ j=2,\ k=3,\ p=3$$

$$D_{131}=D_{113}\ ,\ D_{132}=D_{123}$$

$$D_{133}=Tr(U_{333}J_3U_{31}^T)\ \text{... for } i=1,\ j=3,\ k=3,\ p=3$$

$$D_{211}=Tr(U_{211}J_2U_{22}^T)+Tr(U_{311}J_3U_{32}^T)\ \text{... for } i=2,\ j=1,\ k=1,\ p=2,3$$

$$D_{212}=Tr(U_{212}J_2U_{22}^T)+Tr(U_{312}J_3U_{32}^T)\ \text{... for } i=2,\ j=1,\ k=2,\ p=2,3$$

$$D_{213}=Tr(U_{313}J_2U_{32}^T)\ \text{... for } i=2,\ j=1,\ k=3,\ p=3$$

$$D_{221}=D_{212}$$

$$D_{222}=Tr(U_{222}J_2U_{22}^T)+Tr(U_{322}J_3U_{32}^T)\ \text{... for } i=2,\ j=2,\ k=2,\ p=2,3$$

$$D_{223}=Tr(U_{323}J_3U_{32}^T)\ \text{... for } i=2,\ j=2,\ k=3,\ p=3$$

$$D_{231}=D_{213}\ ,\ D_{232}=D_{223}$$

$$D_{311}=Tr(U_{311}J_3U_{33}^T)\ \text{... for } i=3,\ j=1,k=1,\ p=3$$

$$D_{312}=Tr(U_{312}J_3U_{33}^T)\ \text{... for } i=3,\ j=1,k=2,\ p=3$$

$$D_{313}=Tr(U_{313}J_3U_{33}^T)\ \text{... for } i=3,\ j=1,k=3,\ p=3$$

$D_{321}=D_{312}$

$D_{322}=Tr(U_{322}J_3U_{33}^T)$ ... for $i=3, j=2, k=2, p=3$

$D_{323}=Tr(U_{323}J_3U_{33}^T)$ ... for $i=3, j=2, k=3, p=3$

$D_{331}=D_{313}$, $D_{332}=D_{323}$

$D_{333}=Tr(U_{333}J_3U_{33}^T)$ ... for $i=3, j=3, k=3, p=3$

C. *Gravitational force N-vector*

G(q) = [Di]

$$D_i = \sum_{p=i}^{n} -m_p g^T U_{pi} \bar{r}_p \tag{7}$$

where $\bar{r}_i$ = the position of the center of mass of link i with respect to the i[th] coordinate system

g = the gravity row vector,

g^T = [gx gy gz 0] was the gravity matrix

|g| = 9.8062 m/s^2.

By substituting the values of *i* and *p*, which varies from 1 to 3 following formulations was obtained

$$D_1 = -\left[m_1 g^T U_{11} \bar{r}_1 + m_2 g^T U_{21} \bar{r}_2 + m_3 g^T U_{31} \bar{r}_3 \right] \quad \text{... for } i = 1, p = 1, 2, 3$$

$$D_2 = -\left[m_2 g^T U_{22} \bar{r}_2 + m_3 g^T U_{32} \bar{r}_3 \right] \quad \text{... for } i = 2, p = 2, 3$$

$$D_3 = -\left[m_3 g^T U_{33} \bar{r}_3 \right] \quad \text{... for } i = 3, p = 3$$

The equation (3) when expanded for 3 axis revolute robot shall be as follows:

$$\begin{aligned}\tau_i &= D_{i1}\ddot{\theta}_1 + D_{i2}\ddot{\theta}_2 + D_{i3}\ddot{\theta}_3 + I_{i(actuator)}\ddot{\theta}_i \\ &+ D_{i11}\dot{\theta}_1^2 + D_{i22}\dot{\theta}_2^2 + D_{i33}\dot{\theta}_3^2 \\ &+ D_{i12}\dot{\theta}_1\dot{\theta}_2 + D_{i13}\dot{\theta}_1\dot{\theta}_3 + \\ &+ D_{i21}\dot{\theta}_2\dot{\theta}_1 + D_{i23}\dot{\theta}_2\dot{\theta}_3 + \\ &+ D_{i31}\dot{\theta}_3\dot{\theta}_1 + D_{i32}\dot{\theta}_3\dot{\theta}_2 + D_i\end{aligned}$$

Substituting values of i i.e 1 to 3 for three links, in equations (3) following formulation for torque was obtained for each link.

$$\begin{aligned}\tau_1 &= D_{11}\ddot{\theta}_1 + D_{12}\ddot{\theta}_2 + D_{13}\ddot{\theta}_3 + I_{1(actuator)}\ddot{\theta}_1 \\ &+ D_{111}\dot{\theta}_1^2 + D_{122}\dot{\theta}_2^2 + D_{133}\dot{\theta}_3^2 \\ &+ 2D_{112}\dot{\theta}_1\dot{\theta}_2 + 2D_{113}\dot{\theta}_1\dot{\theta}_3 + \\ &+ 2D_{123}\dot{\theta}_2\dot{\theta}_3 + D_1\end{aligned} \tag{8}$$

$$\begin{aligned}\tau_2 &= D_{21}\ddot{\theta}_1 + D_{22}\ddot{\theta}_2 + D_{23}\ddot{\theta}_3 + I_{2(actuator)}\ddot{\theta}_2 \\ &+ D_{211}\dot{\theta}_1^2 + D_{222}\dot{\theta}_2^2 + D_{233}\dot{\theta}_3^2 + \\ &+ 2D_{212}\dot{\theta}_1\dot{\theta}_2 + 2D_{213}\dot{\theta}_1\dot{\theta}_3 + \\ &+ 2D_{223}\dot{\theta}_2\dot{\theta}_3 + D_2\end{aligned} \tag{9}$$

$$\begin{aligned}\tau_3 &= D_{31}\ddot{\theta}_1 + D_{32}\ddot{\theta}_2 + D_{33}\ddot{\theta}_3 + I_{3(actuator)}\ddot{\theta}_3 \\ &+ D_{311}\dot{\theta}_1^2 + D_{322}\dot{\theta}_2^2 + D_{333}\dot{\theta}_3^2 \\ &+ 2D_{312}\dot{\theta}_1\dot{\theta}_2 + 2D_{313}\dot{\theta}_1\dot{\theta}_3 \\ &+ 2D_{323}\dot{\theta}_2\dot{\theta}_3 + D_3\end{aligned} \tag{10}$$

Substituting the values derived in equations (4),(5),(6) and (7) in these equations (8)(9) and (10) yields the equations of motion for the robot. These equations show how each term can affect the dynamics of the robot [9].

For formulating the virtual model of control manipulators dynamics the equation (2) may be written as

$$\ddot{\Theta} = (\tau - C(\Theta, \dot{\Theta}) - G(\Theta)) / D(\Theta)$$

Alternatively equation (3) can be rearranged to get the joint angle acceleration as

$$\ddot{q}_j = (\tau_i - D_i - \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q}_j\dot{q}_k) / \sum_{j=1}^{n} D_{ij} \tag{11}$$

Integrating the equation (11) the velocity of joint angle was obtained as

$$\Rightarrow \dot{q}_j = \int \ddot{q}_j = \int (\tau_i - D_i - \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q}_j\dot{q}_k) / \sum_{j=1}^{n} D_{ij} \tag{12}$$

Again integrating the equation (12) the value of joint angle was obtained as

$$\Rightarrow q_j = \int \dot{q}_j = \int\int (\tau_i - D_i - \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q}_j\dot{q}_k) / \sum_{j=1}^{n} D_{ij} \tag{13}$$

III. SIMULATION

In order to study the effects of each term prior to the implementation of controller, behaviour of the robot must be verified by some method. Here to study the robots behavioural characteristics a virtual model in MATLAB & SIMULINK has been developed. As the s-function in MATLAB updates the instantaneous configuration in run time, the same is used to determine the dynamics in terms of the generalized joint velocities and coordinates, represented as state-variables. The robots parameters were taken from [10] for simulation purpose only. The each joint was controlled by a separate PID (Proportional, Integral, and Derivative) controller [11]. The output of s-function was the joint angle and joint velocity of respective joints.

The control architecture is as shown in figure 1.The tuning of PID controller was done in MATLAB by Autotuning facility. The unit step was given as input to the virtual model and the simulation results are obtained for the three links. The open loop control as shown in figure 2 was investigated. Also the same was simulated under closed loop control strategy.

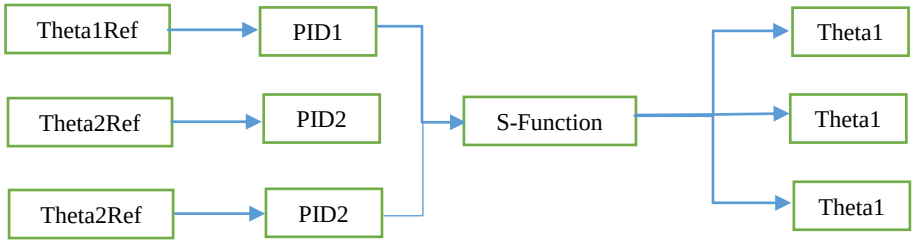


Figure 1. Control architecture

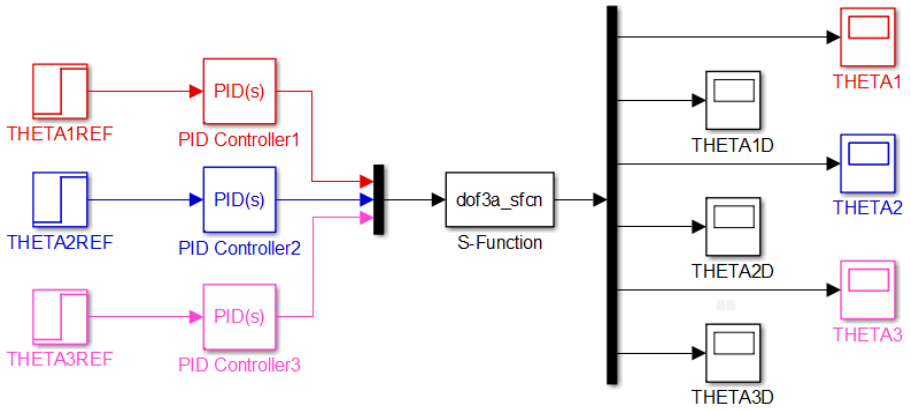


Figure 2. Open loop simulation

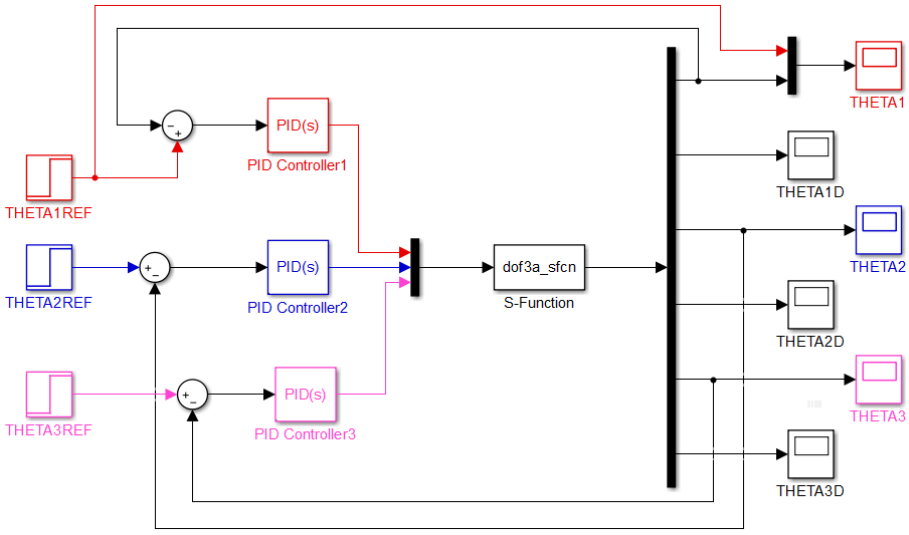
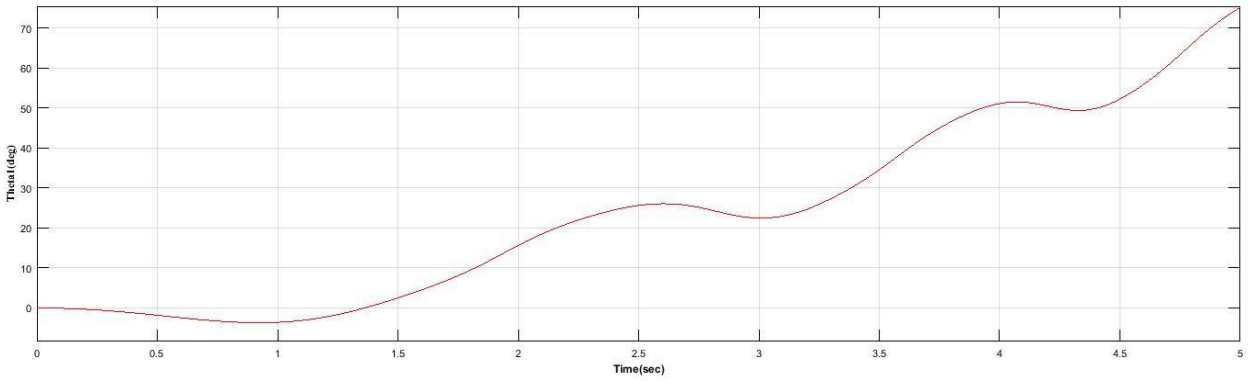
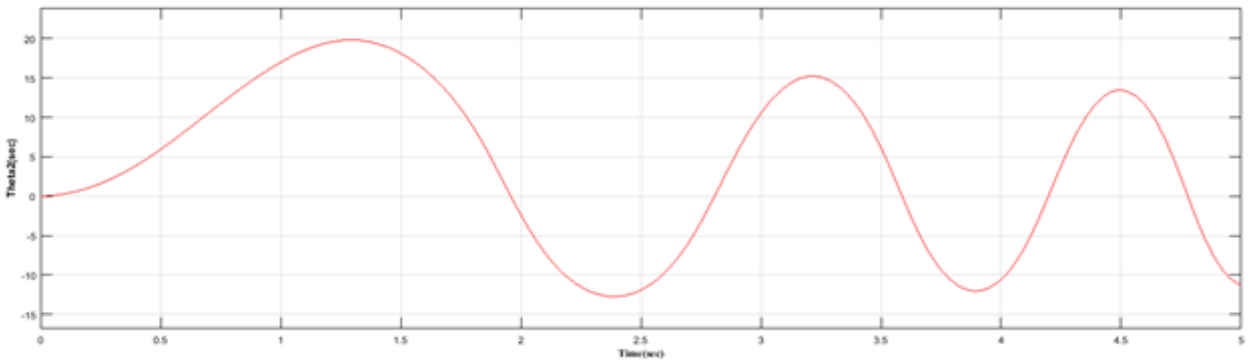


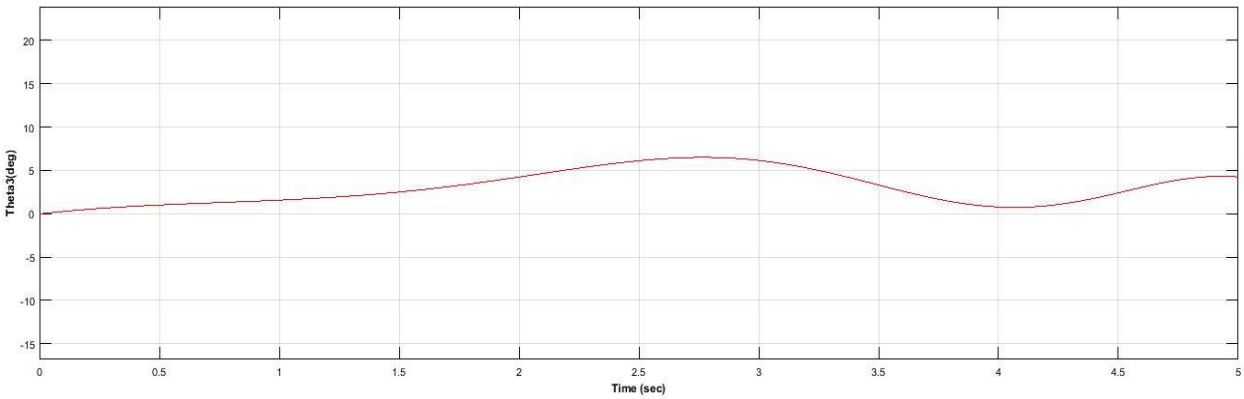
Figure 2. Simulation in Closed Loop



(a) Theta1



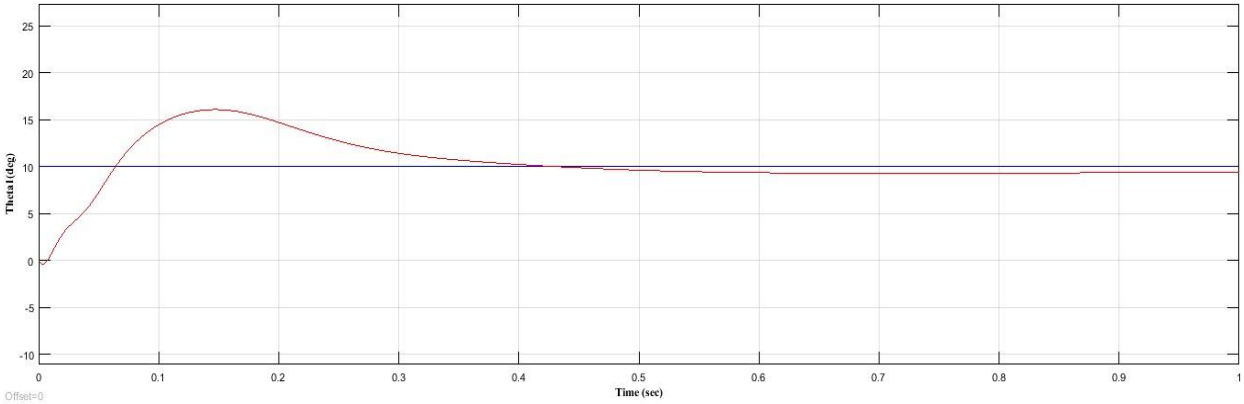
(b) Theta2



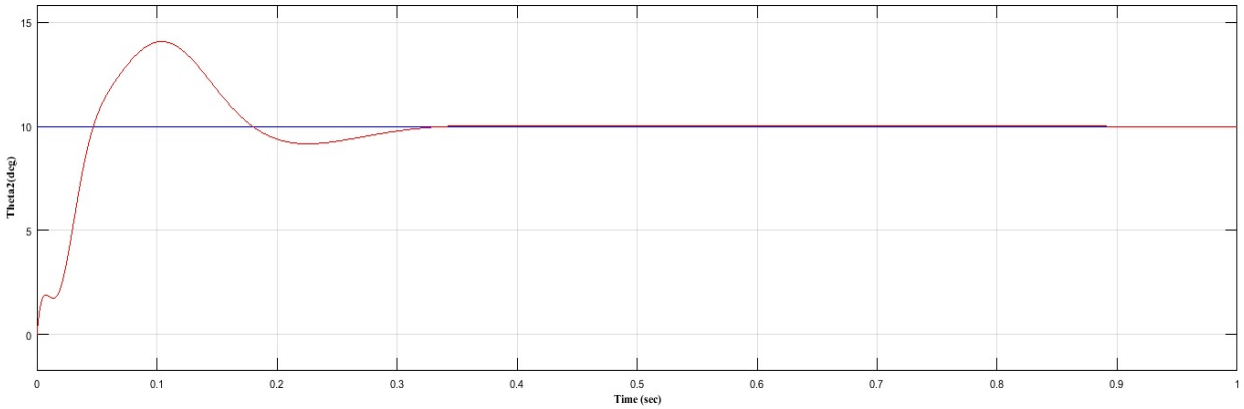
(b) Theta3

Figure 4. Open loop response

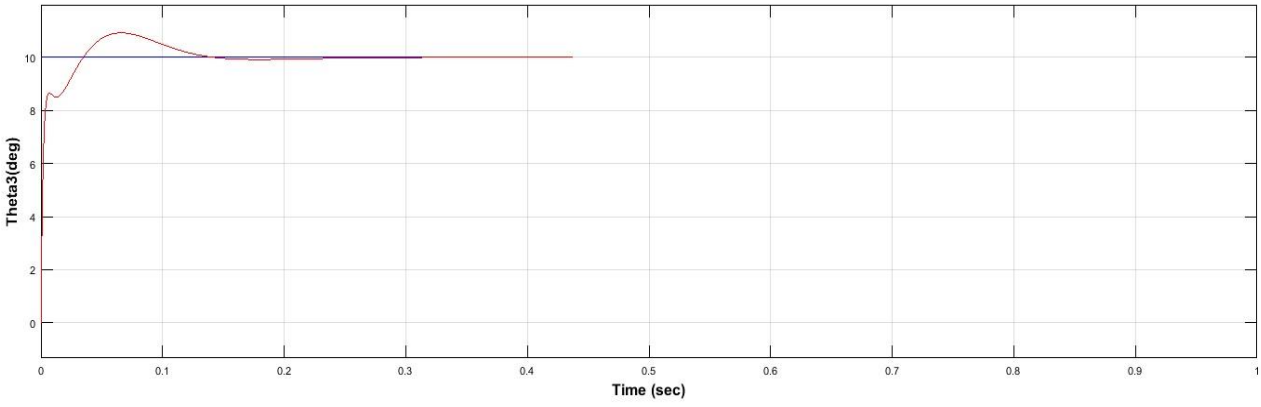
The closed loop responses obtained were shown in figure 5.



(a) Theta1



(b) Theta2



(c)Theta3

Figure 5. Close loop response

IV. RESULTS

The response of the open loop obtain was unstable because the open loop control system does not monitor the condition of the output signals whether the set point is reached or not. Whereas the open loop response was observed to be unstable, the closed loop response is found to be stable because it automatically regulate the output and keep it within the systems desired input value or set point. This regulation was controlled by the feedback of the error to the controller. As the error becomes positive the output is regulated by negative control signal and vice versa. The results obtained for input of 10 degrees to all three joints. The overshoots for theta1, theta2 are observed to be 40.5% and 31.5% and 20% respectively. The rise time for theta1, theta2 and theta3 are observed to be 29.98, 8.67 and 6.89 milliseconds respectively. The steady state error was 0.05 degree for all the joint angles. A little mountain during the rise was observed may be due to the gravity effect.

V. CONCLUSIONS

The forward dynamics of 3 DOF robot manipulator was formulated successfully using the Lagrangian-Euler principle. The forward dynamics problem by L-E method was solved directly by computing the joint accelerations from the joint torque, and joint positions. The formulations obtained were in matrix form and well structured. It was structured in the sense that it was in the matrix form which eases the formulation consequently which eases the computer programming and simulation. The effect of Inertia vector, Centrifugal force, Coriolis acceleration and Gravity vector, in the final expression can be observed in the simulated results. The form of the model presented in this work facilitates neglecting or compensating the terms which are not applicable in a particular operating conditions. For example in the space application the effect of gravity is negligible, the Gravity vector may be conveniently deleted without affecting the structure of dynamics. Similarly, the values of centrifugal force, Coriolis acceleration may be tuned conveniently as per the requirements of the operations. Hence the developed model is very suitable for modelling of robot controller.

The 3dof articulated robot arm was investigated for preparing the virtual model. The forward kinematics and dynamics based on Lagrangian-Euler form was obtained. The virtual model was prepared in SIMULINK. The results of the open loop control and closed loop PID control was presented.

The procedure followed to obtain the dynamics of the 3 DOF articulated robots may be extended to ‘n’ link robot. Similarly the virtual model can be further implied for simulating ‘n’ link articulated robot. The tuning of PID controller may be carried out to achieve the desired accuracy and faster response time.

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