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Actuation scheme based Robustness Index of 3R and 4R Planar Serial Manipulators

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Abstract: The kinematic performance measures of 3R and 4R planar serial manipulators are assessed by considering the actuation schemes. All the conceivable structures in view of actuators area are distinguished. The robustness index is found for various transmission ratios for each structure of the serial manipulators. Finally the nearly isotropic structures are identified.

Keywords: Serial Manipulators, Robustness Index, Transmission Ratios, Actuation schemes, Jacobian Matrix

1. Introduction

The concept of robust design was introduced by Taguchi. He proposed the concept of parameter design to improve the quality of a product whose manufacturing process involves significant variability or noise. Every engineering design is subject to variations that can arise from a variety of sources, including manufacturing operations, variations in material properties, and the operating environment. When variations are ignored, nonrobust designs can result, which are expensive to produce or fail in service. Besides, the robustness of a mechanism is important when calibration is necessary because the lower the sensitivity of the mechanism to dimensional variations, the easier its calibration6.2 Planar 3R Serial Manipulator.Hollerbach and Suh (1) analyzed the methods for resolving kinematic redundancies of manipulators by the impact on joint torques. Kazuhiro Kosuge and Katsuhisa Furuta (2) utilized the backwards of the robustness index of Jacobian Matrix as a measure of kinematic controllability.Mansouri and Ouali (3) proposed another performance index of robot manipulators, the power manipulability. It is completely homogeneous and constitutes a physically reliable framework.Merlet (4) demonstrated that the robustness index and manipulability don't precisely mirror the position accuracy of the robot. Forebearing M.Gosselin (5) proposed new dexterity indices for spatial manipulators. These indices depend on the condition number of the Jacobian Matrix of the manipulators. He detailed kinematic conditions such that prompts dexterity indices that are frame invariant, where as existing indices are influenced by a scaling of the manipulator.Jun Wu et al (6) proposed the condition number of inertia matrix of dynamic equation for assessing the dynamic dexterity of a 2-dof serial manipulator.Mayorga et al (7) exhibited a basic performance index for robot manipulator kinematic outline optimization and best stance assurance. This index is gotten from a homogenized isotropy state of an appropriately weighted Jacobian matrix. The proposed index is connected to the kinematic plan optimization and best stance assurance of a redundant manipulator. Ghosal and Ravani (8) introduced a differential-geometric way to deal with analyze the singularities of assignment space point trajectories of two and three-degree-of-freedom serial manipulators. They have demonstrated that the degeneracies of the velocity ellipsoid gives a basic geometric picture of the conceivable errand space speeds at a singular configuration.

2. Planar 3R Serial Manipulator

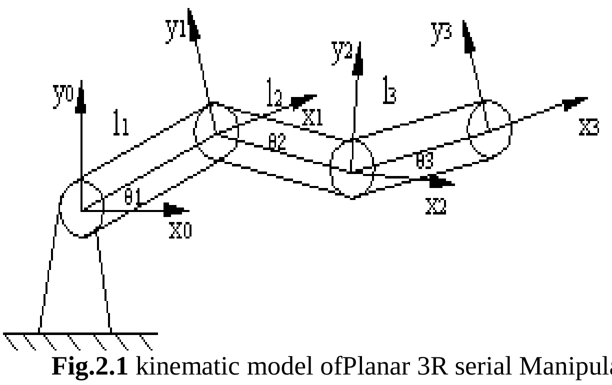

Fig.2.1 kinematic model ofPlanar 3R serial Manipulator

The Jacobian matrix expressed with respect to the coordinate frame located at the manipulator tip (third co-ordinate frame) is given by

$$^{3}J = \begin{bmatrix} l_2 s_3 + l_1 s_{23} & l_2 s_3 & 0 \\ l_2 + l_2 c_3 + l_1 c_{23} & l_3 + l_2 c_3 & l_3 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.1}$$

Considered 3×3 *Matrix then*

$$T_{3\times 3} = \begin{bmatrix} r_1 & 0 & 0 \\ e_{21} r_2 & r_2 & 0 \\ e_{31} r_3 & e_{32} r_3 & r_3 \end{bmatrix} \tag{2.2}$$

$$T^{-1} = \frac{Adj[T]}{Det[T]} \tag{2.3}$$

$$T^{-1} = \frac{1}{r_1 r_2 r_3} \begin{bmatrix} r_2 r_3 & 0 & 0 \\ -e_{21} r_2 r_3 & r_1 r_3 & 0 \\ r_3 r_2 (e_{21} e_{32} - e_{31}) & -e_{32} r_1 r_3 & r_1 r_2 \end{bmatrix} \tag{2.4}$$

$$T^{-1} = \begin{bmatrix} \frac{1}{r_1} & 0 & 0 \\ \frac{-e_{21}}{r_1} & \frac{1}{r_2} & 0 \\ \frac{e_{21} e_{32} - e_{31}}{r_1} & \frac{-e_{32}}{r_2} & \frac{1}{r_3} \end{bmatrix} \tag{2.5}$$

Jacobian of planar 3R serial manipulator based on actuator’s location is given by

$$J_a = J_q T^{-1} \tag{2.6}$$

6.2.1 Actuator Location Based Structures of 3R Planar Serial Manipulator

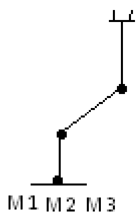

Fig.2.2 3R Structure 1
$e_{21}=1$, $e_{31}=1$
$e_{32}=1$, $t_{21}=-1$
$t_{31}=0$, $t_{32}=-1$

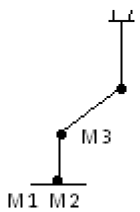

Fig.2.3 3R Structure 2
$e_{21}=1$, $e_{31}=0$
$e_{32}=1$, $t_{21}=-1$
$t_{31}=1$, $t_{32}=-1$

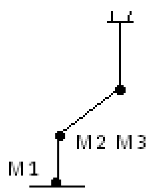

Fig.2.4 3R Structure 3
$e_{21}=0$, $e_{31}=0$
$e_{32}=1$, $t_{21}=0$
$t_{31}=0$, $t_{32}=-1$

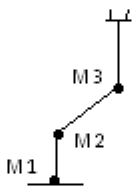

Fig.2.5 3R Structure 4
$e_{21}=0$, $e_{31}=0$
$e_{32}=0$, $t_{21}=0$
$t_{31}=0$, $t_{32}=0$

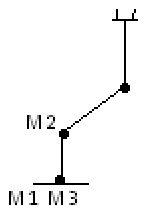

Fig.2.63R Structure 5
$e_{21}=0$, $e_{31}=1$
$e_{32}=1$, $t_{21}=0$
$t_{31}=-1$, $t_{32}=-1$

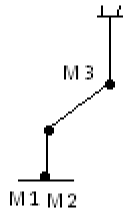

Fig.2.7 3R Structure 6
$e_{21}=1$, $e_{31}=0$
$e_{32}=0$, $t_{21}=-1$
$t_{31}=0$, $t_{32}=0$

2.2Results 3R Serial manipulator

The performance indices are evaluated for the structure 1 with five different sets of transmission ratios and the results are given below

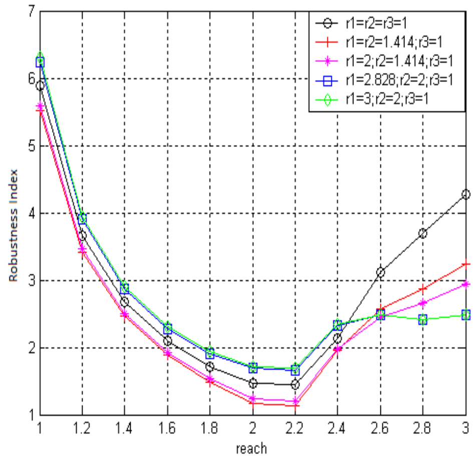

Fig 2.8 Variation of robustness index at various Transmission ratio for a 3R Manipulator

2.3 Planar 4R Serial Manipulator

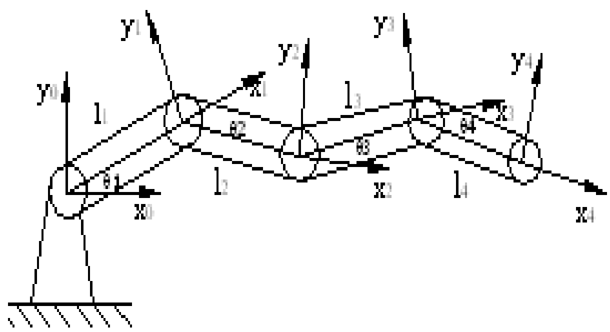

Fig2.9kinematic model of Planar 4R serial Manipulator

The Jacobian matrix expressed with respect to the coordinate frame located at the manipulator tip (fourth co-ordinate frame) is given by

$$^4\mathrm{J} = \begin{bmatrix} l_3 s_4 + l_2 s_{34} + l_1 s_{234} & l_3 s_4 + l_2 s_{34} & l_3 s_4 & 0 \\ l_4 + l_3 c_4 + l_2 c_{34} + l_1 c_{234} & l_4 + l_3 c_4 + l_2 c_{34} & l_4 + l_3 c_4 & l_4 \end{bmatrix} \quad (2.7)$$

consider 4 X 4 matrix

$$T_{4x4} = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ e_{21} r_1 & r_2 & 0 & 0 \\ e_{31} r_3 & e_{32} r_3 & r_3 & 0 \\ e_{41} r_4 & e_{42} r_4 & e_{43} r_4 & r_4 \end{bmatrix} \quad (2.8)$$

Determinant of (T) matrix is = $r_1 r_2 r_3 r_4$

Inverse of Matrix T_{4x4} matrix

$$T^{-1} = \begin{bmatrix} \frac{1}{r_1} & 0 & 0 & 0 \\ \frac{t_{21}}{r_1} & \frac{1}{r_2} & 0 & 0 \\ \frac{t_{31}}{r_1} & \frac{t_{32}}{r_2} & \frac{1}{r_3} & 0 \\ \frac{t_{41}}{r_1} & \frac{t_{42}}{r_2} & \frac{t_{43}}{r_3} & \frac{1}{r_4} \end{bmatrix} \quad (2.9)$$

$$J_a = J_q T^{-1}\ (or)\ J_q = J_a T \quad (2.10)$$

2.3.1 Actuator Location Based Structures of Planar 4R Serial Manipulator

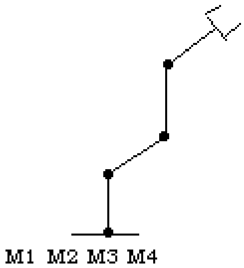

Fig.2.10 4R Structure 1
$e_{21}=1, e_{31}=1, e_{32}=1$
$e_{41}=1, e_{42}=1, e_{43}=1$
$t_{21}=-1, t_{31}=0, t_{32}=-1$
$t_{41}=0, t_{42}=0, t_{43}=-1$

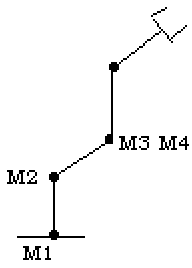

Fig.2.114R Structure 2
$e_{21}=0, e_{31}=0, e_{32}=0$
$e_{41}=0, e_{42}=0, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=0$
$t_{41}=0, t_{42}=0, t_{43}=-1$

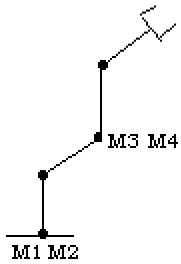

Fig.2.12 4R Structure 3
$e_{21}=1, e_{31}=0, e_{32}=0$
$e_{41}=0, e_{42}=0, e_{43}=1$
$t_{21}=-1, t_{31}=0, t_{32}=0$
$t_{41}=0, t_{42}=0, t_{43}=-1$

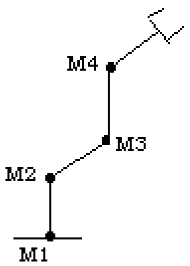

Fig.2.13 4R Structure 4
$e_{21}=0, e_{31}=0, e_{32}=0$
$e_{41}=0, e_{42}=0, e_{43}=0$
$t_{21}=0, t_{31}=0, t_{32}=0$
$t_{41}=0, t_{42}=0, t_{43}=0$

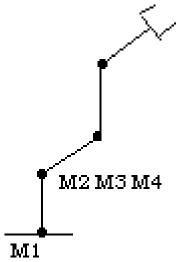

Fig.2.14 4R Structure 5
$e_{21}=0, e_{31}=0, e_{32}=1$
$e_{41}=0, e_{42}=1, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=-1$
$t_{41}=0, t_{42}=0, t_{43}=-1$

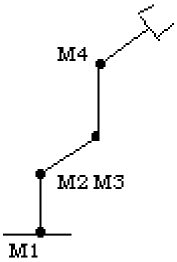

Fig.2.15 4R Structure 6
$e_{21}=0, e_{31}=0, e_{32}=1$
$e_{41}=0, e_{42}=0, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=-1$
$t_{41}=0, t_{42}=0, t_{43}=0$

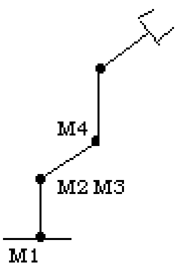

Fig.2.16 4R Structure 7
$e_{21}=0, e_{31}=0, e_{32}=1$
$e_{41}=0, e_{42}=0, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=-1$
$t_{41}=0, t_{42}=1, t_{43}=-1$

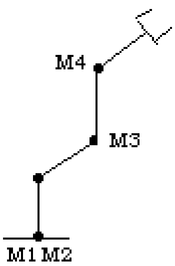

Fig.2.17 4R Structure 8
$e_{21}=1, e_{31}=0, e_{32}=0$
$e_{41}=0, e_{42}=0, e_{43}=0$
$t_{21}=-1, t_{31}=0, t_{32}=0$
$t_{41}=0, t_{42}=0, t_{43}=0$

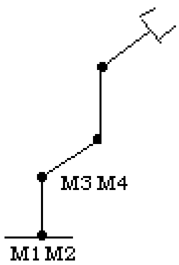

Fig.2.18 4R Structure 9
$e_{21}=1, e_{31}=0, e_{32}=1$
$e_{41}=0, e_{42}=1, e_{43}=1$
$t_{21}=-1, t_{31}=1, t_{32}=-1$
$t_{41}=0, t_{42}=0, t_{43}=-1$

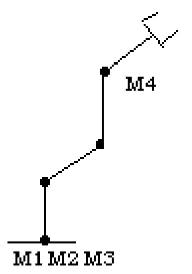

Fig.2.19 4R Structure 10
$e_{21}=1, e_{31}=1, e_{32}=1$
$e_{41}=0, e_{42}=0, e_{43}=0$
$t_{21}=-1, t_{31}=0, t_{32}=-1$
$t_{41}=0, t_{42}=0, t_{43}=0$

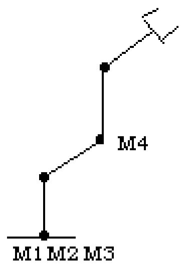

Fig.2.20 4R Structure 11
$e_{21}=1, e_{31}=1, e_{32}=1$
$e_{41}=0, e_{42}=0, e_{43}=1$
$t_{21}=-1, t_{31}=0, t_{32}=-1$
$t_{41}=0, t_{42}=1, t_{43}=-1$

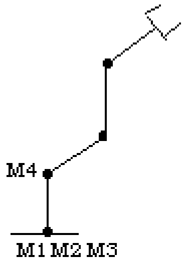

Fig.2.21 4R Structure 12
$e_{21}=1, e_{31}=1, e_{32}=1$
$e_{41}=0, e_{42}=1, e_{43}=1$
$t_{21}=-1, t_{31}=0, t_{32}=-1$
$t_{41}=1, t_{42}=0, t_{43}=-1$

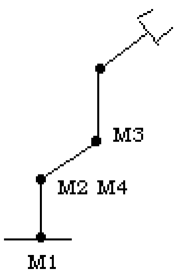

Fig.2.22 4R Structure 13
$e_{21}=0, e_{31}=0, e_{32}=0$
$e_{41}=0, e_{42}=1, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=0$
$t_{41}=0, t_{42}=-1, t_{43}=-1$

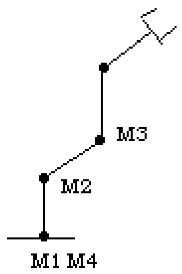

Fig.2.23 4R Structure 14
$e_{21}=0, e_{31}=0, e_{32}=0$
$e_{41}=1, e_{42}=1, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=0$
$t_{41}=-1, t_{42}=-1, t_{43}=-1$

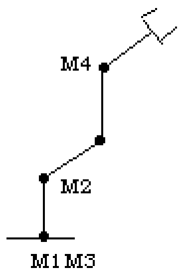

Fig.2.24 4R Structure 15
$e_{21}=1, e_{31}=1, e_{32}=1$
$e_{41}=0, e_{42}=1, e_{43}=1$
$t_{21}=-1, t_{31}=0, t_{32}=-1$
$t_{41}=1, t_{42}=0, t_{43}=-1$

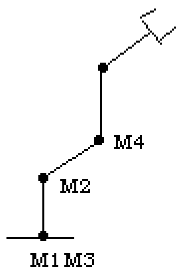

Fig.2.25 4R Structure 16
$e_{21}=0, e_{31}=1, e_{32}=1$
$e_{41}=0, e_{42}=0, e_{43}=1$
$t_{21}=0, t_{31}=-1, t_{32}=-1$
$t_{41}=1, t_{42}=1, t_{43}=-1$

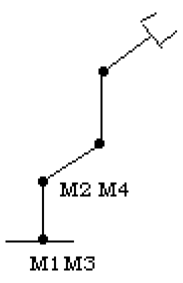

Fig.2.26 4R Structure 17
$e_{21}=0, e_{31}=1, e_{32}=1$
$e_{41}=0, e_{42}=1, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=-1$
$t_{41}=0, t_{42}=0, t_{43}=-1$

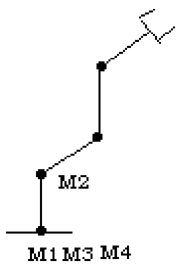

Fig.2.27 4R Structure 18
$e_{21}=0, e_{31}=1, e_{32}=1$
$e_{41}=1, e_{42}=1, e_{43}=1$
$t_{21}=0, t_{31}=-1, t_{32}=-1$
$t_{41}=0, t_{42}=0, t_{43}=-1$

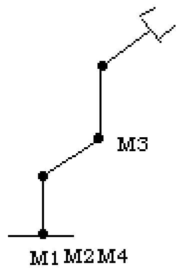

Fig.2.28 4R Structure 19
$e_{21}=1, e_{31}=0, e_{32}=0$
$e_{41}=1, e_{42}=1, e_{43}=1$
$t_{21}=-1, t_{31}=0, t_{32}=0$
$t_{41}=0, t_{42}=-1, t_{43}=-1$

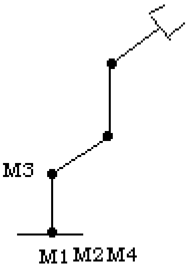

Fig.2.29 4R Structure 20
$e_{21}=1, e_{31}=0, e_{32}=1$
$e_{41}=1, e_{42}=1, e_{43}=1$
$t_{21}=-1, t_{31}=1, t_{32}=-1$
$t_{41}=-1, t_{42}=0, t_{43}=-1$

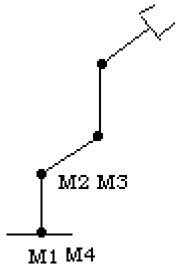

Fig.2.30 4R Structure 21
$e_{21}=0, e_{31}=0, e_{32}=1$
$e_{41}=1, e_{42}=1, e_{43}=1$
$t_{21}=0, t_{31}=0, t_{32}=-1$
$t_{41}=-1, t_{42}=0, t_{43}=-1$

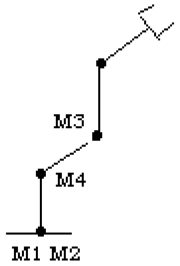

Fig.2.31 4R Structure 22
$e_{21}=1, e_{31}=0, e_{32}=0$
$e_{41}=0, e_{42}=1, e_{43}=1$
$t_{41}=1, t_{42}=-1, t_{43}=-1$

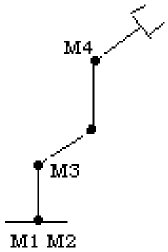

Fig.2.32 4R Structure 23
$e_{21}=1, e_{31}=0, e_{32}=1$
$e_{41}=0, e_{42}=0, e_{43}=0$
$t_{41}=0, t_{42}=0, t_{43}=0$

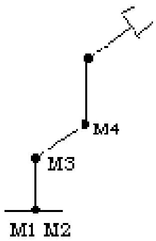

Fig.2.33 4R Structure 24
$e_{21}=1, e_{31}=0, e_{32}=1$
$e_{41}=0, e_{42}=0, e_{43}=1$
$t_{41}=-1, t_{42}=1, t_{43}=-1$

6.3.3 Results of 4R manipulator

The performance indices are evaluated for the structure 1 with five different sets of transmission ratios and the results are given below.

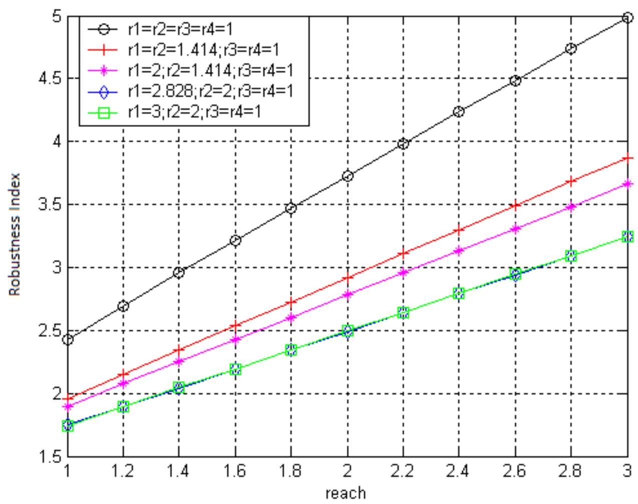

Fig 6.37 Variation of Condition Number at various Transmission ratio for a 4-Link Manipulator

Conclusion

The kinematic performance measures of 3R and 4R planar serial manipulators are assessed by considering the actuation schemes. All the conceivable structures in light of actuators' area are distinguished. For various transmission ratios in each structure the performance measures are resolved. Nearly isotropic configurations are acquired for the 3R manipulator for the transmission ratio (r1= r2=r3 =1) and reachis 2.2 and for the 4R manipulator for the transmission ratio (r1= 3, r2=2, r3=r4= 1) and reach is 1.0

References

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