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Progress of Plastic Zones in Elasto-Plastic Material of a Circular Plate with the Opening

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Abstract*— The literature on the load bearing capacities of circular plates with opening is very scant. There is a concentration and complex distribution of stresses around the openings upon loading even though the material is elastic. These stresses get redistributed as the material starts being plastic and plasticity progresses with increasing loads. In this study, an elasto-plastic analysis is carried out to determine maximum non-dimensional deformations and corresponding collapse loads for clamped circular plates with the opening. Von Mises yield criterion has been considered. The progress of plastic zones under uniformly distributed load (UDL) or point load (PL) is plotted and studied for the elasto-plastic material used.

***Keywords*— Elasto-Plastic; Bending Analysis; Circular Plate; Opening; Plastic Zone**

I. INTRODUCTION

The existence of opening or hole in solids is often encountered in the design of manholes, concrete ducts, openings in slab, automobile parts, mechanical manufacture, pressure vessels, underground excavations, etc. In such cases, the designers are interested in the stress concentration at the opening edges and the maximum deformation.

The elastic designing results in overdesigning of the component and wastage of the material. This limitation led to conduct elasto-plastic analysis of the yield loads for the elements. In this study the main focus is on the spread of plasticity due to incremental increase in loading and the maximum deformation of the circular plate with various thicknesses and having a circular opening.

II. LITERATURE SURVEY

The elastic design is highly uneconomical, whereas in elasto-plastic design, the whole material is stressed to its yield limit and even beyond that, which makes it very economical. The redistribution of the stresses occurs, when the structure is loaded beyond the elastic limit. The calculation of this redistribution is not easy, and limit analysis has been the usual option for such analyses. The limit analysis is being employed from a long time for the plastic analysis of beams and frames. However, such analysis leads to collapse loads instead of gradual elasto-plastic behavior. Plates, due to their two-dimensional idealization, make the problem complicated and limit analysis gives the upper and lower bounds solutions.

For studying the proper progress of plasticity in the material of plates, an accurate elastic theory gives an accurate foundation to the analysis. A higher order theory based on 3-D Hooke's law gives more realistic quadratic variation of transverse shear strains and cubic variation of transverse normal strain through the plate thickness [1]. The finite element formulation for bending analysis of plates based on this theory was developed and performance of a 9-noded Lagrangian quadrilateral element for very thick to extremely thin plates under different loading and support condition was evaluated [2]. The pure bending of simply supported circular plate of transversely loaded, isotropic and functionally graded material was studies [3]. Uniaxial, biaxial and shear stresses for anisotropic pate under in-plane loading around hole were determined [4]. A plate with circular hole under tensile stress condition of loading was evaluated and a through-the-thickness stress concentration factor was derived and its variation was studied [5]. The analysis of thick plates using incremental finite element formulation and higher order shear deformation theory was done [6].

Looking to the need for investigation of elasto-plastic behaviour of material upto the collapse of plates from the above literature review, the study of circular plate with opening was undertaken.

III. THEORETICAL FORMULATION

In the present investigation, a higher order shear deformation theory (HOSDT) [1, 2] is used for the analysis of plates. The theory dealt here considers deformations, which account for the warping of the cross-sections. Hence, shear correction coefficients are not required and parabolic distribution of transverse shear strain over the thickness of the plate is obtained. Thus, limitations of usual Kirchhoff's theory as well as Reissner-Mindlin type first order theories are eliminated.

3.1 Theoretical Formulation Based on HOSDT.

In order to approximate a three-dimensional elasticity problem to a two-dimensional one, the three-dimensional displacement components *u* (*x, y, z*), *v* (*x, y, z*) and *w* (*x, y, z*) of any point in the plate space can be expressed in terms of thickness coordinate '*z*' using a Taylor's series expansion, viz.

$$\left.\begin{aligned}
u(x,y,z) &= u(x,y,0) + z\left(\frac{\partial u}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 u}{\partial z^2}\right)_0 + \frac{1}{3!}z^3\left(\frac{\partial^3 u}{\partial z^3}\right)_0 + \dots\dots\dots + \infty \\
v(x,y,z) &= v(x,y,0) + z\left(\frac{\partial v}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 v}{\partial z^2}\right)_0 + \frac{1}{3!}z^3\left(\frac{\partial^3 v}{\partial z^3}\right)_0 + \dots\dots\dots + \infty \\
w(x,y,z) &= w(x,y,0) + z\left(\frac{\partial w}{\partial z}\right)_0 + \frac{1}{2!}z^2\left(\frac{\partial^2 w}{\partial z^2}\right)_0 + \frac{1}{3!}z^3\left(\frac{\partial^3 w}{\partial z^3}\right)_0 + \dots\dots\dots + \infty
\end{aligned}\right\} \quad (1)$$

Using these expressions, the three-dimensional elasticity problem of plate bending can be reduced to a two-dimensional formulation without any considerable loss of accuracy. Lo et al. [7] and Kant [2] have further shown that the retention of only first four terms for the in-plane displacements *u* and *v* and first three terms for the transverse displacement *w* is sufficient.

Retaining only the flexural behaviour representing terms from the above expressions, our displacement model for the plate bending can be expressed as:

$$\begin{aligned}
u &= z\,\theta_x + z^3\,\theta_x^* & \text{where}\quad \theta_x^* &= \frac{1}{3!}\frac{\partial^3 u}{\partial z^3} \\
v &= z\theta_y + z^3\,\theta_y^* & \theta_y^* &= \frac{1}{3!}\frac{\partial^3 v}{\partial z^3} \\
w &= w_0
\end{aligned} \quad (2)$$

θ_x and θ_y are the rotations of the normal to the mid-plane about y and x axes respectively. The expressions of '*u*' and '*v*' represent a non-linear variation of in-plane displacements through the thickness. Thus, warping of transverse cross-sections is automatically incorporated.

3.2 Strain Displacement Relationship

As per the theory of elasticity [8], strain displacement relationship given :

$$\left.\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} = z\chi_x + z^3\,\chi_x^* \\
\varepsilon_y &= \frac{\partial v}{\partial y} = z\chi_y + z^3\,\chi_y^* \\
\varepsilon_z &= 0 \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = z\chi_{xy} + z^3\,\chi_{xy}^* \\
\gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \phi_y + z^2\,\phi_y^* \\
\gamma_{zx} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi_x + z^2\,\phi_x^*
\end{aligned}\right\} \quad (3)$$

Where,

$$\left.\begin{aligned}
\left(\chi_x,\chi_y,\chi_{xy}\right) &= \left(\frac{\partial\theta_x}{\partial x},\ \frac{\partial\theta_y}{\partial y},\ \frac{\partial\theta_x}{\partial y}+\frac{\partial\theta_y}{\partial x}\right)\\
\left(\chi_x^*,\chi_y^*,\chi_{xy}^*\right) &= \left(\frac{\partial\theta_x^*}{\partial x},\ \frac{\partial\theta_y^*}{\partial y},\ \frac{\partial\theta_x^*}{\partial y}+\frac{\partial\theta_y^*}{\partial x}\right)\\
\left(\phi_x,\phi_y,\phi_x^*,\phi_y^*\right) &= \left(\theta_x+\frac{\partial w_0}{\partial x},\ \theta_y+\frac{\partial w_0}{\partial y},\ 3\theta_x^*,\ 3\theta_y^*\right)
\end{aligned}\right\} \tag{4}$$

The flexural and transverse shear strains in the plate can be written in the concise matrix form as:

$$\begin{aligned}
\varepsilon_f &= \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = z\begin{Bmatrix}\chi_x\\ \chi_y\\ \chi_{xy}\end{Bmatrix} + z^3\begin{Bmatrix}\chi_x^*\\ \chi_y^*\\ \chi_{xy}^*\end{Bmatrix} = z\chi + z^3\chi^*\\
\varepsilon_s &= \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = \begin{Bmatrix}\phi_y\\ \phi_x\end{Bmatrix} + z^2\begin{Bmatrix}\phi_y^*\\ \phi_x^*\end{Bmatrix}
\end{aligned} \tag{5}$$

3.3 Stress Strain Relationship

The stress tensor and strain tensor of a three dimensional continuums are related by the generalised Hooke's law:

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} \tag{6}$$

Where σ_{ij} is the stress tensor, ε_{kl} is the strain tensor and C_{ijkl} is a fourth order tensor field relating stresses with strains. With the adopted nonlinear displacement model eq.(2) the stresses can be expressed as:

$$\begin{aligned}
\sigma_x &= \frac{E}{1-\nu^2}\left[\left(z\chi_x+z^3\chi_x^*\right)+\nu\left(z\chi_y+z^3\chi_y^*\right)\right]\\
\sigma_y &= \frac{E}{1-\nu^2}\left[\left(z\chi_y+z^3\chi_y^*\right)+\nu\left(z\chi_x+z^3\chi_x^*\right)\right]\\
\tau_{xy} &= G\left(z\chi_{xy}+z^3\chi_{xy}^*\right)\\
\tau_{yz} &= G\left(\phi_y+z^2\phi_y^*\right)\\
\tau_{xz} &= G\left(\phi_x+z^2\phi_x^*\right)
\end{aligned} \tag{7}$$

3.4 Yielding of Plate: Von Mises Yield Criterion

The complete elasto-plastic deformation process of any structure essentially consists of the elastic portion, a point indicating the start of plastic portion and the plastic portion of deformation. A yield criterion is a law defining the limit of elastic behaviour under any possible combination of stresses. Von Mises suggested from purely theoretical considerations that the yielding occur when the second deviatoric stress invariant 'J_2' attains a critical value i.e.

$$J_2 = \kappa^2 \tag{8}$$

Expressing J_2 in terms of generalized stresses, we have

$$\frac{1}{6}\left[\left(\sigma_x-\sigma_y\right)^2+\left(\sigma_y-\sigma_z\right)^2+\left(\sigma_z-\sigma_x\right)^2+6\left(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2\right)\right] = \kappa^2 \tag{9}$$

Based on the earlier researches, superiority of von Mises criterion over Tresca criterion has been established. Hence, von Mises yield criterion has been used in all the numerical examples.

IV. NUMERICAL EXAMPLE

The computer program based on the above formulation has been developed for the numerical experimentation. The results are in good agreement with results available in the literature [9, 10], which validate our theory and computer program developed. The elasto-plastic analysis of circular plate with central circular opening has been carried out for various parameters.

Example 1: Clamped circular plate having an opening with 10% of area subjected to uniformly distributed load for *R/h* = 2.5, 5, 10 and 100, where '*h*' is the thickness of plate, Modulus of Elasticity (*E*) = 10.92 units, Modulus of Rigidity (*G*) = 4.2, Radius of plate (*R*) =10.0 units and Radius of opening (*r*) = 3.162 units.

The non-dimensional parameters used in the formulation are:

For non-dimensional displacement : $W_c = W_0 \dfrac{Eh^3}{12\left(1-\nu^2\right)\left(R^2-r^2\right)M_p}$

For non-dimensional uniformly distributed load: $\overline{q} = \dfrac{q_0\left(R^2-r^2\right)}{M_p}$

For non-dimensional point load: $\overline{P} = \dfrac{P_0}{M_p}$

Example 2: Clamped circular plate having an opening with 10% of area subjected to point load for *R/h* = 2.5, 5, 10 and 100, where '*h*' is the thickness of the plate, Modulus of elasticity (*E*) = 10.92 units, Modulus of Rigidity (*G*) = 4.2, Radius of plate (*R*) =10.0 units and Radius of opening (*r*) = 3.162 units. The non-dimensional parameters are same as above.

V. RESULT AND DISCUSSION

Table 1. Non-dimensional displacement and non-dimensional load for clamped circular plate

Type of load	*R/h*	Actual load app.		Non-dim. Disp.		Non-dim. Load	
		Elastic	Collapse	Elastic	Collapse	Elastic	Collapse
UDL	2.5	435.0	631.5	0.23206	0.72505	9.7877	14.209
	5	104.0	162.5	0.15914	0.50104	9.3602	14.6253
	10	27.0	41.25	0.14871	0.54764	9.7202	14.8503
	100	0.25	0.35	0.13192	0.21512	9.0002	12.6002
Point Load	2.5	9600.0	19200.0	0.20897	0.47213	2.4	4.8
	5	2720.0	2880.0	0.15589	0.17684	2.72	2.88
	10	640.0	800.0	0.12866	0.16273	2.56	3.2
	100	0.64	0.96	0.01075	0.014	0.256	0.384

A clamped circular plate having central circular opening is analyzed with higher order shear deformation theory and is presented here for various thicknesses. The other geometric and material properties have been kept same. Two load cases; first, a uniformly distributed load over the plate and second, edge loading on opening periphery, have been considered separately. The load has been applied incrementally. The start and gradual spread of plasticity has been depicted in the form of tabulated results and plan diagrams of the plate. The table 1 shows maximum elastic load and collapse load with their non-dimensional values and the effect of thickness variation can also be seen from this table. Fig: 1 and 2 show non-dimensional displacement versus non-dimensional load plot for uniformly distributed load and point loaded opening cases respectively for various thicknesses for an opening having 10% area of plate.

Fig: 3 and 4 shows gradual spread of plastic zones with increasing uniformly distributed load for a plate having an opening of 10% area of plate for thick and moderately thick plate. Fig: 5 and 6 also depicted the spread of plasticity on the similar plate but with increasing point load on opening edge for thick and thin plate.

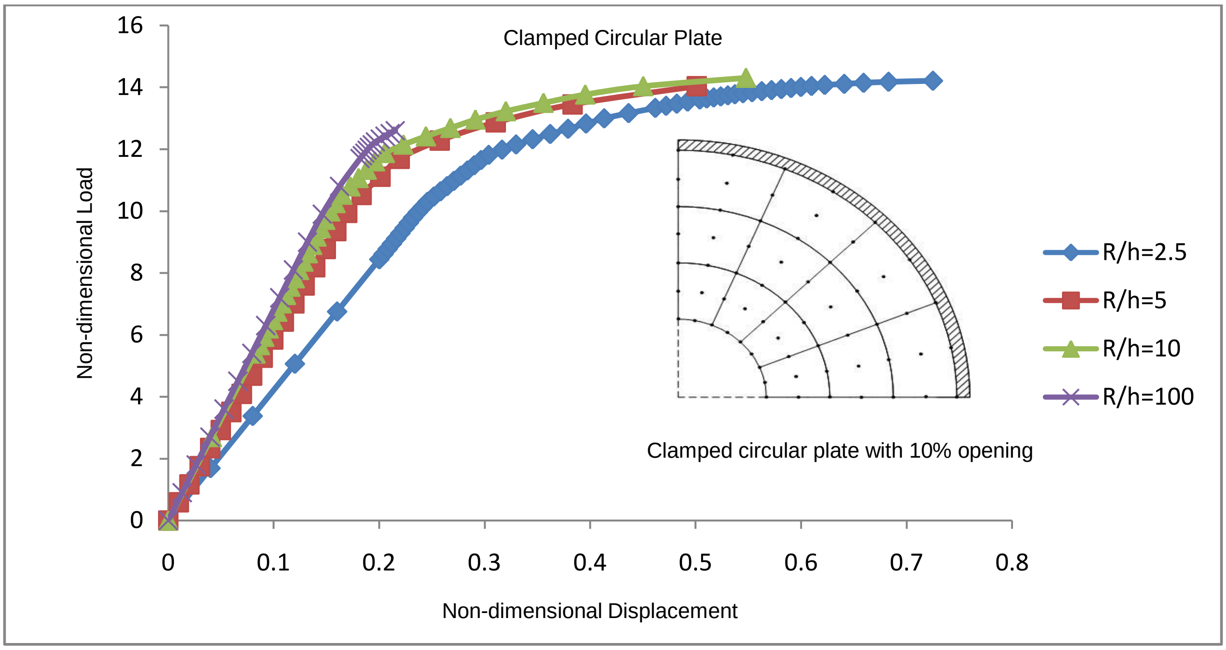

Fig: 1. Circular clamped plate having 10% opening under UDL

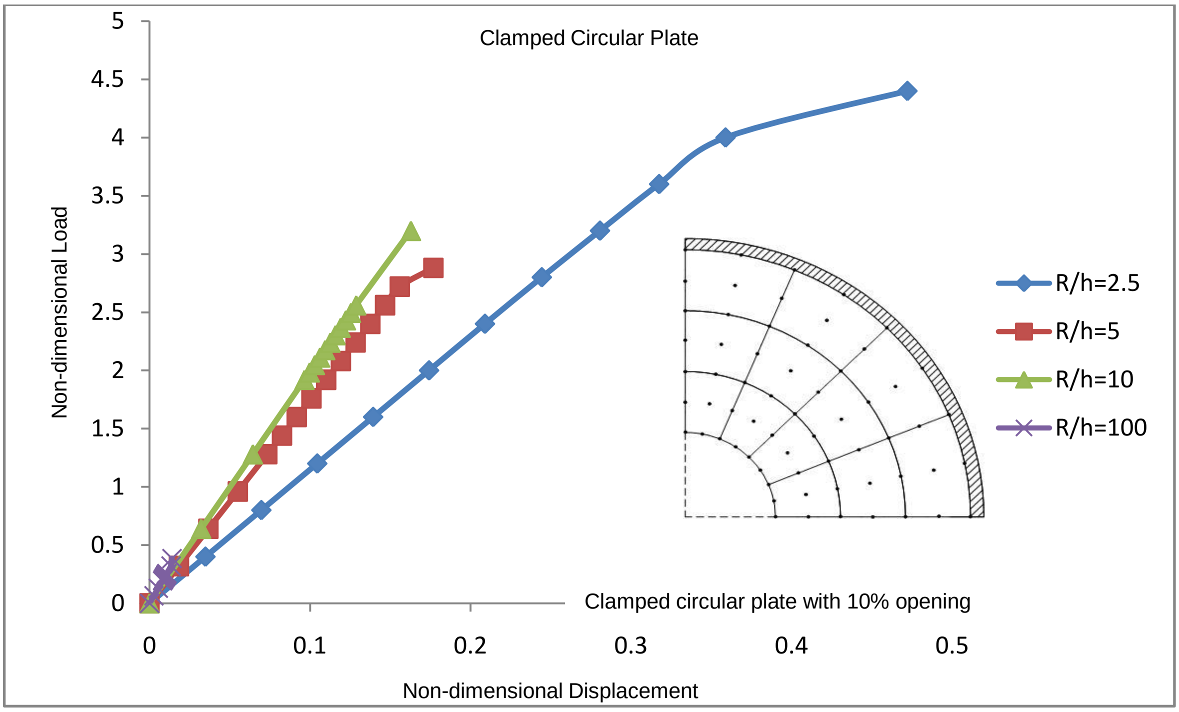

Fig: 2. Circular clamped plate having 10% opening under PL

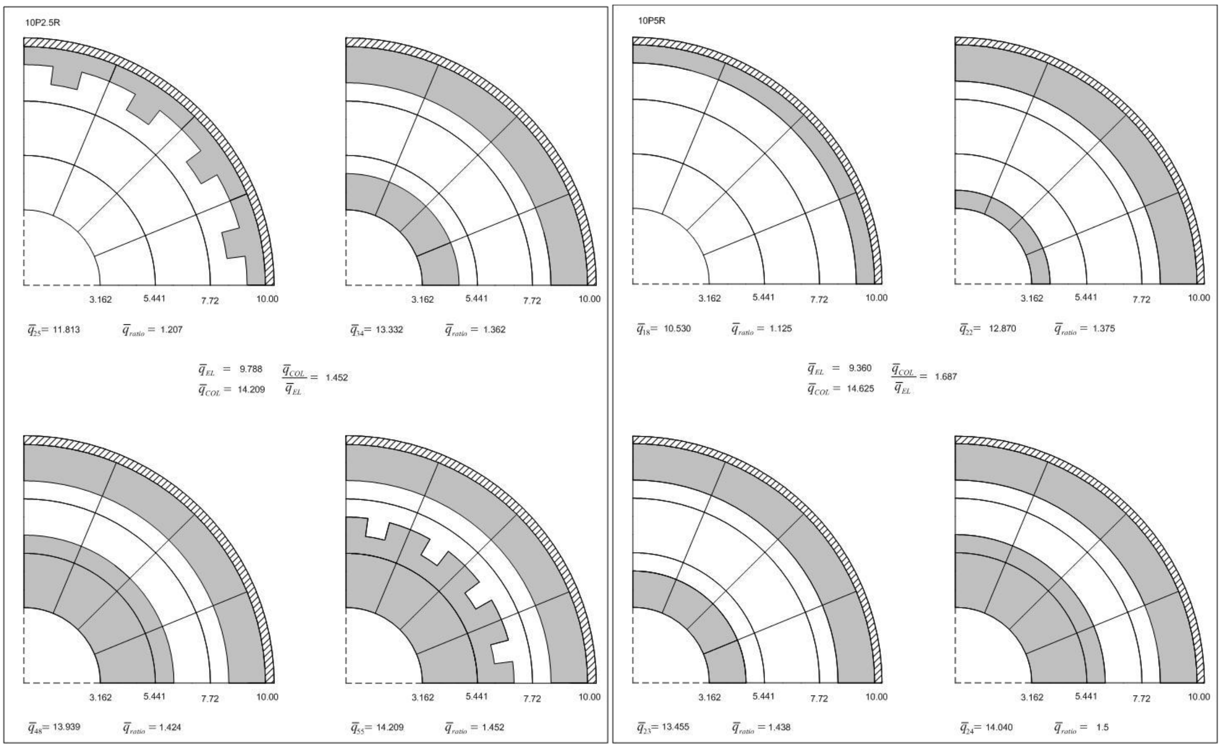

Fig: 3. Progress of plastic zone: Clamped circular plate with 10% opening for *R/h*= 2.5 and 5 under UDL

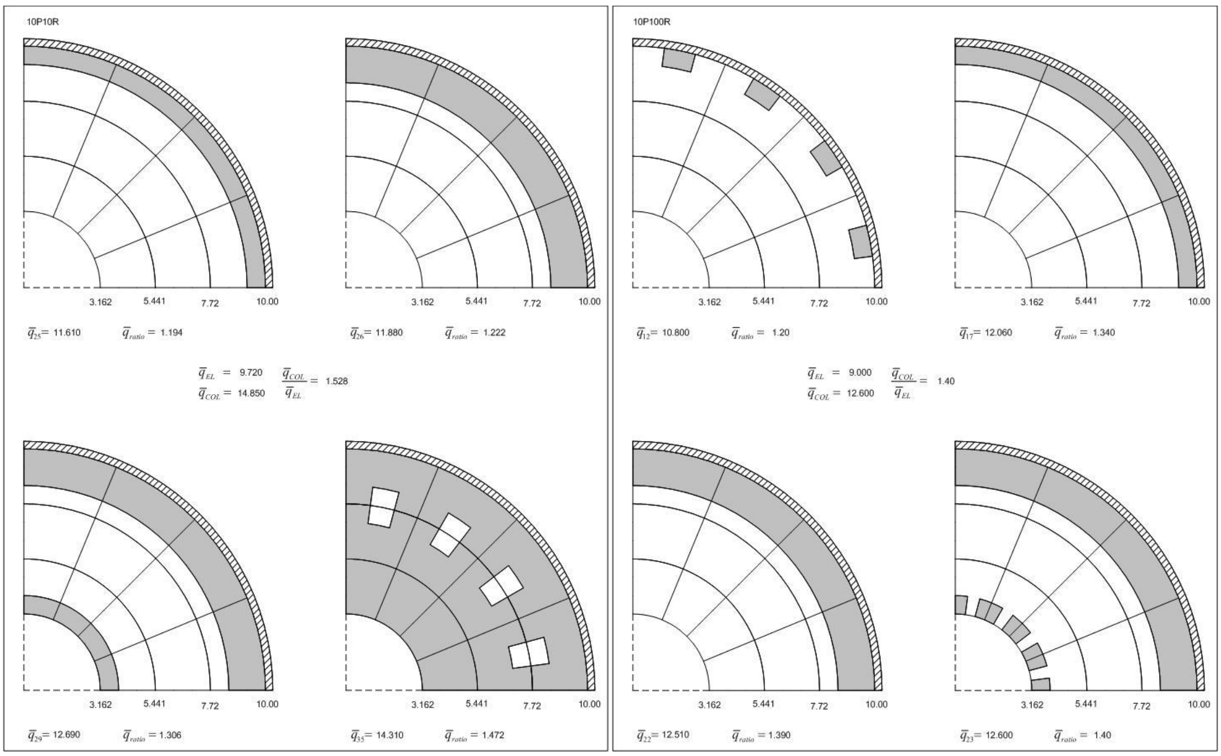

Fig: 4. Progress of plastic zone: Clamped circular plate with 10% opening for *R/h*= 10 and 100 under UDL

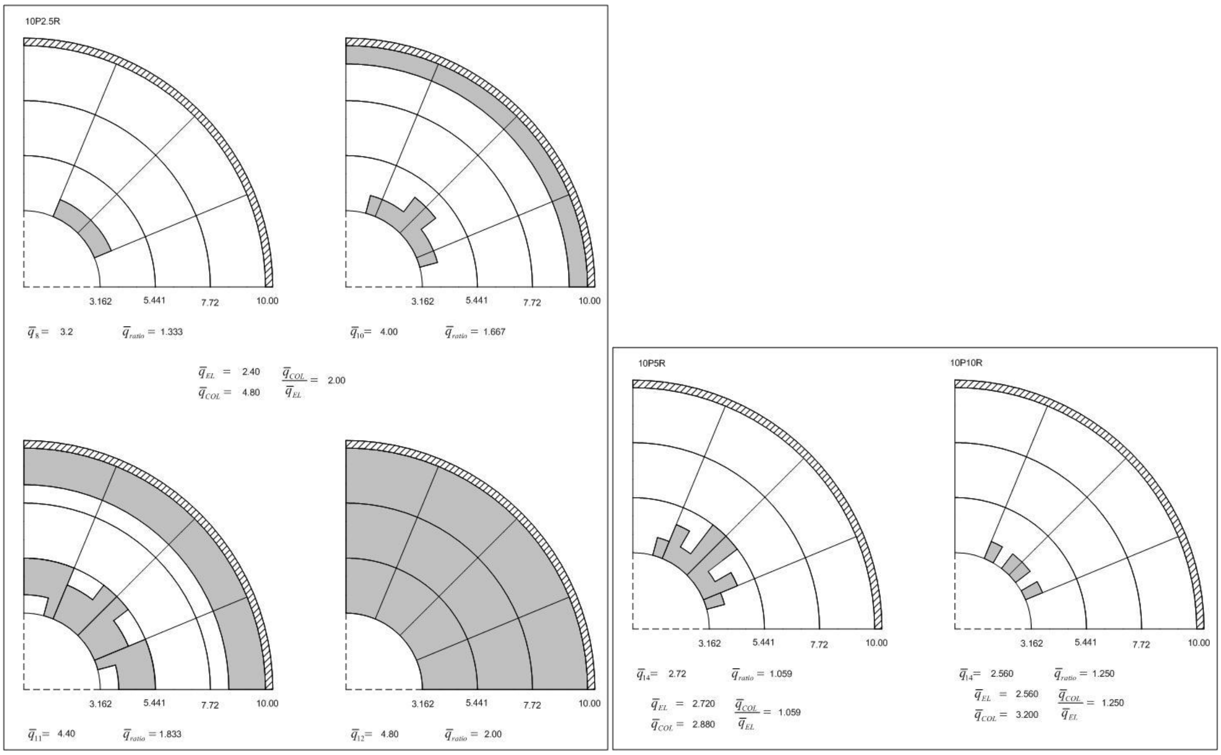

Fig: 5. Progress of plastic zone: Clamped circular plate with 10% opening for *R/h*= 2.5, 5 and 10 under PL

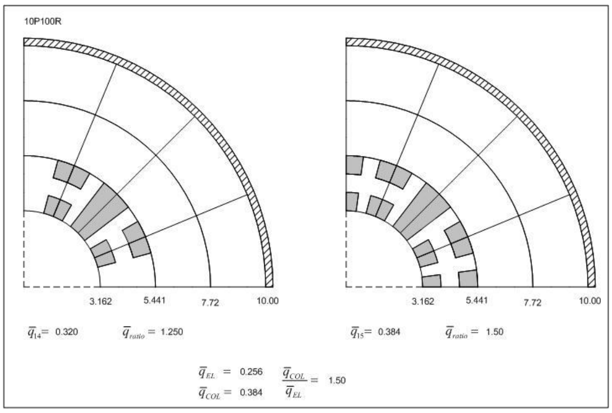

Fig: 6. Progress of plastic zone: Clamped circular plate with 10% opening for *R/h*= 100 under PL

VI. CONCLUSION

The actual collapse load in thick plate is very large than thin plate due to its larger stiffness. On non-dimensionalization the collapse loads become comparable.

In case of circular plate under uniformly distributed load, initially the yielding starts from the clamped support and then from the edge of opening and these plastic zones spread towards each other. However, in case of point load, plasticity begins near the edge of opening and then from the support. These zones meet only for very thick (*R/h*=2.5) plate, while plates with all other thicknesses fail due to excessive displacement before these zones meet.

The progress of yielded zones is gradual in case of uniformly distributed loads as opposed to the point loaded opening case. The yielding starts near the clamped edge at approximately 70% of collapse load and another zone starts near the edge of opening at approximately 85% of collapse load in the distributed load case, barring the exception of very thin (*R/h*=100) plate.

REFERENCES

[1] T. Kant, "Numerical analysis of thick plates," Computer Methods in Applied Mechanics and Engineering, 1982, **31**, pp. 1-18.

[2] T. Kant, D. R. J. Owen and O. C. Zienkiewicz, "A refined higher-order C^0 plate bending element," Computers and Structures, 1982,**15**(2), pp. 177-183.

[3] Xiang-yu L, Hao-jiang D. and C. Wei-qiu, "Pure bending of simply supported circular plate of transversely isotropic functionally graded material," Journal of Zhejiang University Science A: **7** 2006, (8): pp. 1324-1328.

[4] V. G. Ukadgaonker and D. K. N. Rao, "A general solution for stresses around holes in symmetric laminates under inplane loading," Composite Structures, 2000, **49**, pp. 339-354.

[5] M. A. Vaz, J. C. R. Cyrino and G. G. da Silva, "Three-dimensional stress concentration factor in finite width plates with a circular hole," World Journal of Mechanics, 2013, **3**: pp. 153-159.

[6] T. Kant, R. K. Tripathi and, V. Rode, "Elasto-Plastic behaviour of thick plates with a higher-order shear deformation theory," Proc. Indian Natn. Sci. Acad, **79**(4), Spl. Issue Part A, 2013, pp. 563-574.

[7] Lo K. H., Christensen, R. M. and E. M. Wu, "A high-order theory of plate deformation-part I: Homogeneous plates," ASME Journal of Applied Mechanics, 1977, **99**, pp. 663-668.

[8] A. Mendelson, "Plasticity: Theory and Applications," Macmillan Publishing Co, 1968 New York.

[9] V. R. Rode and T. Kant, "A program and user's manual for elasto-plastic analyses of plates based on a higher-order shear deformation theory," Rep. CE/SW/95/1, April 1995, Department of Civil Engineering, Indian Institute of Technology, Bombay.

[10] S. Savithri, "Linear and nonlinear analysis of thick homogeneous and laminated plates," Ph. D. Thesis, Indian Institute of Technology, 1991, Madras.